ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 4, Issue 6, December 2014

Theoretical Study on Vertical Free Vibration Analysis of Self-anchored Cable-stayed Suspension Bridge

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Abstract*—Based on Large-displacement Non-linear Elastic Generalized Variational Principle, coupling effect of axial and flexural action, shearing strain energy, torsional strain energy of stiffening girder were considered, the large-displacement incomplete generalized potential energy functional of space coupling free vibration of a three-span self-anchored cable-stayed suspension bridge was presented. By constraint variation, fundamental differential equations of vertical flexural vibration were formulated. The linear free vibration differential equation was obtained when the nonlinear items were discarded. Set a self-anchored cable-stayed suspension bridge with a main span of 100m as an example, the natural frequency of vertical vibration equation was obtained. By contrast, it finds that analytical results agree well with numerical results. This approach provides theoretical basis for analysis of natural vibration character of self-anchored cable-stayed suspension bridges.

***Index Terms*—Self-anchored cable-stayed suspension bridge, Coupling of flexural and axial deformation, Vertical free vibration, Functional, Generalized variation.**

I. INTRODUCTION

The natural vibration characteristics of bridge structures are the basis of seismic response analysis, wind-resistant and the vibration effects caused by vertical load. The study for space free vibration of bridge structures has matured, which has shown some success in the field of research on vibration of large span bridge. Siu Sai Koizumi and Baidan Cheng [1], Abdel-Ghaffar [2-6], Koishi.I and Shiraishi [7, 8], as well as some domestic scholars have published literatures of the study on space free vibration of self-anchored suspension bridge. Though the analytic theory of the self-anchored cable-stayed suspension bridge was very similar to the suspension bridge's, there were also vertical, lateral torsional and longitudinal vibration coupling, as well as horizontal and vertical vibration of the main tower. And stiffening girder existed bending coupling effect at the same time[9-12]. Zhuanghe Construction Bridge, opened to traffic in 2008, is the world's first self-anchored cable-stayed suspension bridge at present. Generalized potential energy function with large deflection was established; then differential equations of vertical vibration were derived. The linear free vibration differential equation was obtained when the nonlinear terms are discarded. Set Zhuanghe Constructing Bridge as an example to obtain the analytical results and numerical results. By contrast, it finds that analytical results agree well with numerical results, which could provide a theoretical basis for static analysis of such type bridge. Based on the Hamilton variational principle[13], the effect of shearing strain energy and axial compressive energy of stiffening girder were considered; the incomplete

II. THE STRAIN ENERGY OF STIFFENING GIRDER UNDER THE COUPLING EFFECT OF AXIAL AND FLEXURAL ACTION

The stiffening girder is a continuous beam with constant cross section which belongs to bending member, as the horizontal tension of main cables acts on the stiffening girder. Taking a stiffening girder unit as an example, as shown in Fig. 1 and Fig. 2.

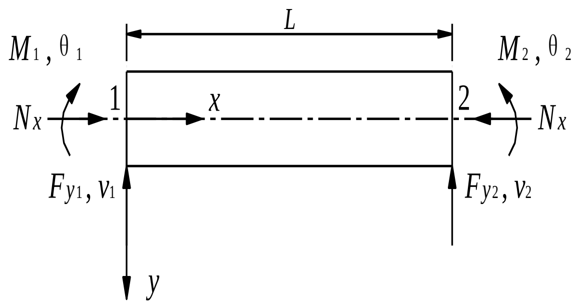

Fig. 1 Load distribution of stiffening girder element

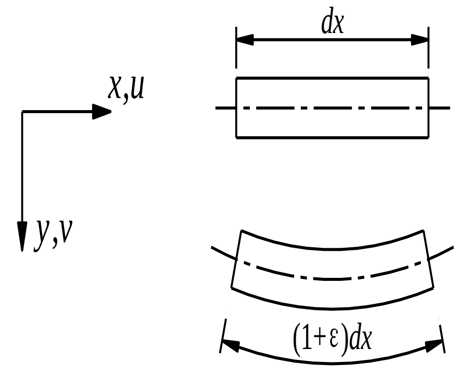

Fig. 2 Finite element deformation of stiffening girder

If only compression and bending deformation were considered, axial strain can be expressed as (1).

$$\varepsilon_x = \frac{\partial u_b}{\partial x} - y\frac{\partial^2 v_b}{\partial x^2} + \frac{1}{2}\left(\frac{\partial v_b}{\partial x}\right)^2 \quad (1)$$

The axial strain energy of the stiffening girder can be expressed as (2).

