ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 4, Issue 6, December 2014

Thermal Stresses of Two-Dimensional Transient Thermoelastic Problem for A Hallow Cylinder

Varsha K. Dange
Shree . Shivaji Arts Commerce and Science College Rajura

Abstract— ***The present paper deals with the determination of temperature distribution, thermoelastic displacement function and thermal stresses of hallow cylinder occupying the space D: $a \le r \le b,\ 0 \le z \le h,$ with stated boundary conditions. We apply integral transform techniques to find the thermoelastic solution. The results are obtained as series of Bessel's functions. Numerical calculations are carried out for hallow cylinder made of aluminum metal and illustrated graphically.***

Key words: **Thermo elastic problem, hallow cylinder, temperature distribution, thermal stresses, integral transform.**

I. INTRODUCTION

Nowacki [2] has determined steady-state thermal stresses in a thick circular plate subjected to an axisymmetric temperature distribution on the upper face with zero temperature on the lower face and circular edge. **Wankhede [5]** has determined the quasi-static thermal stresses in circular plate subjected to arbitrary initial temperature on the upper face with lower face at zero temperature. However, there are not many investigations on transient state. **Roy, Choudhuri [4]** have succeeded in determining the quasi-static thermal stresses in a circular plate subjected to transient temperature along the circumference of circular upper face with lower face at zero temperature and the fixed circular edge thermally insulated. In a recent work, some problems have been solved by **Noda et al. [3]** and **Deshmukh et al. [1]**. This paper concerned with direct transient thermoelastic problem of hallow cylinder occupying the space D: $a \le r \le b,\ 0 \le z \le h,$ with stated boundary conditions. The direct heat conduction equation is solved by using Marchi – Zgrablich transform and Laplace transform techniques. The temperature distribution, thermoelastic displacement function and thermal stresses are obtained in terms of infinite series of Bessel's functions. Numerical calculations are carried out for hallow cylinder made of aluminum metal.

II. STATEMENT OF THE PROBLEM

Consider the hallow cylinder occupying the space D: $a \le r \le b,\ 0 \le z \le h,$ where a and b are the internal and external radii respectively and (r,z,t) are cylindrical coordinates. The hallow cylinder subjected to third kind boundary conditions are maintained at $g(r,t)$ on the lower surface $z=0$, $u(r,t)$ on the upper surface at $z=h$ and zero at inner and outer curved surface of the hallow cylinder. Under these more realistic prescribed conditions the temperature distribution, unknown interior temperature $f(r,t)$ within the region $a \le r \le b$, thermoelastic displacement function $U(r,z,t)$ and thermal stresses are required to be determined.

The differential equation governing the displacement function $U(r,z,t)$ is

$$\frac{\partial^2 U}{\partial r^2} + \frac{1}{r}\frac{\partial U}{\partial r} = (1+\nu)\, a_t T \tag{1}$$

With $U = 0$ at $r = a$ and $r = b$ (2)

Where ν and a_t are Poison's ratio and the linear coefficient of thermal expansion of the material of the hallow cylinder respectively.

The equation for $T(r,z,t)$, the temperature, in cylindrical coordinates, is:

$$\kappa\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial z^2}\right] = \frac{\partial T}{\partial t} \tag{3}$$

Subject to the initial and boundary conditions

$$M_t(T,1,\ 0,0) = 0 \qquad \text{For all } a \le r \le b\ , \tag{4}$$

$$M_r(T,1,\ k_1,a) = 0\ , \qquad M_r(T,1,\ k_2,b) = 0 \qquad \text{for all } 0 \le z \le h\ ,\ t>0 \tag{5}$$

$$M_z(T,1,c,0) = g(r,t), \quad \text{for all } a \le r \le b\ ,\ t>0 \tag{6}$$

$$M_z(T,1,c,h) = u(r,t), \quad \text{for all } a \le r \le b\ ,\ t>0 \tag{7}$$

$$M_z(T,1,c,\xi) = f(r,t), \quad \text{for all } a \le r \le b\ ,\ t>0 \tag{8}$$

The most general expression for these conditions can be given by

$$M_\nu(f,k,\bar{\bar{k}},\$) = (\bar{k}f + \bar{\bar{k}}\hat{f})_{\nu=\$}$$

Where the prime (^) denotes differentiation with respect to ν , k_1 k_2 and c are radiation constants on the curved surfaces and upper and lower surface of the hallow cylinder respectively.

