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Thermal stress analysis of a thick annular disc due to heat generation

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Abstract- ***In this paper, an attempt has been made to determine the temperature distribution, displacement function and thermal stresses of a thick annular disc occupying the space*** $D: a \le r \le b,\ -h \le z \le h,$ ***due to heat generation. Here we apply integral transform techniques to find the thermoelastic solution.***

Keywords: **Thermoelastic problem, annular disc, Thermal Stresses, integral transform.**

I. INTRODUCTION

Khobragade et al. [3 - 12] have derived temperature distribution, displacement function, thermal stresses and thermal deflection of a thick and thin annular disc. Further Khobragade et al. [13] have established displacement function, temperature distribution and stresses and deflection of a triangular plate.

This paper is concerned with transient thermoelastic problem of a thick circular plate occupying the space $D: a \le r \le b,\ -h \le z \le h,$ due to heat generation.

II. STATEMENT OF THE PROBLEM

Consider thick annular disc of thickness $2h$ occupying the space $D: a \le r \le b,\ -h \le z \le h,$ the material is homogenous and isotropic. The differential equation governing the displacement potential function $\phi(r,z,t)$ as Nowacki [1] is

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial z^2} = \left(\frac{1+\nu}{1-\nu}\right)\alpha_t T \qquad (1)$$

where ν and a_t are Poisson's ratio and linear coefficient of thermal expansion of the material of the disc and T is the temperature of the disc satisfying the differential equation as Noda [2] is

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} + \frac{g(r,z,t)}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad (2)$$

Subject to initial condition

$$M_t(T,1,0,0) = F(r,z) \qquad a \le r \le b, -h \le z \le h. \qquad (3)$$

The boundary conditions are

$$\left.\begin{aligned} M_r(T,0,1,a) &= g_1(z,\ t) \\ M_r(T,0,1,b) &= g_2(z,\ t) \end{aligned}\right\}, \quad -h \le z \le h,\ t>0 \qquad (4)$$

$$\left.\begin{aligned} M_z(T,1,k_1,h) &= f_1(r,\ t) \\ M_z(T,1,k_2,-h) &= -\frac{Q_0}{\lambda} f_2(r,\ t) \end{aligned}\right\}, \quad a \le r \le b,\ t>0 \qquad (5)$$

where k is thermal diffusivity of material of the disc.

The displacement function in the cylindrical coordinate system are represented by Michell's function as Noda [2] are

$$u_r = \frac{\partial \phi}{\partial r} - \frac{\partial^2 M}{\partial r \partial z} \qquad (6)$$

$$u_z = \frac{\partial \phi}{\partial z} + 2(1-\nu)\nabla^2 M - \frac{\partial^2 M}{\partial z^2} \qquad (7)$$

The Michell's function [2] must satisfy

$$\nabla^2 \nabla^2 M = 0 \qquad (8)$$

where

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$$

The component of stresses are represented by the thermoelastic displacement potential ϕ and Michell's function M as Noda [2] are

$$\sigma_{rr} = 2G\left\{\left(\frac{\partial^2 \phi}{\partial r^2} - \nabla^2 \phi\right) + \frac{\partial}{\partial z}\left(\nu \nabla^2 M - \frac{\partial^2 M}{\partial r^2}\right)\right\} \qquad (9)$$

$$\sigma_{\theta\theta} = 2G\left\{\left(\frac{1}{r}\frac{\partial \phi}{\partial r} - \nabla^2 \phi\right) + \frac{\partial}{\partial z}\left(\nu \nabla^2 M - \frac{1}{r}\frac{\partial^2 M}{\partial r^2}\right)\right\} \qquad (10)$$

$$\sigma_{zz} = 2G\left\{\left(\frac{\partial^2 \phi}{\partial z^2} - \nabla^2 \phi\right) + \frac{\partial}{\partial z}\left\{\left((z-\nu)\nabla^2 M - \frac{\partial^2 M}{\partial z^2}\right)\right\}\right\} \qquad (11)$$

$$\sigma_{rz} = 2G\left\{\frac{\partial^2 \phi}{\partial r \partial z} + \frac{\partial}{\partial r}\left\{\left((1-\nu)\nabla^2 M - \frac{\partial^2 M}{\partial z^2}\right)\right\}\right\} \qquad (12)$$

For traction free surface stress function

$\sigma_{\theta z} = \sigma_{r\theta} = 0$ at $z = \pm h$ for thick Disc.