ISSN: 2277-3754
ISO 9001:2008 Certified
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$$U_b = \frac{1}{2}\iiint_{vom} E\varepsilon_x^2 d(vom) \quad (2)$$

Substituted equation (1) into equation (2):

$$U_b = \frac{1}{2}\int_{x_1}^{x_4}\int_{A_b}\left[\left(\frac{\partial u_b}{\partial x}\right)^2 + y^2\left(\frac{\partial^2 v_b}{\partial x^2}\right)^2 + \frac{1}{4}\left(\frac{\partial v_b}{\partial x}\right)^4 - 2y\left(\frac{\partial u_b}{\partial x}\right)\left(\frac{\partial^2 v_b}{\partial x^2}\right) - y\left(\frac{\partial^2 v_b}{\partial x^2}\right)\left(\frac{\partial v_b}{\partial x}\right)^2 + \left(\frac{\partial u_b}{\partial x}\right)\left(\frac{\partial v_b}{\partial x}\right)^2\right]EdAdx$$

Noting that $\int_{A_b} dA = A_b$, $\int_{A_b} ydA = 0$, $\int_{A_b} y^2 dA = I_z$, equation (3) can be expressed into equation (4).

$$U_b = \frac{1}{2}\int_{x_1}^{x_4}\left[A_b\left(\frac{\partial u_b}{\partial x}\right) + I_{bz}\left(\frac{\partial^2 v_b}{\partial x^2}\right)^2 + \frac{A_b}{4}\left(\frac{\partial v_b}{\partial x}\right)^4 + A_b\left(\frac{\partial u_b}{\partial x}\right)\left(\frac{\partial v_b}{\partial x}\right)^2\right]Edx$$

As $N_{bx} = EA_b\frac{\partial u_b}{\partial x}$, ignoring the infinitely small quantity of higher order, equation (4) can be expressed into equation (5).

$$U_b = \frac{1}{2}\int_{x_1}^{x_4}\left[EA_{bx}\left(\frac{\partial u_b}{\partial x}\right)^2 + EI_{bz}\left(\frac{\partial^2 v_b}{\partial x^2}\right)^2 + N_{bx}\left(\frac{\partial v_b}{\partial x}\right)^2\right]dx$$

In equation (5), the first term is strain energy generated by axial force; the second term is stain energy generated by moment; the third term is strain energy generated by coupling effect of axial force and moment, which can be ignored in linear analysis of the structure.

III. THE INCOMPLETE GENERALIZED POTENTIAL ENERGY FUNCTION WITH LARGE DEFLECTION

Set Dalian Bay bridge as an example (in Fig. 3), based on the structural characteristics, the following were assumed [10,14-15]:

(1) All materials are in accordance with Hooke's law.

(2) In finished bridge state, dead load was distributed uniformly along the span, and main cables were of parabolic shape; stay cables were of linear shape.

(3) The stiffening girder has vertical supports. It was a continuous beam with constant cross section, located in the tower.

(4) Torsional deformation of tower was unconsidered, only longitudinal and axial compressive deformation was considered.

(5) The stay cables and hangers were dense arranged. Hangers can be viewed as a uniform film with vertical resistance, without axial expansion. Stayed cables can be viewed as a uniform film with axial resistance, without axial expansion.

(6) The stiffening girder has longitudinal, vertical, lateral and torsional vibration directions. The main tower has longitudinal, horizontal and vertical vibration directions.

(7) Considered torsion warping deformation of stiffening girder, and ignored the distortion of the section. It was Units assumed that diaphragms were dense and with infinity shear stiffness. While the dead load on beam, main cable and tower is q_b, q_c, q_t respectively, defined displacement of various parts as following:

$u_{lc}, w_{lc}, v_{lc}, u_{rc}, w_{rc}, v_{rc}$ are the longitudinal, transverse and vertical of left and right main cable respectively;

u_t, w_t, v_t are the longitudinal, lateral and vertical displacement of main tower;

u_b, w_b, v_b, θ are the longitudinal, transverse, vertical displacement and torsion angle of beam.

