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Thermal Stress Analysis of a Thick Circular Plate

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Abstract- ***In this paper, an attempt has been made to determine the temperature distribution, displacement function and thermal stresses of a thick circular plate occupying the space*** $D: 0 \le r \le a,\ -h \le z \le h,$ ***due to heat generation with stated boundary conditions. Here we apply integral transform techniques to find the thermoelastic solution.***

Keywords: **Thermal stresses, thick circular plate, Thermal Stresses, integral transform.**

I. INTRODUCTION

Khobragade et al. [3 - 12] have derived temperature distribution, displacement function, thermal stresses and thermal deflection of a thick and thin circular plate. This paper is concerned with transient thermo elastic problem of a thick circular plate occupying the space $D: 0 \le r \le a,\ -h \le z \le h,$ due to heat generation with radiation type boundary conditions.

II. STATEMENT OF THE PROBLEM

Consider thick circular plate of thickness $2h$ occupying the space $D: 0 \le r \le a,\ -h \le z \le h,$ the material is homogenous and isotropic. The differential equation governing the displacement potential function $\phi(r,z,t)$ as Nowacki [1] is

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial z^2} = \left(\frac{1+\nu}{1-\nu}\right)\alpha_t T \qquad (1)$$

Where ν and a_t are Poisson's ratio and linear coefficient of thermal expansion of the material of the plate and T is the temperature distribution of the plate satisfying the differential equation as Noda [2] is

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} + \frac{g(r,z,t)}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad (2)$$

Subject to initial condition

$$M_t(T,1,0,0) = 0 \qquad 0 \le r \le a, -h \le z \le h. \qquad (3)$$

The boundary conditions are

$$M_r(T,1,0,a) = g_1(z,\ t), \qquad -h \le z \le h,\ t > 0 \qquad (4)$$

$$\left.\begin{array}{l} M_z(T,1,k_1,h) = f_1(r,\ t) \\ M_z(T,1,k_2,-h) = f_2(r,\ t) \end{array}\right\},\ 0 \le r \le a,\ t > 0 \qquad (5)$$

where k is thermal diffusivity of material of the plate. The displacement function in the cylindrical coordinate system are represented by Michell's function as Noda [2] are

$$u_r = \frac{\partial \phi}{\partial r} - \frac{\partial^2 M}{\partial r \partial z} \qquad (6)$$

$$u_z = \frac{\partial \phi}{\partial z} + 2(1-\nu)\nabla^2 M - \frac{\partial^2 M}{\partial z^2} \qquad (7)$$

The Michell's function [2] must satisfy

$$\nabla^2 \nabla^2 M = 0 \qquad (8)$$

Where

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$$

The component of stresses are represented by the thermo elastic displacement potential ϕ and Michell's function M as Noda [2] are

$$\sigma_{rr} = 2G\left\{\left(\frac{\partial^2 \phi}{\partial r^2} - \nabla^2 \phi\right) + \frac{\partial}{\partial z}\left(\nu \nabla^2 M - \frac{\partial^2 M}{\partial r^2}\right)\right\} \qquad (9)$$

$$\sigma_{\theta\theta} = 2G\left\{\left(\frac{1}{r}\frac{\partial \phi}{\partial r} - \nabla^2 \phi\right) + \frac{\partial}{\partial z}\left(\nu \nabla^2 M - \frac{1}{r}\frac{\partial^2 M}{\partial r^2}\right)\right\} \qquad (10)$$

$$\sigma_{zz} = 2G\left\{\left(\frac{\partial^2 \phi}{\partial z^2} - \nabla^2 \phi\right) + \frac{\partial}{\partial z}\left\{\left((2-\nu)\nabla^2 M - \frac{\partial^2 M}{\partial z^2}\right)\right\}\right\} \qquad (11)$$

$$\sigma_{rz} = 2G\left\{\frac{\partial^2 \phi}{\partial r \partial z} + \frac{\partial}{\partial r}\left\{\left((1-\nu)\nabla^2 M - \frac{\partial^2 M}{\partial z^2}\right)\right\}\right\} \qquad (12)$$

For traction free surface stress function

$\sigma_{\theta z} = \sigma_{r\theta} = 0$ at $z = \pm h$ for thick plate.