The stress function σ_{rr} and $\sigma_{\theta\theta}$ are given by

$$\sigma_{rr} = -2\mu\frac{1}{r}\frac{\partial U}{\partial r} \tag{9}$$

$$\sigma_{\theta\theta} = -2\mu\frac{\partial^2 U}{\partial r^2} \tag{10}$$

Where μ is the Lame's constant, while each of the stress functions σ_{rz} , σ_{zz} and $\sigma_{\theta z}$ are zero within the hallow

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 4, Issue 6, December 2014

cylinder in the plane state of stress. The equations (1) to (10) constitute the mathematical formulation of the problem under consideration.

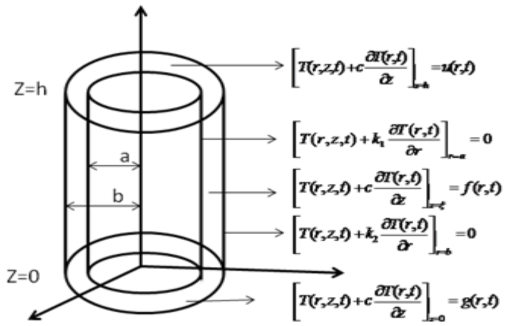

Fig The geometry of the problem

III. SOLUTION OF THE PROBLEM

Results Required

First introduce the integral transform of order n over the variable r. Let n be the parameter of the transform, then the integral transform and its inversion theorem are written as

$$\bar{g}(n)=\int_a^b r\, g(r)\, S_p(k_1,k_2,\mu_n r)\,dr, \qquad (11)$$

$$g(r)=\sum_{n=1}^{\infty}(\bar{g}_p(n)/C_n)\,S_p(k_1,k_2,\mu_n r) \qquad (12)$$

Where $\bar{g}_p(n)$ is the transform of $g(r)$ with respect to nucleus $S_p(k_1,k_2,\mu_n r)$ which is defined as.

$$S_p(k_1,k_2,\mu_n r)=J_p(\mu_n r)[Y_p(k_1,\mu_n a)+Y_p(k_2,\mu_n b)] - Y_p(\mu_n r)[J_p(k_1,\mu_n a)+J_p(k_2,\mu_n b)]$$

$$\left.\begin{aligned} J_p(k_i,\mu_n r)&=J_p(\mu_n r)+k_i\,\mu_n\,J_p'(\mu_n r)\\ Y_p(k_i,\mu_n r)&=Y_p(\mu_n r)+k_i\,\mu_n\,Y_p'(\mu_n r)\end{aligned}\right\} \quad \text{for} \quad i=1,2$$

and

$$C_n=\int_a^b r[S_p(k_1,k_2,\,\mu_n b)]^2\,dr \qquad (13)$$

in which $J_p(\mu_n r)$ and $Y_p(\mu_n r)$ are Bessel functions of first and second kind of order p respectively.

Determination Temperature Function $T(r,z,t)$ **:**

By applying finite Marchi – Zgrablich transform as defined in (11) to the equations (3) and (4) (6) and (7) and using (5) to reduce differential equation in Marchi – Zgrablich transform domain and then applying laplace transform and making use of respective inversion as in (12) and Sneddon over the heat conduction equation one obtains the expression for temperature distribution function as

$$T(r,z,t)=\sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\frac{k}{c^2}\sum_{m=1}^{\infty}\frac{\left[\sinh\frac{z}{c}-\cosh\frac{z}{c}\right]-\left[\sinh(\frac{z-h}{c})-\cosh(\frac{z-h}{c})\right]}{\sinh(h/c)}\right.$$

$$\int_0^t \bar{u}(n,t')\,e^{-k\left[\frac{1-c^2\mu_n^2}{c^2}\right](t-t')}dt-\frac{2k\pi}{h^2}\sum_{m=1}^{\infty}\left[\frac{m}{\cos m\pi}\right]$$

$$\times\frac{\left[\sin\left(\frac{m\pi}{h}\right)z-\left(\frac{cm\pi}{h}\right)\cos\left(\frac{m\pi}{h}\right)z\right]+\left[\sin\left(\frac{m\pi}{h}\right)(z-h)-\left(\frac{cm\pi}{h}\right)\cos\left(\frac{m\pi}{h}\right)(z-h)\right]}{\left[1+(cm\pi/h)^2\right]}$$

$$\left.\int_0^t \bar{g}(n,t')e^{-k\left[\mu_n^2+\left(\frac{m\pi}{h}\right)^2\right](t-t')}dt'\right\}S_0(k_1,k_2,\mu_n r) \qquad (14)$$