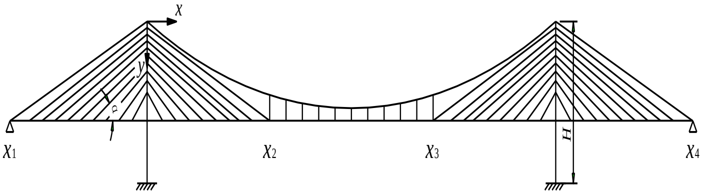

Fig.3 Illustration of self-anchored cable-stayed suspension bridge

U_{be}, U_{te}, U_{ce} are the stiffening girder and tower, the cable strain energy respectively, U_{bg}, U_{cg} are gravitational potential energy, T_b, T_t, T_c are Kinetic energy, λ_j is the Lagrange multiplier, f_j is the constraint condition of boom and cable-stayed.

$$\Pi^* = \int_{x_1}^{x_4}(T_b + T_t + T_c - U_{be} - U_{te} - U_{ce} - U_{bg} - U_{cg} - \sum_{i=1}^{5}\sum_{j=1}^{4}\lambda_j f_j dx_i)dt$$

Then the large-displacement partition incomplete generalized potential energy functional $\prod^*$ [16][17] is:

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
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$$\prod^{*}=\int_{t_1}^{t_2}\{\frac{1}{2}\int_{x_1}^{x_4}\frac{q_b}{gA_b}[I_{by}\left(\frac{\partial^2 w_b}{\partial x\partial t}\right)^2+I_{bz}\left(\frac{\partial^2 v_b}{\partial x\partial t}\right)^2+\left(I_{by}+I_{bz}\right)\left(\frac{\partial\theta}{\partial t}\right)^2+A_b\left(\frac{\partial u_b}{\partial t}\right)^2+A_b\left(\frac{\partial v_b}{\partial t}\right)^2$$

$$+A_b\left(\frac{\partial w_b}{\partial t}\right)^2]dx_i+\frac{1}{2}\int_0^H\frac{q_t}{gA_t}[I_{ty}\left(\frac{\partial^2 u_t}{\partial y\partial t}\right)^2+I_{tz}\left(\frac{\partial^2 w_t}{\partial y\partial t}\right)^2+A_t\left(\frac{\partial u_t}{\partial t}\right)^2+A_t\left(\frac{\partial w_t}{\partial t}\right)^2+$$

$$A_t\left(\frac{\partial v_t}{\partial t}\right)^2]dy_i+\frac{1}{2}\int_{x_1}^{x_4}\frac{q_c}{2g}[\left(\frac{\partial u_{lc}}{\partial t}\right)^2+\left(\frac{\partial v_{lc}}{\partial t}\right)^2+\left(\frac{\partial w_{lc}}{\partial t}\right)^2+\left(\frac{\partial u_{rc}}{\partial t}\right)^2+\left(\frac{\partial v_{rc}}{\partial t}\right)^2+\left(\frac{\partial w_{rc}}{\partial t}\right)^2]dx_i$$

$$-\frac{L_c}{E_cA_c}[\frac{1}{2}H_q(H_{lc}+H_{rc})+\frac{1}{2}(H_{lc}^2+H_{rc}^2)]-\frac{1}{2}[\int_{x_1}^{x_4}\beta E_1 J_w\left(\frac{\partial^2\theta}{\partial x^2}\right)^2dx_i+\int_{x_1}^{x_4}GJ_t\left(\frac{\partial\theta}{\partial x}\right)^2dx_i$$

$$+\int_{x_1}^{x_4}EI_{by}\left(\frac{\partial^2 w_b}{\partial x^2}\right)^2dx_i+\int_{x_1}^{x_4}EI_{bz}\left(\frac{\partial^2 v_b}{\partial x^2}\right)^2dx_i+\int_{x_1}^{x_4}EA_{bx}\left(\frac{\partial u_b}{\partial x}\right)^2dx_i+\int_{x_1}^{x_4}N_{bx}\left(\frac{\partial v_b}{\partial x}\right)^2dx_i$$