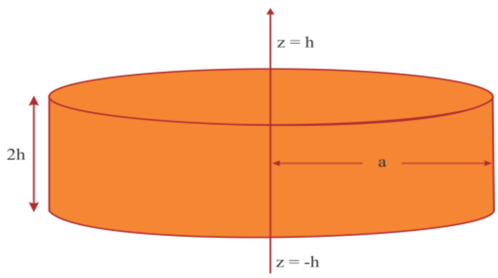

Fig. 1: The geometry of the problem

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Equations (1) to (12) constitute the mathematical formulation of the problem under consideration.

III. SOLUTION OF THE PROBLEM

Applying Hankel transform defined in [13] to the equation (2), we get

$$-\mu_m^2\bar{T}(\mu_m,z,t)+\frac{d^2\bar{T}}{dz^2}(\mu_m,z,t)+\frac{\bar{g}(\mu_m,z,t)}{k}=\frac{1}{\alpha}\frac{d\bar{T}}{dt} \tag{13}$$

Again applying Marchi-Fasulo transform defined in [14] to above equation, we obtain

$$\frac{d\bar{T}^*}{dt}+\alpha p^2\bar{T}^*=\Psi \tag{14}$$

where

$$p^2=\mu_m^2+\lambda_n^2$$

Equation (14) is a 1st order differential equation whose solution is given by

$$\bar{T}^*(\mu_m,n,t)=\int_0^t \Psi e^{-\alpha p^2(t-t')}dt' \tag{15}$$

where

$$\psi=\alpha\left(\frac{P_n(n)}{k_1}\bar{f}_1-\frac{P_n(-n)}{k_2}\bar{f}_2-\frac{\bar{g}^*}{k}\right)$$

Applying inverse finite Marchi-Fasulo transform to the equation (15), we get

$$\bar{T}(\mu_m,z,t)=\sum_{n=1}^{\infty}\frac{P_n(z)}{\lambda_n}\int_0^t \Psi e^{-\alpha p^2(t-t')}dt' \tag{16}$$

Applying inverse finite Hankel transform to the equation (16), we get

$$T(r,z,t)=\frac{2}{a^2}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{J_0(r\mu_m)}{[J_1(a\mu_m)]^2}\frac{P_n(z)}{\lambda_n}\Omega \tag{17}$$

where $\Omega=\left[\int_0^t \Psi e^{-\alpha p^2(t-t')}dt'\right]$

This is the desired solution of the given problem.

Let us assume Michell's function M, which satisfy condition (8) as

$$M(r,z)=\frac{2}{a^2}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{J_0(r\mu_m)}{[J_1(a\mu_m)]^2}\frac{P_n(z)}{\lambda_n} \tag{18}$$

Using (1) and (17), we get displacement potential ϕ *as*

$$\phi=A\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{J_0(r\mu_m)}{[J_1(a\mu_m)]^2}\frac{P_n(z)}{\lambda_n}\Omega \tag{19}$$

where $A=\left(\frac{1+\nu}{1-\nu}\right)\frac{a_t}{a^2}$

IV. DETERMINATION OF DISPLACEMENT FUNCTION

Substituting equations (18) and (19) in equation (6) and (7), we get

$$u_r=A\sum_m\sum_n\frac{\mu_m J_1(\mu_m r)}{[J_1(a\mu_m)]^2}\frac{P_n(z)}{\lambda_n}\Omega-\frac{2}{a^2}\sum_m\sum_n\frac{\mu_m J_1(\mu_m r)}{[J_1(a\mu_m)]^2}\frac{P_n'(z)}{\lambda_n} \tag{20}$$

$$u_z=A\sum_m\sum_n\frac{\mu_m J_0(\mu_m r)}{[J_1(a\mu_m)]^2}\frac{P_n'(z)}{\lambda_n}\Omega-\frac{2}{a^2}\sum_m\sum_n\frac{J_0(\mu_m r)}{[J_1(a\mu_m)]^2}\frac{P_n''(z)}{\lambda_n} \tag{21}$$