Determination Unknown Interior Temperature f (r, t):

Substituting the value of $T(r,z,t)$ from equation (14) to equation (8) one obtains the unknown temperature f(r,t) as,

$$f(r,t)=\sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\frac{k}{c^2}\sum_{m=1}^{\infty}\frac{\left[\sinh\frac{\xi}{c}-\cosh\frac{\xi}{c}\right]-\left[\sinh(\frac{\xi-h}{c})-\cosh(\frac{\xi-h}{c})\right]}{\sinh(h/c)}\right.$$

$$\int_0^t \bar{u}(n,t')\,e^{-k\left[\frac{1-c^2\mu_n^2}{c^2}\right](t-t')}dt-\frac{2k\pi}{h^2}\sum_{m=1}^{\infty}\left[\frac{m}{\cos m\pi}\right]$$

$$\times\frac{\left[\sin\left(\frac{m\pi}{h}\right)\xi-\left(\frac{cm\pi}{h}\right)\cos\left(\frac{m\pi}{h}\right)\xi\right]+\left[\sin\left(\frac{m\pi}{h}\right)(\xi-h)-\left(\frac{cm\pi}{h}\right)\cos\left(\frac{m\pi}{h}\right)(\xi-h)\right]}{\left[1+(cm\pi/h)^2\right]}$$

$$\left.\int_0^t \bar{g}(n,t')e^{-k\left[\mu_n^2+\left(\frac{m\pi}{h}\right)^2\right](t-t')}dt'\right\}S_0(k_1,k_2,\mu_n r) \qquad (15)$$

Determination of Thermoelastic Displacement Function $\cup(r,z,t)$ ***as:***

Substituting the value of $T(r,z,t)$ from equation (14) to equation (1) one obtains the thermoelastic displacement function $\cup(r,z,t)$ as.

$$U(r,z,t)=\alpha_t(1+\nu)\sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\frac{k}{c^2}\sum_{m=1}^{\infty}\frac{\left[\sinh\frac{z}{c}-\cosh\frac{z}{c}\right]-\left[\sinh(\frac{z-h}{c})-\cosh(\frac{z-h}{c})\right]}{\mu_n^2\sinh(h/c)}\right.$$

$$\int_0^t \bar{u}(n,t')\,e^{-k\left[\frac{1-c^2\mu_n^2}{c^2}\right](t-t')}dt-\frac{2\pi}{h^2}\sum_{m=1}^{\infty}\left[\frac{m}{\cos m\pi}\right]$$

$$\times\frac{\left[\sin\left(\frac{m\pi}{h}\right)z-\left(\frac{cm\pi}{h}\right)\cos\left(\frac{m\pi}{h}\right)z\right]+\left[\sin\left(\frac{m\pi}{h}\right)(z-h)-\left(\frac{cm\pi}{h}\right)\cos\left(\frac{m\pi}{h}\right)(z-h)\right]}{\left[1+(cm\pi/h)^2\right]\mu_n^2}$$

$$\left.\int_0^t \bar{g}(n,t')e^{-k\left[\mu_n^2+\left(\frac{m\pi}{h}\right)^2\right](t-t')}dt'\right\}S_0(k_1,k_2,\mu_n r) \qquad (16)$$

Determination of Stress Functions σ_{rr} ***and*** $\sigma_{\theta\theta}$ **:**

Using equation (16)) to equation (9) and (10) one obtains the stress function σ_{rr} and $\sigma_{\theta\theta}$ as,

$$\sigma_{rr}=-2\mu k\alpha_t(1+\nu)\sum_{n=1}^{\infty}\frac{1}{rC_n}\left\{\frac{1}{c^2}\sum_{m=1}^{\infty}\frac{\left[\sinh\frac{z}{c}-\cosh\frac{z}{c}\right]-\left[\sinh(\frac{z-h}{c})-\cosh(\frac{z-h}{c})\right]}{\mu_n^2\sinh(h/c)}\right.$$

$$\int_0^t \bar{u}(n,t')\,e^{-k\left[\frac{1-c^2\mu_n^2}{c^2}\right](t-t')}dt-\frac{2\pi}{h^2}\sum_{m=1}^{\infty}\left[\frac{m}{\cos m\pi}\right]$$