$$+\int_{x_1}^{x_4}\mu GA_b\left(\frac{\partial u_b}{\partial y}+\frac{\partial v_b}{\partial x}\right)^2dx_i+\int_{x_1}^{x_4}\mu GA_b\left(\frac{\partial u_b}{\partial z}+\frac{\partial w_b}{\partial x}\right)^2dx_i]-\frac{1}{2}[\int_0^H EI_{tx}\left(\frac{\partial^2 w_t}{\partial y^2}\right)^2dy_i$$

$$+\int_0^H EI_{tz}\left(\frac{\partial^2 u_t}{\partial y^2}\right)^2dy_i+\int_0^H EA_{ty}\left(\frac{\partial v_t}{\partial y}\right)^2dy_i+\int_0^H N_{ty}\left(\frac{\partial u_t}{\partial y}\right)^2dy_i+\int_0^H\mu GA_t\left(\frac{\partial u_t}{\partial y}\right)^2dy_i$$

$$+\int_0^H\mu GA_t\left(\frac{\partial w_t}{\partial y}\right)^2dy_i]-[\int_{x_1}^{x_4}q_b v_b dx_i+\int_{x_1}^{x_4}\frac{1}{2}q_c\left(v_{lc}+v_{rc}\right)dx_i]-\sum_{i=1}^{5}\sum_{j=1}^{4}\int_{x_1}^{x_4}\lambda_j f_j dx_i\}dt$$

IV. THE SELF-ANCHORED CABLE-STAYED SUSPENSION BRIDGE'S BASIC DIFFERENTIAL EQUATION OF VERTICAL FREE VIBRATION AND SIMPLIFIED SOLUTION

A. *Basic differential equation of vertical free vibration*

Performing variation of the generalized function $\prod^*$ on u_b, w_b, v_b, θ, u_{lc}, w_{lc}, v_{lc}, u_{rc}, w_{rc}, v_{rc}, u_t, w_t, v_t, λ_j respectively and correspondent Euler equations can be obtained, i.e. [18-21]

$$\delta\prod^{*}=\frac{\partial\prod^{*}}{\partial\phi_i}\delta\phi_i=0,\ \phi_i=u_b、w_b、v_b、\theta、u_{lc}、w_{lc}、v_{lc}、u_{rc}、w_{rc}、v_{rc}、u_t、w_t、v_t、\lambda_j$$

.

Set the stiffening girder vertical vibration as an example, the vertical vibration equation was listed.

Vertical vibration equation of stiffening girder:

$$\frac{q_b}{g}\ddot{v}_b-\frac{q_b}{g}I_{bz}\frac{\partial^4 v_b}{\partial x^2\partial t^2}+EI_{bz}\frac{\partial^4 v_b}{\partial x^4}+N_{bx}\frac{\partial^2 v_b}{\partial x^2}-\mu GA_b\left(\frac{\partial^2 v_b}{\partial x^2}+\frac{\partial u_b}{\partial y}\frac{\partial^2 v_b}{\partial x^2}\right)+\frac{q_c}{2g}\left(\ddot{v}_{lc}+\ddot{v}_{rc}\right)$$

$$+\frac{q_c}{H_q}\left(H_{lc}+H_{rc}\right)-\frac{1}{2}H_q\left(\frac{\partial^2 v_{lc}}{\partial x^2}+\frac{\partial^2 v_{rc}}{\partial x^2}\right)-H_{lc}\frac{\partial^2 v_{lc}}{\partial x^2}-H_{rc}\frac{\partial^2 v_{rc}}{\partial x^2}=0$$

B. *Simplified solution of the vertical free vibration equation*

The stiffening girder under vertical vibration purely, applicated the thought of Xiaoxi, Baishi, etc. in this paper, $v_{lc}=v_{rc}=v_b, H_{lc}=H_{rc}, H_v=H_{lc}+H_{rc}$. Ignoring the nonlinear terms and coupling terms, equation (8) can be expressed into equation (9).

$$\frac{q_b}{g}\ddot{v}_b+EI_{bz}\frac{\partial^4 v_b}{\partial x^4}+N_{bx}\frac{\partial^2 v_b}{\partial x^2}+\frac{q_c}{H_q}H_v-H_q\frac{\partial^2 v_b}{\partial x^2}=0$$

