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Thermoelastic Problem of a Thick Clamped Rectangular Plate

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Abstract*- This Paper is concerned with thermal deflection of a thick clamped rectangular plate occupying the space D: $-a \le x \le a, 0 \le y \le b, 0 \le z \le h$, with known boundary conditions. In this paper an attempt has been made to determine the temperature distribution and deflection of a thick clamped rectangular plate with the help of integral transform techniques. The result are obtained in terms Bessel's function in the form of infinite series and depicted graphically.

***Key words*-Thermoelastic Response, thick rectangular Plate, Thermal deflection, integral transform.**

I. INTRODUCTION

Khobragade et al. [4] have derived thermal deflection of a thin rectangular plate due to partially distributed heat supply. Khobragade et al. [6,7] have investigated displacement function, temperature distribution and stresses of a thin rectangular plate and Khobragade, Durge [3] have established displacement function, temperature distribution and stresses of a thick rectangular plate. In the present paper, an attempt is made to determine the temperature distribution and thermal deflection at any point of the plate occupying the space D: $-a \le x \le a, 0 \le y \le b, 0 \le z \le h$ with the known boundary conditions. Finite Fourier sine transform and Laplace transform techniques have been used to find the solution of the problem. Numerical estimate for the temperature distribution is obtained and depicted graphically.

II. STATEMENT OF THE PROBLEM

Consider a thick clamped rectangular plate occupying the space *D*. The differential equation satisfied by the deflection ω *(x,y, t)*, as Khobragade et al. [4] is

$$D\nabla^4 \omega(x,y,t) = \frac{-\nabla^2 M_T(x,y,t)}{1-\nu} \quad (1)$$

Where ν is the Poisson's ratio of the plate material, M_T denote the thermal momentum of the plate and *D* denote the flexural rigidity,

Where

$$\nabla^2 = \frac{d^2}{dx^2} + \frac{d^2}{dy^2}$$

and the resultant thermal momentum M_T is defined as

$$M_T(x,y,t) = \alpha E \int_0^h z T(x,y,z,t)\,dz \quad (2)$$

where α ,*E* are the linear coefficient of thermal expansion of the material, and Young's modulus respectively. Since the edge of the rectangular plate is fixed and clamped,

$$\omega = \frac{\partial^2 \omega}{\partial x^2} = \frac{\partial^2 \omega}{\partial y^2} = 0 \text{ at } x = a \text{ and } y = b \quad (3)$$

The temperature distribution of the plate at time t satisfying the differential equation as [4] is

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{k}\frac{\partial T}{\partial t} \quad (4)$$

where *k* is the thermal diffusivity of the material of the plate, subject to the initial and boundary conditions

$$T(x,y,z,0) = 0 \quad (5)$$

$$\left[T(x,y,z,t) + k_1 \frac{\partial T(x,y,z,t)}{\partial x}\right]_{x=-a} = 0 \quad (6)$$

$$\left[T(x,y,z,t) + k_2 \frac{\partial T(x,y,z,t)}{\partial x}\right]_{x=a} = 0 \quad (7)$$

$$\left[T(x,y,z,t)\right]_{y=0} = 0 \quad (8)$$

$$\left[T(x,y,z,t)\right]_{y=b} = 0 \quad (9)$$

$$\left[\frac{\partial T(x,y,z,t)}{\partial z}\right]_{z=0} = 0 \quad (10)$$

$$\left[\frac{\partial T(x,y,z,t)}{\partial z}\right]_{z=h} = f(x,y,t) \quad (11)$$

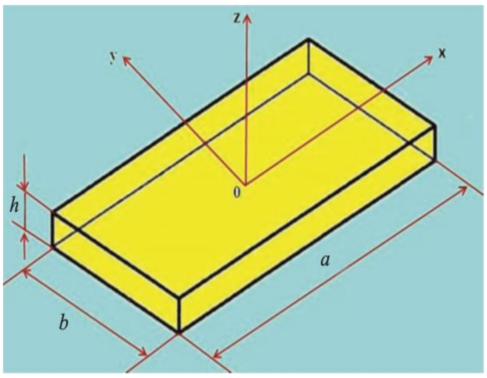

Fig. 1: Shows the geometry of the problem

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Equations (1) to (11) constitute the mathematical formulation of the problem under consideration.

III. SOLUTION OF THE PROBLEM

By applying finite Marchi-Fasulo transform defined in [4], Fourier sine and cosine transform defined in [1], w.r.t x, y and z respectively and using their inversion to the equation (4), using boundary conditions (5) to (11), one obtains the expression for temperature distribution as

$$T(x,y,z,t)=\frac{4k}{bh}\sum_{l=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}(-1)^l\frac{P_n(x)}{\mu_n}$$

$$\times\sin(q_m y)\cos(r_l z)$$

$$\times\int_0^t \bar{\bar{f}}(m,n,t')e^{-k[\mu_n^2+q_m^2+r_l^2](t-t')}dt' \quad (12)$$

where l, m, n are the positive integers and

$$r_l=\frac{l\pi}{h},\quad q_m=\frac{m\pi}{b}$$

$$\bar{\bar{f}}(m,n,t)=\int_{-a}^{a}\int_0^b f(x,y,t)\sin(q_m y)\frac{P_n(x)}{\mu_n}dxdy$$