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 4, Issue 6, December 2014

$$\times\frac{\left[\sin\left(\frac{m\pi}{h}\right)z-\left(\frac{cm\pi}{h}\right)\cos\left(\frac{m\pi}{h}\right)z\right]+\left[\sin\left(\frac{m\pi}{h}\right)(z-h)-\left(\frac{cm\pi}{h}\right)\cos\left(\frac{m\pi}{h}\right)(z-h)\right]}{\left[1+(cm\pi/h)^2\right]\mu_n^2}$$

$$\int_0^t \bar{g}(n,t')e^{-k\left[\mu_n^2+\left(\frac{m\pi}{h}\right)^2\right](t-t')}dt'\}S_0'(k_1,k_2,\mu_n r) \tag{17}$$

$$\sigma_{\theta\theta}=-2\mu k\alpha_t(1+\nu)\sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\frac{1}{c^2}\sum_{m=1}^{\infty}\frac{\left[\sinh\frac{z}{c}-\cosh\frac{z}{c}\right]-\left[\sinh(\frac{z-h}{c})-\cosh(\frac{z-h}{c})\right]}{\mu_n^2\sinh(h/c)}\right.$$

$$\int_0^t \bar{u}(n,t')\,e^{-k\left[\frac{1-c^2\mu_n^2}{c^2}\right](t-t')}dt-\frac{2\pi}{h^2}\sum_{m=1}^{\infty}\left[\frac{m}{\cos m\pi}\right]$$

$$\times\frac{\left[\sin\left(\frac{m\pi}{h}\right)z-\left(\frac{cm\pi}{h}\right)\cos\left(\frac{m\pi}{h}\right)z\right]+\left[\sin\left(\frac{m\pi}{h}\right)(z-h)-\left(\frac{cm\pi}{h}\right)\cos\left(\frac{m\pi}{h}\right)(z-h)\right]}{\left[1+(cm\pi/h)^2\right]\mu_n^2}$$

$$\int_0^t \bar{g}(n,t')e^{-k\left[\mu_n^2+\left(\frac{m\pi}{h}\right)^2\right](t-t')}dt'\}S_0''(k_1,k_2,\mu_n r) \tag{18}$$

IV. NUMERICAL RESULTS, DISCUSSION AND REMARKS

To interpret the numerical computations we consider material properties of Aluminum metal, which can be commonly used in both wrought and cast forms. The low density of aluminum results in its extensive use in the aerospace industry.

Poisson ratio, υ	0.281
Thermal expansion coefficient, α_t (cm/cm-^{0}C)	25.5×10^{-6}
Thermal diffusivity, κ (cm^2/sec)	0.86
Inner radius, a (cm)	1
Outer radius, b (cm)	2
Height h (cm)	2

Table 1: Material properties and parameters used in this study

In the foregoing analysis are performed by setting the radiation constants, $c=k_1=k_2=1$ and ξ=1.5.

set

$$u(r,t)=(1-e^t)\delta(r)e^h$$

$$g(r,t)=(1-e^t)\delta(r)$$

Equations (14) and (15) becomes

$$T(r,z,t)=\sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\frac{k}{c^2}\sum_{m=1}^{\infty}\frac{\left[\sinh\frac{z}{c}-\cosh\frac{z}{c}\right]-\left[\sinh(\frac{z-h}{c})-\cosh(\frac{z-h}{c})\right]}{\sinh(h/c)}\right.$$

$$e^h\left[\frac{k\left[\frac{1-c^2\mu_n^2}{c^2}\right](1+e^t)-e^{k\left[\frac{1-c^2\mu_n^2}{c^2}\right]}+1}{k\left[\frac{1-c^2\mu_n^2}{c^2}\right]\left[k\left[\frac{1-c^2\mu_n^2}{c^2}\right]+1\right]}\right]-\frac{2k\pi}{h^2}\sum_{m=1}^{\infty}\left[\frac{m}{\cos m\pi}\right]$$