Substituting equations (18) and (19) in equations (9) to (12), we obtain

$$\sigma_{rr}=2G\left\{A\sum_m\sum_n\left[\frac{\mu_m J_1(r\mu_m)}{[J_1(a\mu_m)]^2}\frac{P_n(z)}{\lambda_n}+\frac{J_0(r\mu_m)}{[J_1(a\mu_m)]^2}\frac{P_n'(z)}{\lambda_n}\right]\Omega+\left((\nu-1)\left(\frac{2}{a^2}\sum_m\sum_n\frac{\mu_m{}^2 J_0''(r\mu_m)}{[J_1(a\mu_m)]^2}\frac{P_n'(z)}{\lambda_n}\right)+\frac{2}{a^2}\sum_m\sum_n\left[\frac{1}{r}\frac{\mu_m J_1(r\mu_m)}{[J_1(a\mu_m)]^2}\frac{P_n(z)}{\lambda_n}+\frac{J_0(r\mu_m)}{[J_1(a\mu_m)]^2}\frac{P_n'(z)}{\lambda_n}\right]\right\} \tag{22}$$

$$\sigma_{\theta\theta}=2G\left\{-A\sum_m\sum_n\left[\frac{\mu_m{}^2 J_1'(r\mu_m)}{[J_1(a\mu_m)]^2}\frac{P_n(z)}{\lambda_n}+\frac{\mu_m J_0(r\mu_m)}{[J_1(a\mu_m)]^2}\frac{P_n(z)}{\lambda_n}+\left[\frac{\mu_m J_0(r\mu_m)}{[J_1(a\mu_m)]^2}\frac{P_n'(z)}{\lambda_n}\right]\Omega+\left(\nu\left(\frac{2}{a^2}\sum_m\sum_n\frac{\mu_m{}^2 J_0''(r\mu_m)}{[J_1(a\mu_m)]^2}\frac{P_n(z)}{\lambda_n}\right)\right.\right.\right.$$

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$$+\left[\frac{1}{r}\frac{\mu_m J_1(r\mu_m)}{[J_1(a\mu_m)]^2}\frac{P_n(z)}{\lambda_n}+\frac{J_0(r\mu_m)}{[J_1(a\mu_m)]^2}\frac{P_n'(z)}{\lambda_n}\right]\Bigg]$$

$$+\frac{(1+\nu)}{(1-\nu)}a_t\sum_m\sum_n\frac{\mu_m J_1'(r\mu_m)}{[J_1(a\mu_m)]^2}\frac{P_n'(z)}{\lambda_n} \qquad (23)$$

$$\sigma_{zz}=2G\left\{A\sum_m\sum_n\frac{J_0(r\mu_m)}{[J_1(a\mu_m)]^2}\frac{P_n'(z)}{\lambda_n}\Omega\right.$$

$$-A\sum_m\sum_n\left[\frac{\mu_m{}^2 J_1'(r\mu_m)}{[J_1(a\mu_m)]^2}\frac{P_n(z)}{\lambda_n}+\frac{\mu_m J_0(r\mu_m)}{[J_1(a\mu_m)]^2}\frac{P_n(z)}{\lambda_n}\right]$$

$$+\left[\frac{\mu_m J_0(r\mu_m)}{[J_1(a\mu_m)]^2}\frac{P_n'(z)}{\lambda_n}\right]\Omega$$

$$+\left((2-\nu)\left(\frac{2}{a^2}\sum_m\sum_n\frac{\mu_m{}^2 J_0''(r\mu_m)}{[J_1(a\mu_m)]^2}\frac{P_n'(z)}{\lambda_n}\right)\right.$$

