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On Statistical Adapting the Order Filters for Periodic Signals Processing under Condition of Preserving their Waveform

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Abstract*— In this paper, restoration of the quality of noise periodic and frequency-modulated signals under condition of preservation of the waveform of the original signal is considered. For this purpose, it is offered to use the weighted order statistics (WOS) filters. However, the analytical estimation of the behavior of the WOS filters is complicated enough because of their nonlinearity, what allows us to consider the WOS filter response as a casual event. For this reason, the statistical trials method seems to be promising in selecting more qualitative WOS filters projects. For solving the corresponding task, a set of the standard WOS and a special class of the CoPh WOS filters (filters bank) is attracted under condition of a variation of work frequencies of the CoPh WOS filters. The facility used is a specialized interactive graphical computer system interface. We intend to propose some experience gained in this field of research.

***Index Terms*—Order statistics, restoring noise periodic signals, statistical trials.**

I. INTRODUCTION

We will consider some issues of adaptation of the weighted order statistics (WOS) filters to processing the periodic signals. The periodic and the frequency-modulated (FM) signals are widely used. Let a periodic signal be a one-dimensional time series $Y=\{y_1,\ldots, y_N\}$ recorded at discrete instants of time $t_1,\ldots, t_N$, $(t_{i-1}-t_i=\Delta t=const, i=2,\ldots, N)$. Specific features of the order filters were considered in [1]. They include the following aspects: 1) a remarkable ability of the impulse noise removal; 2) noise robustness; 3) preservation of steps for a signal in the form of a telegraphic sequence; 4) the response of the filter tends to zero while the filter length (n) approaches an integer number of signal periods; 5) nonlinearity. Taking into account the restriction on the length of the WOS filters in the processing of periodic signals, a special attention was given in [2] to the development of the co-phased WOS (CoPh WOS) filters**.** An informal definition of such filters is illustrated in Fig. 1. It is the following: the weights of all the terms on the filter input except the weight of the central element (CE) and the weights of such terms, which are apart from one another by the length of the period harmonic function (co-phased), are equalized to zero provided the filter length is equal to m periods (where m is even (m=2,4, etc.).

Owing to nonlinearity, the analytical estimation of the WOS filers behavior is a very complicated process. It appreciably depends, on the one hand, on the filter project (values of scales, the size of the aperture or a window of the analysis, a sequence of operations in the multistage process of filtration) and, on the other hand, it depends on the form of a signal and specificity of noise. So, the results of the research in [3] show the importance of the filter design for attaining high attenuation levels of noise without causing a considerable signal distortion. Because of the complication of the analytical estimation of the behavior of the WOS filters, we can suppose that their response in the general case is a casual event. Thus, the task of interest is the processing of periodic and signals of other forms and monitoring the quality of the restored signals depending on using different projects of filters and their parameters. The basic points of this approach were discussed in [4]. The numerical method of solving mathematical tasks with attracting the modeling of casual events, or statistical trials, for selecting the most effective project of the WOS filter is of interest. For this purpose, we propose to use a specialized computer system [5] and to attract some set of the WOS filters pooled in the corresponding filters bank.

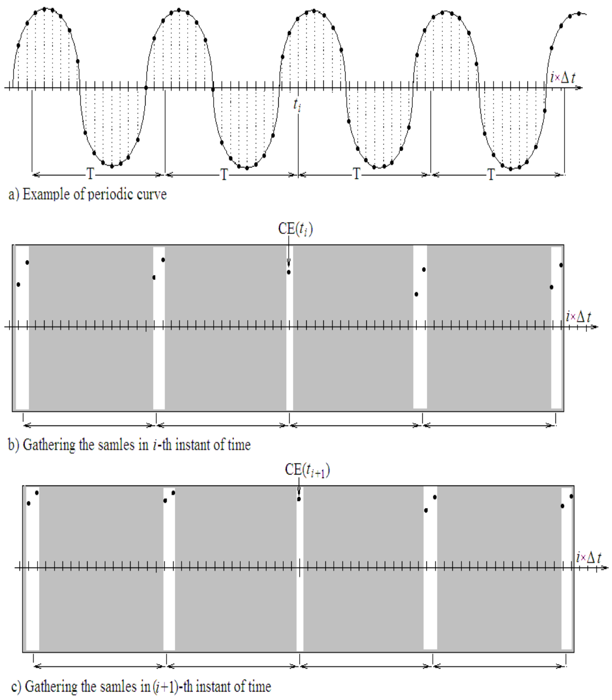

Fig. 1. Sampling of periodic signal values closest to the CE phase at different instants of time.