$$\times\frac{\left[\sin\left(\frac{m\pi}{h}\right)z-\left(\frac{cm\pi}{h}\right)\cos\left(\frac{m\pi}{h}\right)z\right]+\left[\sin\left(\frac{m\pi}{h}\right)(z-h)-\left(\frac{cm\pi}{h}\right)\cos\left(\frac{m\pi}{h}\right)(z-h)\right]}{\left[1+(cm\pi/h)^2\right]}$$

$$\left[\frac{k\left[\mu_n^2+\left(\frac{m\pi}{h}\right)^2\right](1+e^t)-e^{k\left[\mu_n^2+\left(\frac{m\pi}{h}\right)^2\right]}+1}{k\left[\mu_n^2+\left(\frac{m\pi}{h}\right)^2\right]\left[k\left[\mu_n^2+\left(\frac{m\pi}{h}\right)^2\right]+1\right]}\right]\}rS_0^2(k_1,k_2,\mu_n r) \tag{19}$$

$$f(r,t)=\sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\frac{k}{c^2}\sum_{m=1}^{\infty}\frac{\left[\sinh\frac{\xi}{c}-\cosh\frac{\xi}{c}\right]-\left[\sinh(\frac{\xi-h}{c})-\cosh(\frac{\xi-h}{c})\right]}{\sinh(h/c)}\right.$$

$$e^h\left[\frac{k\left[\frac{1-c^2\mu_n^2}{c^2}\right](1+e^t)-e^{k\left[\frac{1-c^2\mu_n^2}{c^2}\right]}+1}{k\left[\frac{1-c^2\mu_n^2}{c^2}\right]\left[k\left[\frac{1-c^2\mu_n^2}{c^2}\right]+1\right]}\right]-\frac{2k\pi}{h^2}\sum_{m=1}^{\infty}\left[\frac{m}{\cos m\pi}\right]$$

$$\times\frac{\left[\sin\left(\frac{m\pi}{h}\right)\xi-\left(\frac{cm\pi}{h}\right)\cos\left(\frac{m\pi}{h}\right)\xi\right]+\left[\sin\left(\frac{m\pi}{h}\right)(\xi-h)-\left(\frac{cm\pi}{h}\right)\cos\left(\frac{m\pi}{h}\right)(\xi-h)\right]}{\left[1+(cm\pi/h)^2\right]}$$

$$\left[\frac{k\left[\mu_n^2+\left(\frac{m\pi}{h}\right)^2\right](1+e^t)-e^{k\left[\mu_n^2+\left(\frac{m\pi}{h}\right)^2\right]}+1}{k\left[\mu_n^2+\left(\frac{m\pi}{h}\right)^2\right]\left[k\left[\mu_n^2+\left(\frac{m\pi}{h}\right)^2\right]+1\right]}\right]\}rS_0^2(k_1,k_2,\mu_n r) \tag{20}$$

The derived numerical results from equation (14) , (16) ,(17)and (18) have been illustrated graphically in figures 1 to 5 as follows.

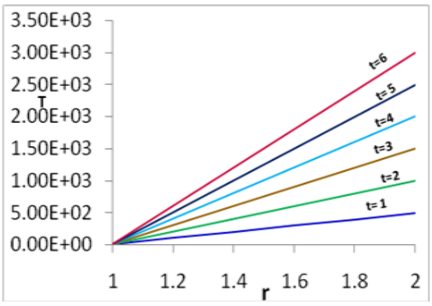

Fig 1: Graph of T versus r for different values of t

Figure 1 represents graph of temperature **T(r,z,t)** versus **r** for different values of **t**. It is observed that that temperature increases uniformly for different values of t .

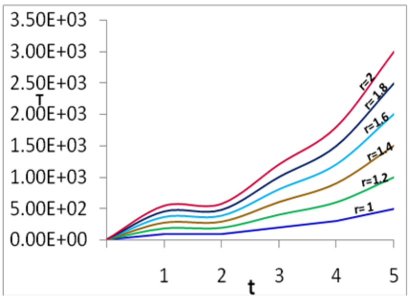

Fig 2: Graph of T(r,z,t) versus t(in seconds) for different values of r

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 4, Issue 6, December 2014

Figure 2: represents graph of temperature **T(r ,z , t)** versus **t** for different values of **r** .It is clear that for fixed t the temperature increases for different values of r .