As $A=EI_{bz}$, $B=\left(N_{bx}-H_q\right)$, $C=\frac{q_c}{H_q}$, $D=\frac{q_b}{g}$

The equation (9) can be expressed into equation (10).

$$A\frac{\partial^4 v_b}{\partial x^4}-B\frac{\partial^2 v_b}{\partial x^2}+CH_v+D\ddot{v}_b=0$$

As $N_{bx}=H_q$, $v(x,t)=\bar{v}(x)e^{i\omega t}, H_v(t)=\bar{H}_v(t)e^{i\omega t}$. Where ω is self vibration frequency, $\bar{v}(x)$ is vibration mode. Separated (10) for time variable, which simplified into a 4 order constant coefficient nonhomogeneour differential equation.[22-24]

$$A\frac{\partial^4\bar{v}_b}{\partial x^4}+C\bar{H}_v-D\omega^2\bar{v}_b=0 \qquad (11)$$

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
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Let $\bar{v}(x)=\exp(\lambda x)$, the corresponding homogeneous equation of (11) can be simplified into (12).

$$A\lambda^4 - D\omega^2 = 0 \quad (12)$$

k_1, k_2, k_3, k_4 can be obtained by solving the characteristic equation of (12)

$$k_1、k_2 = \pm\sqrt[4]{\frac{D\omega^2}{A}}, \; k_3、k_4 = \pm i\sqrt[4]{\frac{D\omega^2}{A}} = \pm i\mu$$

(13)

A general solution of (11) equation's corresponding homogeneous equation can be expressed as (14).

$$\bar{v}(x) = C_1 ch\frac{\mu x}{l} + C_2 sh\frac{\mu x}{l} + C_3\cos\frac{\mu x}{l} + C_4\sin\frac{\mu x}{l} + \frac{C\bar{H}_v}{D\omega^2}$$

For three-span self-anchored cable-stayed suspension bridge, under dead loads, the moment of the stiffening girder became smaller, by changing the tension of suspender and stayed cable. Taking the fulcrum moment = 0 approximately. $\bar{v}(x_1=0)=\bar{v}(x_4=L)=0$, $\bar{v}''(x_1=0)=\bar{v}''(x_4=L)=0$. Integration constant and H_v can be calculated by boundary conditions and the equations of main cable. At last, vibration frequencies and vibration modes can be obtained.

Symmetric vibration mode of main span:

$$\bar{v}(x) = \frac{C}{\Phi^2(Z^2-1)}\left[\frac{Z+1}{2Z}\left(1-\cos\frac{\mu x}{L}-\tan\frac{\mu}{2}\sin\frac{\mu x}{L}\right)+\frac{Z-1}{2Z}\left(1-ch\frac{\mu x}{L}-th\frac{\mu}{2}sh\frac{\mu x}{L}\right)\right]$$

Symmetric vibration mode of side span:

$$\bar{v}(x) = \frac{C}{\Phi_1^2(Z_1^2-1)}\left[\frac{Z_1+1}{2Z_1}\left(1-\cos\frac{\mu x}{L_1}-\tan\frac{\mu}{2}\sin\frac{\mu x}{L_1}\right)+\frac{Z_1-1}{2Z_1}\left(1-ch\frac{\mu x}{L_1}-th\frac{\mu}{2}sh\frac{\mu x}{L_1}\right)\right]$$

(16)

Antisymmetric vibration mode and frequency of main span:

$$\bar{v}(x) = C\sin\frac{n\pi x}{L}, \; \omega = \frac{n\pi^2}{L^2}\sqrt{\frac{gEI_{bz}}{q}}, \; n=1,2,3,\cdots$$

(17)

Antisymmetric vibration mode and frequency of side span:

$$\bar{v}(x) = C\sin\frac{n\pi x}{L_1}, \; \omega = \frac{n\pi^2}{L_1^2}\sqrt{\frac{gEI_{bz1}}{q}}, \; n=1,2,3,\cdots$$

(18)

the following symbols were used above:

$$\Phi = \frac{H_q L}{\sqrt[4]{EI_{bz}}}, \; \Phi_1 = \frac{H_q L_1}{\sqrt[4]{EI_{bz1}}}, \; Z = Z_1 = \frac{2\omega}{H_q^2}\sqrt{\frac{g}{q}}+1, \; \mu = \frac{\Phi}{\sqrt{2}}\sqrt{Z-1}, \; \mu_1 = \frac{\Phi_1}{\sqrt{2}}\sqrt{Z_1-1}$$

, g = acceleration of gravity. For main span, q = dead load intensity, L = span, I_{bz} =vertical moment of inertia; for side span, q_1 = dead load intensity, L_1 = span, I_{bz1} =vertical moment of inertia; C = integral constant.