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II. THE BASIC DEFINITIONS OF THE WOS FILTERS

Before considering the research into projects of the WOS filters, it is worthwhile to give the basic definitions of such objects.

Let a sequence

$Y = \{y_j: j=1, \ldots, n\}$ (1)

Presents a signal that includes n quantities of numerical data or samples. Here, the r^{th}–order statistics $y_{(r,n)}$ is defined as the r^{th} quantity in size. Let a ratio $\alpha=(r–1)/(n–1)$ be a measure of the size of a corresponding sample on the sequence Y. Let this ratio be called the percentile of the WOS filter. Then the idea of the order statistics is transformed to the percentile form. A sample of the sequence Y is called α-order statistics, denoted as $y_{(\alpha,n)}$ if there exists a number $n_\alpha=(n-1)\times\alpha$ of $y_{i(\alpha)}$ values, and a number $n_\beta=(n-1)\times(1-\alpha)= (n-1)\times\beta$ of $y_{i(\beta)}$values, provided:

$$\begin{cases} y_{i(\alpha)} \le y_{(\alpha,n)}, \\ y_{i(\beta)} \ge y_{(\alpha,n)}, \\ y_{(\alpha,n)} \bigcup_{1}^{n_\alpha} y_{i(\alpha)} \bigcup_{1}^{n_\beta} y_{i(\beta)} = Y, \end{cases} \quad (2)$$

and $\alpha+\beta=1$.

Now, a formal definition of the order filtration procedure can be presented as a sequence of the following operations:

1. $Y=\{y_i, i=0, \pm1, \ldots, \pm v,\}$
   is the sampling of $n=2v+1$ signal values, where $Y \subset X$ (n is odd),
2. $\tilde{Y} = \tilde{y}_{c-v} \le \ldots \le \tilde{y}_c \le \ldots \le \tilde{y}_{c+v}$,
   is the construction of a variational row, where the term $\tilde{y}_i$ is the statistics of a corresponding order, $i = \overleftarrow{1, n}$.
3. RANK$(y_1,\ldots, y_{(n-1)/2},\ldots, y_n)= \tilde{y}_r$
   is the operation of replacing the central term (CE) $y_v \in Y$ by the statistics $\tilde{y}_r \in \tilde{Y}$ ($v, c, i\in Z$).

In a special case if $r=(n1)/2$ (i.e., $\alpha=0.5$), a filter is a median one $MED_n(y_1,\ldots, y_{(n-1)/2},\ldots, y_n)= \tilde{y}_c$.

Let us have a set W of the quantities w_i ($i=1,\ldots, n$), each quantity w_i being associated with the sample $y_i\in Y$. This $w_i \in W$ is called a weight and can be treated as a number of copies of the corresponding sample $y_i \in Y$. For example, the weighted median value of a sequence of numbers is defined in [6] as a simple median of the extended sequence formed by repeating each term w_i times. At the same time, weights are usually set symmetric with respect to the central element (CE) of sequence (1). Weights are introduced for emphasizing some elements of a sequence [1], [7]. The extended sequence Y thereby gains a new quality as a set with the number of elements $N = \sum_{i=1}^{n} w_i$. At the same time N is also odd. Then, the form

$$\tilde{y}^{W}_{\alpha,N(w_0,w_1,\ldots,w_R)} = \text{RANK}_{N(W)}(w_1 \times y_1,\ldots,w_{(n-1)/2} \times y_{_{(n-1)/2}},\ldots,w_n \times y_n) \quad (3)$$

in the general case is the definition of the WOS filter. This generalization allows a filter to keep properties of the median one [8], but changes the result of the filtering, i.e., $u_{0.5} \ne u_{0.5}^{(w)}$.

In the general case, a filter as a whole can include some ordered set (a sequence) of separate filters of signal processing, i.e., be the multistage process of filtration. Let such a single filter be called a filter node. At the same time, such a node includes some "filter terms" sequence of the length n_i, where i is the order number of the filter node in the corresponding structure.

As for the processing the periodic signals by the CoPh WOS filters, we should note that the corresponding set W of the CoPh WOS filter includes such a subset $W^*\subset W$ of the weights w^*, where $\forall w^*\in W^*$: $w^*= 0$.