IV. DETERMINATION OF THERMAL DEFLECTION

Substituting the value of $T(x,y,z,t)$ from equation (12) in equation (2) one obtains the thermal momentum as

$$M_T(x,y,t)=\frac{4k\alpha E}{bh}\sum_{l=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}(-1)^l$$

$$\times\left(\int_0^h z\cos(r_l z)dz\right)\frac{P_n(x)}{\mu_n}\sin(q_m y)$$

$$\times\int_0^t \bar{\bar{f}}(m,n,t')e^{-k[\mu_n^2+q_m^2+r_l^2](t-t')}dt'$$

(13)

We assume that the solution of equation (1) satisfying equation (3) as

$$\omega(x,y,t)=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\omega_{mn}(t)\frac{P_n(x)}{\mu_n}\sin(qy) \quad (14)$$

Using the equations (13) and (14) in (1), one obtains

$$\omega_{mn}(t)=\frac{4k\alpha E}{D(1-\nu)bh}\sum_{l=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}(-1)^l$$

$$\times\left(\frac{r_l h\sin(r_l h)+\cos(r_l h)-1}{r_l^2(\mu_n^2+q_m^2)}\right)$$

$$\times\int_0^t \bar{\bar{f}}(m,n,t')e^{-k[\mu_n^2+q_m^2+r_l^2](t-t')}dt' \quad (15)$$

Substituting the value of $\omega_{mn}(t)$ in equation (14), one obtains the expression for thermal deflection as

$$\omega(x,y,t)=\frac{4k\alpha E}{D(1-\nu)bh}\sum_{l=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}(-1)^l$$

$$\times\left(\frac{r_l h\sin(r_l h)+\cos(r_l h)-1}{r_l^2(\mu_n^2+q_m^2)}\right)P_n(x)\sin(q_m y)$$

$$\times\int_0^t \bar{\bar{f}}(m,n,t')e^{-k[\mu_n^2+q_m^2+r_l^2](t-t')}dt' \quad (16)$$

V. SPECIAL CASE AND NUMERICAL RESULTS

Set $f(x,y,t)=(1-e^{-t})(x^2-ax)(y^2-by)$,

(17) $\beta=\frac{32k}{abh}$, $\gamma=\frac{16k\alpha E}{D(1-\nu)abh}$ a = 1ft, b = 2 ft, h = 0.5 ft, t = 1sec and k= 0.86 in equations (12) and (16) one obtains

$$\frac{T(x,y,z,t)}{\beta}=\sum_{l=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}(-1)^l\left[(-1)^m-1\right]\left[(-1)^n-1\right]$$

$$\times\left(\frac{1}{\mu_n^3 q_m^3}\right)\cos(r_l z)\,P_n(x)\,\sin(q_m y)$$

$$\times\int_0^1(1-e^{-t'})e^{-k[\mu_n^2+q_m^2+r_l^2](t-t')}dt' \quad (18)$$

$$\frac{\omega(x,y,t)}{\gamma}=\sum_{l=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}(-1)^l\left[(-1)^m-1\right]\left[(-1)^n-1\right]$$

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$$\times\left(\frac{r_l h\sin(r_l h)+\cos(r_l h)-1}{r_l^{2}\mu_n^{3}q_m^{3}(\mu_n^{2}+q_m^{2})}\right)P_n(x)\sin(q_m y)$$
$$\times\int_0^1 (1-e^{-t'})e^{-k[\mu_n^{2}+q_m^{2}+r_l^{2}](t-t')}dt' \qquad (19)$$

VI. CONCLUSION

The temperature distribution and thermal deflection of a thick rectangular plate have been obtained, with the aid of finite Marchi-Fasulo transform; Fourier sine transform and cosine transform techniques when the stated boundary conditions are known. The results are obtained in the form of infinite series. The series solutions converge provided we take sufficient number of terms in the series. The expressions are represented graphically. The temperature distribution and deflection that are obtained can be applied to the design of useful structures or machines in engineering applications.

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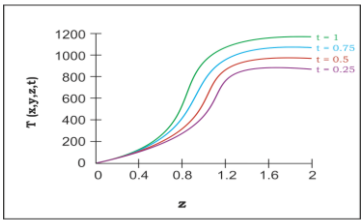

Fig.2 : Graph of temperature distribution versus z

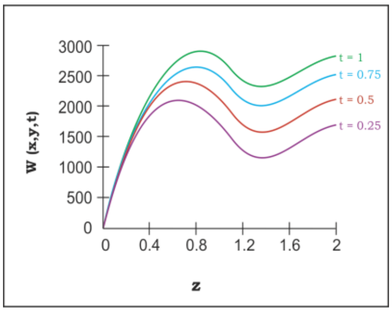

Fig. 3 : Graph of thermal deflection versus z

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