$$+\left[\frac{1}{r}\frac{\mu_m J_1(r\mu_m)}{[J_1(a\mu_m)]^2}\frac{P_n'(z)}{\lambda_n}+\frac{J_0(r\mu_m)}{[J_1(a\mu_m)]^2}\frac{P_n''(z)}{\lambda_n}\right]\Bigg)$$

$$\left.-\frac{2}{a^2}\sum_m\sum_n\frac{J_0(r\mu_m)}{[J_1(a\xi_m)]^2}\frac{P_n''(z)}{\lambda_n}\right)\Bigg\} \qquad (24)$$

$$\sigma_{rz}=2G\left\{A\sum_m\sum_n\frac{\mu_m J_1(r\mu_m)}{[J_1(a\mu_m)]^2}\left[\frac{P_n'(z)}{\lambda_n}\right]\Omega\right.$$

$$+\frac{2(1-\nu)}{a^2}\sum_m\sum_n\frac{\mu_m{}^3 J_1''(r\mu_m)}{[J_1(a\xi_m)]^2}\frac{P_n(z)}{\lambda_n}$$

$$+\left[\frac{1}{r}\frac{\mu_m^2 J_1'(r\mu_m)}{[J_1(a\mu_m)]^2}\frac{P_n(z)}{\lambda_n}-\frac{1}{r^2}\frac{\mu_m J_1(r\mu_m)}{[J_1(a\mu_m)]^2}\frac{P_n(z)}{\lambda_n}+\frac{\mu_m J_1(r\mu_m)}{[J_1(a\mu_m)]^2}\frac{P_n'(z)}{\lambda_n}\right]$$

$$\left.+\frac{2}{a^2}\sum_m\sum_n\frac{\mu_m J_1(r\mu_m)}{[J_1(a\mu_m)]^2}\left[\frac{P_n'(z)}{\lambda_n}\right]\right\} \qquad (25)$$

V. SPECIAL CASE

Set $f_1(r,t)=f_2(r,t)=B(1-e^{-t})r$,

$g_1(z,t)=C(1-e^{-t})z$,

$$g(r,z,t)=\delta(r-r_0)\delta(z-z_0)\delta(t-t_0) \quad (26)$$

Applying finite Hankel transform, one obtains

$$\overline{f_1}(\mu_m,t)=\left[\frac{a}{\mu_m}J_1(a\mu_m)-\frac{a(a^2\mu_m^2-4)}{\mu_m^3}J_1(a\mu_m)-\frac{2a^2}{\mu_m^2}J_0(a\mu_m)\right]$$

$$\times B(1-e^{-t})$$

$$=\overline{f_2}(\mu_m,t) \qquad (27)$$

Using equation (27) in equation (17), one obtains

$$T(r,z,t)=\frac{2}{a^2}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{J_0(r\mu_m)}{[J_1(a\mu_m)]^2}\frac{P_n(z)}{\lambda_n}\Omega \quad (28)$$

VI. NUMERICAL RESULTS

Set $a=2, k=15.9\times10^6, t=1$ sec. in equation (28), we get

$$T(r,z,t)=(0.5)\sum_m\sum_n\frac{J_0(r\xi_m)}{[J_1(a\xi_m)]^2}\frac{P_n(z)}{\lambda_n}$$

$$\times e^{-(15.9\times10^6)P^2t}\int_0^1\Psi e^{(15.9\times10^6)P^2t^1}\,dt^1$$

$$\times\left(\frac{2}{\mu_m}J_1(2\mu_m)-\frac{2(4\mu_m^2-4)}{\mu_m{}^3}J_1(2\mu_m)-\frac{2}{\mu_m^2}J_0(2\mu_m)\right)$$

(29)

VII. CONCLUSION

In this article, the temperature distribution, displacement and thermal stresses of a thick circular plate are investigated with known boundary conditions. Finite integral transform techniques are used to obtain numerical results. The results are obtained in terms of Bessel's function in the form of infinite series. Any particular cases of special interest can be derived by assigning suitable values to the parameters and functions in expressions. The results that are obtained can be useful to the design of structure or machines in engineering applications.

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