Equations (1) to (12) constitute the mathematical formulation of the problem under consideration.

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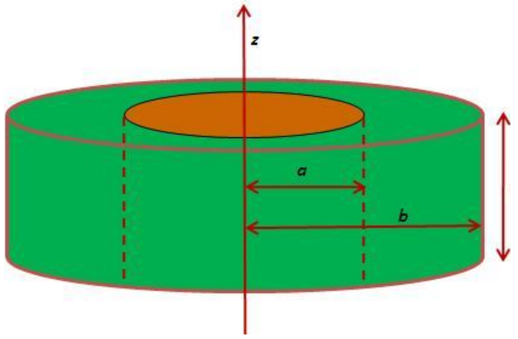

Fig. 1: The geometry of the problem

III. SOLUTION OF THE PROBLEM

Applying Hankel transform defined in [13] to the equation (2), we get

$$-\mu_m^2\bar{T}(\mu_m,z,t)+C+\frac{d^2\bar{T}}{dz^2}(\mu_m,z,t)+\frac{\bar{g}}{k}(\mu_m,z,t)=\frac{1}{\alpha}\frac{d\bar{T}}{dt} \tag{13}$$

Again applying Marchi-Fasulo transform defined in [14] to above equation, we obtain

$$\frac{d\bar{T}^*}{dt}+\alpha p^2\bar{T}^*=\Psi \tag{14}$$

where

$$\psi=\frac{P_n(h)}{k_1}\bar{f}_1\frac{P_n(-h)}{k_2}\bar{f}_2-\frac{\bar{g}^*}{k}$$

$$p^2=\mu_m^2+\lambda_n^2$$

Equation (14) is a first order differential equation whose solution is given by

$$\bar{T}^*=e^{-\alpha p^2 t}\left[\bar{F}^*+\int_0^t \psi e^{\alpha p^2 t'}dt'\right] \tag{15}$$

Applying inverse Marchi-Fasulo transform to the equation (15), we get

$$\bar{T}(\xi_m,n,t)=\sum_{n=1}^{\infty}\frac{P_n(z)}{\lambda_n}e^{-\alpha p^2 t}\left[\bar{F}^*+\int_0^t \psi e^{\alpha p^2 t'}dt'\right] \tag{16}$$

Applying inverse finite Hankel transform to the equation (16), we get

$$T(r,z,t)=\sum_m\sum_n \chi(a,b)\times\frac{P_n(z)}{\lambda_n}\Pi(\mu_m r,b)\,\Omega \tag{17}$$

where

$$\Pi(\mu_m r,b)=[J_0(\mu_m r)G_0(\mu_m b)-J_0(\mu_m b)G_0(\mu_m r)]$$

$$\chi(a,b)=\frac{2\mu_m{}^2 J_0{}^2(\mu_m a)}{[J_0{}^2(\mu_m b)-J_0{}^2(\mu_m a)]},$$

$$\Omega=e^{-\alpha p^2 t}\left[\bar{F}^*(m,n)+\int_0^t \Psi e^{\alpha p^2 t'}dt'\right]$$

Equation (17) is the desired solution of the given problem.

Let us assume Michell's function *M*, which satisfy condition (8) as

$$M(r,z)=\sum_m\sum_n \chi(a,b)\frac{P_n(z)}{\lambda_n}\Pi(\mu_m r,b) \tag{18}$$

Using (1) and (17), we get displacement potential ϕ *as*

$$\phi=A\sum_m\sum_n \chi(a,b)\times\frac{P_n(z)}{\lambda_n}\Pi(\mu_m r,b)\,\Omega \tag{20}$$

where $A=\left(\frac{1+\nu}{1-\nu}\right)a_t$

IV. DETERMINATION OF DISPLACEMENT FUNCTION

Substituting equations (18) and (20) in equation (6), (7) we get

$$u_r=A\sum_m\sum_n \mu_m\chi(a,b)\times\frac{P_n(z)}{\lambda_n}\Pi'(\mu_m r,b)\,\Omega-\sum_m\sum_n \mu_m\chi(a,b)\times\frac{P_n'(z)}{\lambda_n}\Pi'(\mu_m r,b) \tag{21}$$

$$u_z=A\sum_m\sum_n \mu_m\chi(a,b)\times\frac{P_n'(z)}{\lambda_n}\Pi(\mu_m r,b)\,\Omega$$

$$-2(1-\nu)\sum_m\sum_n \mu_m{}^2\chi(a,b)\times\frac{P_n(z)}{\lambda_n}\Pi''(\mu_m r,b)$$

$$+\frac{2(1-\nu)}{r}\sum_m\sum_n \mu_m\chi(a,b)\times\frac{P_n(z)}{\lambda_n}\Pi'(\mu_m r,b)$$