Dalian zhuanghe Construction Bridge was a self-anchored cable-stayed suspension bridge, which have been built with a full-length of 260m. The main bridge was designed as a self-anchored cable-stayed suspension bridge with span arrangement (41.6+100+41.6)m. The main beam was solid concrete edge girder with a width of 28.6m, a height of 2.17m and 1.5% bidirectional cross slope. Main cables were made by 409× φ 7.1mm galvanized high-strength parallel wires, with the span ratio of 1/5.2. Suspenders were made by 109× φ 7.1mm galvanized high-strength parallel wires, while stay cables were made by 163× φ 7.1mm galvanized high-strength parallel wires, with the strength of 1670MPa. Structural arrangement was shown in Fig. 4. Dead load intensity = 20kN/m^2, Material and section properties of structure components were shown in Tab. 1, vibration modes were shown in Tab. 2.

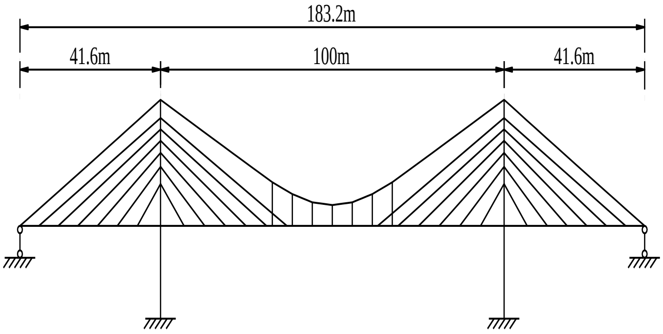

Fig.4 Dalian Zhuanghe Construction Bridge

Table.1 Material and section properties of structure components

Components

Member	E / (kN / m^2)	A / m^2	I_z / m^4	Q / (kN / m)
main cable	1.95×10^8	1.669×10^{-2}	0	1.40
stay cable	1.95×10^8	3.502×10^{-3}	0	0.29
suspender	1.95×10^8	2.117×10^{-3}	0	0.28
main beam	3.5×10^7	9.02	3.06	234.52
single column of main tower	3.5×10^7	6.85	7.08	178.10

Table.2 Analytic solution and numerical solution

Vibration mode	Natural frequency （Hz）		
	analytical result	numerical result	tolerance (%)
first order (symmetric mode)	1.0270	0.8935	14.3%
first order (antisymmetric mode)	0.6451	0.5755	12.1%

It can be seen in Tab. 2 that there are some tolerances between analytical results and numerical results. It's because that nonlinear terms and coupling terms were discarded in the process of analysis. Analytical results can be reference values in preliminary design. While, finite element numerical solution should be used in the specific design process, as it's easier to get the natural frequency which adapts to actual situation.

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
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V. CONCLUSIONS

1) It can be seen from the basis of differential equations: vibration differential equations are nonlinear. When the impact of the nonlinear terms is ignored, the vertical vibration will not couple with the horizontal and torsional vibration. When the shear deformation of the stiffening girder is ignored, the vertical vibration will not couple with the longitudinal vibration.

2) In the vertical vibration equation, due to the terms of couping effect of axial and flexural, it offsets the terms of horizontal force of main cable under dead load and get the equation further simplified.

3) Analytical results agree well with numerical results. Because of relaxing the constraint near supports, which reduces the stiffness of stiffening girder, further leading the analytical results less than numerical results

ACKNOWLEDGMENT

The work is supported by the Science Research Foundation of the Education Department of Liaoning Province, China (Grant No. L2012442) and the Young Scientists Fund of the National Natural Science Foundation of China (Grant No. 51009015 and 51008047)

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