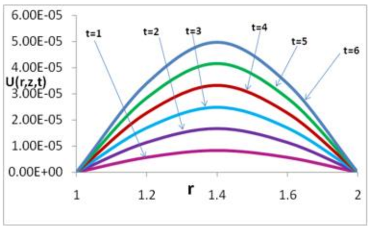

Fig 3 : Graph of $\cup(r,z,t)$ versus r for different values of t

Figure 3: represents graph of $\cup(r,z,t)$ versus **r** for different values of t .We observe that $\cup(r,z,t)$ is zero at both the circular boundaries of the Hallow cylinder at r=1 and r=2.It is clear that $\cup(r,z,t)$ is maximum at r=1.5 of the Hallow cylinder.

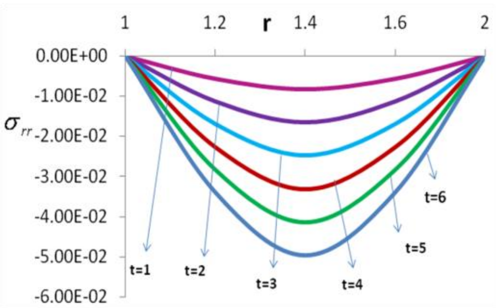

Fig 4: Graph of σ_{rr} versus r for different values of t

Figure 4 represents graph of σ_{rr} versus **r** for different values of **t** .We observe that σ_{rr} is zero at both the circular boundaries of the Hallow cylinder at r=1 and r=2.It is clear that σ_{rr} goes on decreasing to the point r=1.5 then goes on increasing to r=2 of the Hallow cylinder.

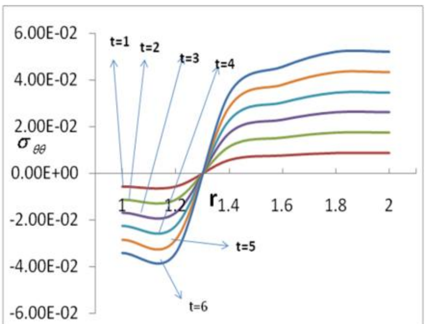

Fig 5: Graph of $\sigma_{\theta\theta}$ versus r for different values of t

Figure 5: It is observed angular stress $\sigma_{\theta\theta}$ develops tensile stress within circular region of the hallow cylinder in the circular region r=1.3to r=2 and compressive stress in circular region r=1 to r=1.3.

V. CONCLUSION

In this study we treated the two-dimensional thermoelastic problem of the hallow cylinder subjected to third kind boundary conditions are maintained at $g(r,t)$ on the lower surface $z=0$, $u(r,t)$ on the upper surface at $z=h$ and zero at inner and outer curved surface of the hallow cylinder. Under these more realistic prescribed conditions the temperature distribution, unknown interior temperature $f(r,t)$ within the region $a \le r \le b$, thermoelastic displacement function $U(r,z,t)$ and thermal stresses have been determined with the help of finite Marchi – Zgrablich transform and Laplace transform techniques . The results are obtained in terms of Bessel's function in the form of infinite series. Moreover, assigning suitable values to the parameters and functions in the equations of temperature, displacements and stresses respectively, expressions of special interest can be derived for any particular case. We may conclude that the system of equations proposed in this study can be adapted to design of useful structures or machines in engineering applications in the determination of thermoelastic behavior and illustrated graphically.

VI. ACKNOWLEDGEMENT

The author is thankful to **prof. Anand Raipure** for kind help in the preparation of this paper.

REFERENCES

[1] Kulkarni, V. S. and Deshmukh, K. C.: Quasi-static transient thermal stresses in a thick annular disc, Sādhanā, 32 (2007) 561–575.

[2] Nowacki, W.: The state of stress in a thick circular plate due to temperature field, Bull. Sci. Acad. Polon Sci. Tech., 5 (1957), 227.

[3] Noda, N; Hetnarski, R. B.and Tanigawa, Y.: Thermal stresses, Second ed. Taylor and Francis, New York (2003), 260.

[4] Roy , Choudhary S. K.: A note on quasi-static thermal deflection of a thin clamped circular plate due to ramp-type heating of a concentric circular region of the upper face, J. of the Franklin Institute, 206 (1973), 213-219.

[5] Wankhede, P. C.: On the quasi-static thermal stresses in a circular plate, Indian J. Pure Appl. Math., Vol. 13 (8) (1982), 1273-1277.