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$$+(1-2\nu)\sum_m\sum_n \chi(a,b)\times\frac{P_n''(z)}{\lambda_n}\Pi(\mu_m r,b) \tag{22}$$

Substituting equations (18) and (20) in equations (9) to (12), we obtain

$$\sigma_{rr}=-2G\left\{\frac{A}{r}\sum_m\sum_n \mu_m\chi(a,b)\frac{P_n(z)}{\lambda_n}\Pi'(\mu_m r,b)\Omega\right.$$

$$\left.+\frac{A}{r}\sum_m\sum_n \chi(a,b)\frac{P_n''(z)}{\lambda_n}\Pi(\mu_m r,b)\Omega\right\}$$

$$+2G\nu\left\{\sum_m\sum_n \mu_m^{\ 2}\chi(a,b)\frac{P_n'(z)}{\lambda_n}\Pi''(\mu_m r,b)\right.$$

$$+\frac{1}{r}\sum_m\sum_n \mu_m\chi(a,b)\frac{P_n'(z)}{\lambda_n}\Pi'(\mu_m r,b)$$

$$+\sum_m\sum_n \chi(a,b)\frac{P_n^{111}(z)}{\lambda_n}\Pi(\mu_m r,b)$$

$$\left.-\sum_m\sum_n \mu_m^{\ 2}\chi(a,b)\frac{P_n'(z)}{\lambda_n}\Pi''(\mu_m r,b)\right\} \tag{22}$$

$$\sigma_{\theta\theta}=-2G\left\{A\sum_m\sum_n \mu_m^{\ 2}\chi(a,b)\frac{P_n(z)}{\lambda_n}\Pi''(\mu_m r,b)\Omega\right.$$

$$\left.+A\sum_m\sum_n \chi(a,b)\frac{P_n''(z)}{\lambda_n}\Pi(\mu_m r,b)\Omega\right\}$$

$$+2G\nu\left\{\sum_m\sum_n \mu_m^{\ 2}\chi(a,b)\frac{P_n'(z)}{\lambda_n}\Pi''(\mu_m r,b)\right.$$

$$+\frac{1}{r}\sum_m\sum_n \mu_m\chi(a,b)\frac{P_n'(z)}{\lambda_n}\Pi'(\mu_m r,b)$$

$$+\sum_m\sum_n \chi(a,b)\frac{P_n^{111}(z)}{\lambda_n}\Pi(\mu_m r,b)$$

$$\left.-\frac{1}{r}\sum_m\sum_n \mu_m^{\ 2}\chi(a,b)\frac{P_n(z)}{\lambda_n}\Pi''(\mu_m r,b)\right\} \tag{23}$$

$$\sigma_{zz}=-2G\left\{A\sum_m\sum_n \mu_m^{\ 2}\chi(a,b)\frac{P_n(z)}{\lambda_n}\Pi''(\mu_m r,b)\Omega\right.$$

$$\left.+\frac{A}{r}\sum_m\sum_n \mu_m\chi(a,b)\frac{P_n(z)}{\lambda_n}\Pi'(\mu_m r,b)\Omega\right\}$$

$$+2G(2-\nu)\left\{\sum_m\sum_n \mu_m^{\ 2}\chi(a,b)\frac{P_n'(z)}{\lambda_n}\Pi''(\mu_m r,b)\right.$$

$$+\frac{1}{r}\sum_m\sum_n \mu_m\chi(a,b)\frac{P_n'(z)}{\lambda_n}\Pi'(\mu_m r,b)$$

$$+\sum_m\sum_n \chi(a,b)\frac{P_n^{111}(z)}{\lambda_n}\Pi(\mu_m r,b)$$

$$\left.-\sum_m\sum_n \chi(a,b)\frac{P_n^{111}(z)}{\lambda_n}\Pi(\mu_m r,b)\right\} \tag{24}$$