According to [2], the algorithm for calculating the weights of the CoPh WOS filter is as follows:

1. Computation of the length of the filter as $\widehat{n}$, where $n=2RT/\Delta t+1$;
2. Realization of assignments: $w_0=1$ for CE and $w_{\pm i}=0$ for other components of the sequence X: $i=1,\ldots,(n-1)/2$.
3. Computation in the loop $j=1,\ldots,R$ of the indices of components with nonzero weights, and computation of the corresponding weights according to the rules:
   i) $V_j = jT / \Delta t$;
   ii) if $V_j = \widehat{V}_j = \breve{V}_j$, then $w_{\pm V_j} = 1$; else:
   iii) $w_{\pm\widehat{V}_j} = V_j - \breve{V}_j$;
   iv) $w_{\pm\breve{V}_j} = \widehat{V}_j - V_j$.

Here, T is the period of a signal, the symbol $\widehat{x}$ denotes the smallest integer (that is greater or equal to x or the nearest from above) and $\breve{x}$ is the largest integer, which is less or equal to x (the nearest from below). The corresponding frequency, which defines values of the filter weights, will be called the work frequency.

III. THE BASIC PROCEDURES OF THE STATISTICAL TRIALS METHOD OF THE WOS FILTERS PROJECTS

It is reasonable to consider the following parameters of the WOS and the CoPh WOS filters that influence the quality of signal processing: values of the weights $W=\{w_1, w_2, \ldots\}$ of the WOS filters as well as the distribution of zero weights which are the functions of frequencies in the case of the CoPh WOS filters; the size of the aperture or the window analysis; a sequence of operations in the multistage process of filtration; the type of operations; percentiles of the WOS and the CoPh WOS filters; swinging the work frequency; the mode of counting the effects of signal digitization. Some explanations will be given for the last two parameters which influence the quality of processing the periodic signals. It is possible to treat a signal using two or three close enough work frequencies and subsequent application of averaging or the median filter

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diagonally (as, for example, in [3]) across the results of the processing.

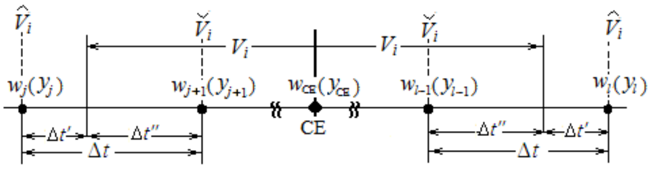

Fig. 2. An example of digitization of a periodic signal and weights distribution of the CoPh WOS filter.

As for the effects of signal digitization, they can be explained by using Fig. 2. Here, $V_i = i\mathrm{T} / \Delta t = i f^{w} \Delta t$ and $\hat{V}_i$ is the nearest from above, $\check{V}_i$ is the nearest from below. The signal data and corresponding weights (filter units) can be processed as they arrive at a filter input, or two neighboring units can be pooled in a certain unit. We propose to attract the pooling in the form:

$$y_j+(y_{j+1} - y_j)\times\Delta t_j', \; w_j= w_j + w_{j+1}. \quad (4)$$

As was established in [2], the CoPh WOS filter of the two-period length of an FM signal will save in its band the frequency zone with a width $\Delta f(\mathrm{CoPh\ WOS})\in 1.0\div 1.5$ Hz. An example of the corresponding processing is depicted in Fig. 3. That is, if the frequency band $\Delta f(\mathrm{FM}) > \Delta f(\mathrm{CoPh\ WOS})$, it is needed to attract a special technique for restoring a source signal in the whole frequency band. Such a case was studied in [9]. The methodology proposed is based on dividing the frequency band of an FM signal into the corresponding number of zones and subsequent processing of a source signal using the corresponding frequencies values. To study properties of the results obtained, using the cluster analysis was proposed [10], [11]. The cluster analysis of the results of separate processing of the source signal in its frequency zones allows us to obtain a distribution of energy in each result obtained. The latter data allow us to restore the source signal in the whole frequency band. But we will not discuss the use of such a technique in this research.

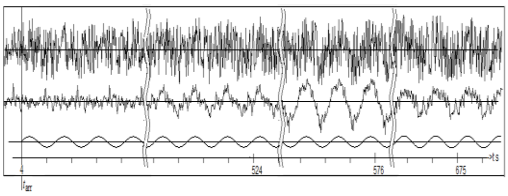

Fig. 3. An image of the model of a noise signal, the results of processing and the sounding signal (from above downward).