$$\sigma_{rz}=2G\left\{A\sum_m\sum_n \mu_m\chi(a,b)\frac{P_n'(z)}{\lambda_n}\Pi'(\mu_m r,b)\Omega\right\}$$

$$+2G(1-\nu)\left\{\sum_m\sum_n \mu_m^{\ 3}\chi(a,b)\frac{P_n(z)}{\lambda_n}\Pi^{111}(\mu_m r,b)\right.$$

$$-\frac{1}{r}\sum_m\sum_n \mu_m^{\ 2}\chi(a,b)\frac{P_n(z)}{\lambda_n}\Pi''(\mu_m r,b)$$

$$+\sum_m\sum_n \mu_m\chi(a,b)\frac{P_n''(z)}{\lambda_n}\Pi'(r,b)$$

$$\left.-\sum_m\sum_n \mu_m\chi(a,b)\frac{P_n''(z)}{\lambda_n}\Pi(\mu_m r,b)\right\} \tag{25}$$

V. SPECIAL CASE

Set $$F(r,z)=z^2\,\frac{\sin(r-a)\sin(b-r)}{r} \tag{26}$$

Applying Marchi-Fasulo transform, are obtain

$$\overline{F}(r,n)=\frac{\sin(r-a)\sin(b-r)}{r}\int_{-h}^{h} z^2 P_n(z)dz$$

$$\overline{F}(r,n)=\frac{\sin(r-a)\sin(b-r)}{r}\left[\frac{2h^2\sin(a_n h)}{a_n}+\frac{4h\cos(a_n h)}{a_n^{\ 2}}-\frac{4\sin(a_n h)}{a_n^{\ 3}}\right] \quad (27)$$

Where

$P_n(z)=Q_n\cos(a_n z)-W_n\sin(a_n z)$,

$Q_n=a_n(\alpha_1+\alpha_2)\cos(a_n h)+(\beta_1-\beta_2)\sin(a_n h)$

$W_n=(\beta_1-\beta_2)\cos(a_n h)+a_n(\alpha_1-\alpha_2)\sin(a_n h)$

Again on applying Hankel transform, we obtain

$$\overline{F}^*(m,n)=\frac{\Lambda}{a\lambda_m}\left[\frac{\cos(a+b)}{2}-\frac{\cos(3b-a)}{2}\right]\frac{J_0(\lambda_n b)}{J_0(\lambda_n a)} \quad (28)$$

Where

$$\Lambda=\Phi\left[\frac{2h^2\sin(a_n h)}{a_n}+\frac{4h\cos(a_n h)}{a_n^{\ 2}}-\frac{4\sin(a_n h)}{a_n^{\ 3}}\right]$$

And

$$\Phi=a_n(\alpha_1+\alpha_2)\cos(a_n h)+(\beta_1-\beta_2)\sin(a_n h).$$

Using equation (27) in equation (17), one obtains

$$T(r,z,t)=\sum_m\sum_n \chi(a,b)\ \times\frac{P_n(z)}{\lambda_n}\Pi(\mu_m r,b)$$

$$\times\left[\int_0^t \Psi e^{-\alpha p^2(t-t')}dt'+\frac{\Lambda}{a\lambda_m}\left[\frac{\cos(a+b)}{2}-\frac{\cos(3b-a)}{2}\right]\frac{J_0(\lambda_n b)}{J_0(\lambda_n a)}\right] \quad (29)$$

VI NUMERICAL RESULTS

Set $a=2, b=3, k=15.9\times10^6, t=1$ second in equation (29), we get

$$T(r,z,t)=\sum_m\sum_n \chi(2,3)\ \times\frac{P_n(z)}{\lambda_n}\Pi(\mu_m r,3)$$

$$\times\left[\int_0^1 \Psi e^{-\alpha p^2(1-t')}dt'+\frac{\Lambda}{2\lambda_m}\left[\frac{\cos(5)}{2}-\frac{\cos(7)}{2}\right]\frac{J_0(\lambda_n 3)}{J_0(\lambda_n 2)}\right]$$

VII. CONCLUSION

In this article, the temperature distribution, displacement function and thermal stresses of a thick annular disc are investigated with known boundary conditions. Finite integral transform techniques are used to obtain numerical results. The results are obtained in terms of Bessel's function in the form of infinite series. Any particular cases of special interest can be derived by assigning suitable values to the parameters and functions in expressions. The results that are obtained can be useful to the design of structure or machines in engineering applications.

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