The algorithm of the corresponding signals processing, which provides a variation of the above data and parameters in the course of signal processing, is presented in Fig. 4. Here, the basic notations are the following:

– {filter$_i$: i=1,…, I} is a filters set,
  – filter(beg)/filter(end) is the 1st/last filters used;
  – filter$_i$={node$_{k(i)}$: $k(i)$=0,…, $K(i)$-1} is a sequence of data of consequent steps of the i-th filter of the corresponding filters bank;

– {f_l: l=1,…, L} is a frequencies set;
– f(beg)/f(end) is the 1st/last work frequency;
– Δf is the step size of frequency updating;
  – $s_{\Delta f}$ is the sequence steps of frequency updating, {s=0,…, $S_{\Delta f}$ -1};
– f^{w} is the current work frequency;
– $\Delta\alpha$ is the step size of procentile updating;
  – $r_{\Delta\alpha}$ is the sequence steps of procentile updating, r=0,…, $R_{\Delta\alpha}$-1;

Other notations are used in the course of the cluster analysis and will not be discussed here.

Here, the basic features of specialized computer system [5], which was utilized in the course of the above trials, will be briefly considered.

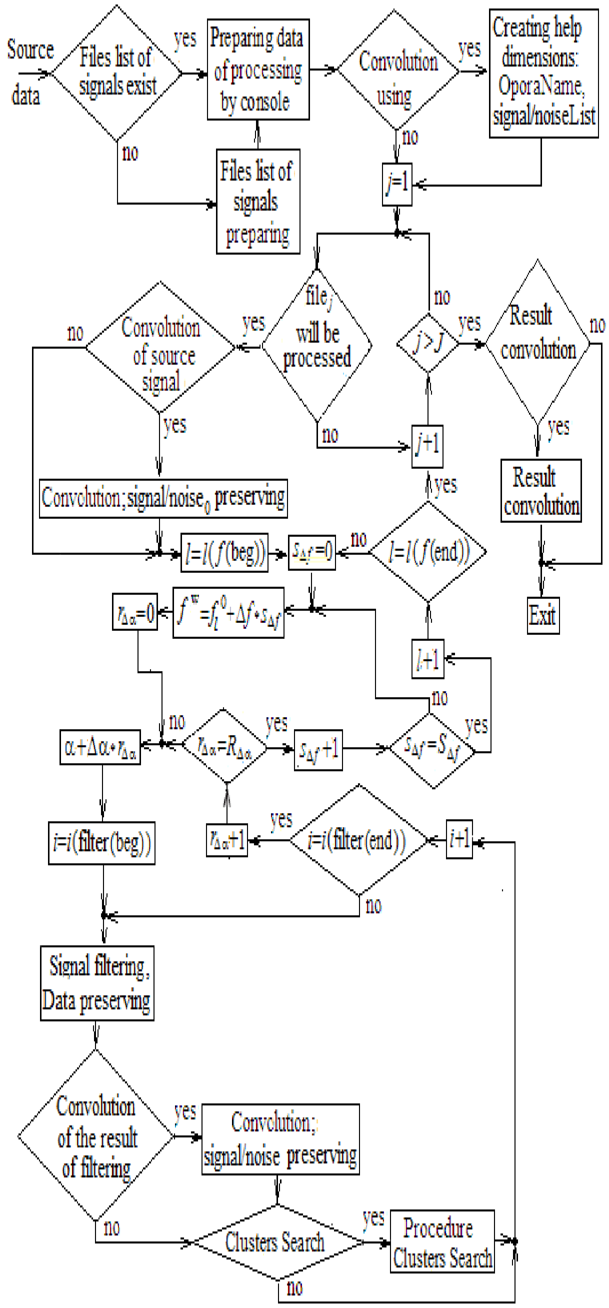

Fig. 4. The algorithm of monitoring the filters projects.

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In the general case, a filter includes a sequence of processing nodes. Here, the list of available filter operations is the following:

0) Transfer: data∈N1=>N3;
1) Data composition: (data∈N1+data∈N2)=>N3;
2) CoPh WOS filter;
3) Coupled CoPh WOS filter;
4) Co-phased average;
5) Coupled co-phased average;
6) Standard WOS filter;
7) Coupled standard WOS filter;
8) Standard average;
9) Coupled standard average.

The filter node as such is a sequence of the following data: $N_{operation}$, L1, *W*1, α1, L2, *W*2, α2, N1, N2, N3. Here, $N_{operation}$ is order number of the current operation; L1, L2 are lengths of the filters; α1, α2 are procentiles of the filters; *W*1, *W*2 are distributions of weights of the filters; N1, N2, N3 are numbers of work files, where N1, N2, N3 ∈ {0, 1,…, 101}. Thus, the user can employ both nonlinear and linear operations. The efficiency of such an approach was demonstrated in [12]. In the course of data processing, each step of action, which is connected with a corresponding node, can be presented by the scheme: processing data of the work file N1 with a consequent transfer of the results obtained to the work file N3. The new work files are created in the course of running the filtering procedure. Some nodes of a filter can include a coupled action of the same type, but with different data. Then, data L2, *W*2, α2 are attracted, and the results of both actions are composed. The work file of the source node of each filter is associated with number “0”. At the same time, the work file of the last filter node is associated with number “101”.

IV. SOME RESULTS OF SIGNAL PROCESSING

The methodology in question was investigated on the models of linear frequency-modulated signals using numerical modeling. Parameters and characteristics of the corresponding signals are the following: the start time of a sounding signal is 0 s; the arrival time of the sounding signal in a noise one is 4 s. At the same time, the bandwidth of the sounding signal is 7.2 Hz ÷8.2 Hz, the digitization frequency being Δt=0.08 s as for a sounding signal and for a noised signal. The white noise with zero average Gaussian distribution was used for obtaining a noise signal model.

Table 1. Results of convolution of a filtered noise signal with a sounding signal.

$\breve{s}/\xi_0$=99.4 – s/n=0.2	$\breve{s}/\xi_0$=87.6 – s/n=0.1	$\breve{s}/\xi_0$=10.1 – s/n=0.01
f Hz – *s*/*ξ*	*f* Hz – *s*/*ξ*	*f* Hz – *s*/*ξ*
7.75 – 125.6	7.7 – 93.0	7.65 – 14.4
7.765 – 127.0	7.71 – 96.8	7.67 – 15.9
7.77 – 147.5	7.72 – 94.1	7.68 – 10.2
7.775 – 137.8	7.73 – 90.4	7.69 – 10.1
7.785 – 145.7	7.74 – 91.0	

The results of the experiments are shown in Table 1, where $\breve{s}/\xi$ is the signal-to-noise ratio obtained by means of convolution of a noise signal with a sounding signal. At the same time, s/n is the estimation of the ratio of the mean square deviation of the sounding signal and the model of noise signal data before the processing.

V. CONCLUSION

In this paper, the approach to the statistical adaptation of the WOS filters to processing the frequency-modulated signals is proposed. The results obtained in the course of the experiments conducted demonstrate the dynamics of a considerable dependence of the quality of signals processing on the value of the work frequency of the CoPhWOS filter and its structure including values of the corresponding data. It can be supposed that the above results demonstrate the possibility of the approach proposed which is rather effective even in the case of the signal-to-noise ratio $s/\xi \ll 1$. In the considered methodology, the selecting of the qualitative projects of the WOS filters is assigned to the user. However, the uses of such methods as dynamic programming, sequential approaches, games theory, or maximum likelihood are of some interest in terms of selecting appropriate results. Meanwhile, In the case of attracting any of the above methods, there can be used two main ways of research in question: 1) selecting an appropriate filter from the existing filters bank with different data of signal processing, 2) consequent constructing a certain filter by means of the gradual complication of the filter structure. In both ways of the research, it is rather desirable to provide computer construction of different WOS filters. Meanwhile, any proposals in this field are absent because the problem becomes complicated owing to nonlinearity of the WOS filters. This circumstance is valid for any selected strategy of the qualitative projects of the WOS filters. To all the other above-mentioned notations, the procedures of the statistical trials as well as the procedures of signal processing by the WOS filters demand high computer costs, i.e., the corresponding realization of the approach proposed demands the use of graphical processors or supercomputers. The importance of an increase in the computation efficiency is also important for the improvement of the quality of the noised FM signals. Really, the width of a frequency zone, in which the waveform of a periodic signal remains, depends on the length of the (as it is noted above, the quality of a periodic signal is preserved in the frequency band of the width 1.0÷1.5 Hz if a length of CoPhWOS filter does not exceed two periods of an FM signal [2]). That is, increasing the filter nodes lengths will decrease a frequency zone, where the sounding signal quality is preserved (see the Section III). Hence, it is required to increase the quantity of the frequency zones which are parameters of filtration for the qualitative processing of a signal. The latter circumstance, in turn, will entail an increase of the computer time costs.

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