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# Thermoelastic solution of a cylinder with Internal heat source

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***Abstract-*** *In this paper an attempt has been made to determine the temperature distribution and thermal deflection of a cylinder in which sources are generated according to the linear function of the temperature, with known boundary conditions. The results are obtained as series of Bessel functions in the form of infinite series. Numerical calculations are carried out for a particular case of a cylinder made of Aluminium metal and the results are depicted graphically.*

***Keywords and phrases:*** *Transient response, cylinder, temperature distribution, thermal deflection, integral transform*

## I. INTRODUCTION

Nowacki [2] has determined steady-state thermal stresses in a circular plate subjected to an axisymmetric temperature distribution on the upper face with zero temperature on the lower face and the circular edge respectively. Roy Choudhary [5] discussed the normal deflection of a thin clamped circular plate due to ramp type heating of a concentric circular region of the upper face. This satisfies the time-dependent heat conduction equation. In this paper an attempt has been made to determine the temperature distribution and thermal deflection of a cylinder occupying the space 'D' $a \le r \le b;\ 0 \le z \le h$. The cylinder is considered having arbitrary initial temperature and subjected to radiation type boundary conditions which are fixed at $(r = a)$ and $(r = b)$. The non homogeneous type boundary conditions are maintained on plane surfaces of the cylinder. The governing heat conduction equation has been solved by using integral transform technique. The results are obtained in series form in terms of Bessel's functions. The results for thermal deflection have been computed numerically and are illustrated graphically.

## II. STATEMENT OF THE PROBLEM

Consider the cylinder of length $h$ occupying the space '$D$' $a \le r \le b;\ 0 \le z \le h$. The cylinder is considered having arbitrary initial temperature and subjected to radiation type boundary conditions which are fixed at $(r = a)$ and $(r = b)$. The non homogeneous type boundary conditions are maintained at plane surfaces of the disc. For time $t > 0$, heat is generated within the cylinder at the rate $g(r, z, t)$. The differential equation satisfying the deflection function $\omega(r, t)$ as Khobragade [16] is given by

$$\nabla^4 \omega = \frac{\nabla^2 M_T}{D(1-\nu)} \tag{1}$$

where $M_T$ is the thermal moment of the cylinder defined as

$$M_T = a_t E \int_0^h T(r, z, t)\, z\, dz \tag{2}$$

$D$ is the flexural rigidity of the cylinder denoted as

$$D = \frac{Eh^3}{12(1-\nu^2)} \tag{3}$$

$a_t$, $E$ and $\nu$ are the coefficients of the linear thermal expansion, Young's modulus and Poisson's ratio of the material respectively and

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} \tag{4}$$

Since the curved surfaces of the cylinder is fixed and clamped,

$$\omega = \frac{\partial \omega}{\partial r} = 0 \text{ at } r = a, b \tag{5}$$

The temperature distribution of the cylinder $T(r, z, t)$ at time $t$ satisfies the differential equation as Noda [5] is

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} + \frac{g(r, z, t)}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{6}$$

With boundary conditions

$$T + k_1 \frac{\partial T}{\partial r} = G_1(z, t) \quad \text{at} \quad r = a,\ t > 0 \tag{7}$$

$$T + k_2 \frac{\partial T}{\partial r} = G_2(z, t) \quad \text{at} \quad r = b,\ t > 0 \tag{8}$$

$$T = f_1(r, t) \text{ at } z = 0,\ t > 0 \tag{9}$$

$$T = f_2(r, t) \text{ at } z = \xi,\ t > 0 \text{ (known)} \tag{10}$$

$$T = F(r, t) \text{ at } z = h,\ t > 0 \text{ (unknown)} \tag{11}$$

and initial condition is

$$T(r, z, t) = T_0 \text{ in } a \le r \le b;\ 0 \le z \le h \text{ for } t = 0 \tag{12}$$

where $k_1$ and $k_2$ are radiation constants on curved surfaces and plane surfaces of the cylinder respectively and $\alpha$ is thermal diffusivity of the material of the cylinder.

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Equations (1) – (12) constitute mathematical formulation of the problem.

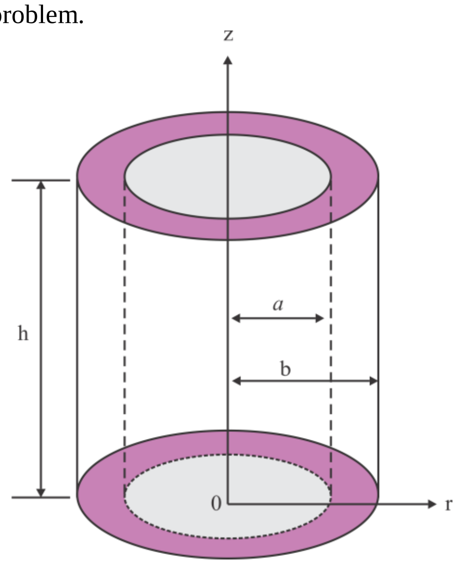


**Fig 1: Geometry of the problem**

## III. SOLUTION OF THE PROBLEM

Applying Marchi-Zgrablich transform defined in [1] to the equation (6) , using conditions (7) and (8) one obtains

$$\frac{d^2\bar{T}}{dz^2}+\frac{\bar{g}}{k}-\mu_m^2\bar{T}+\psi=\frac{1}{\alpha}\frac{d\bar{T}}{dt} \tag{13}$$

Where $\bar{T}$ is the Marchi-Zgrablich transform of $T$ and $m$ is the Marchi-Zgrablich transform parameter.

Where

$$\psi(z,t)=\frac{b}{k_2}S_0(\alpha,\beta,\mu_m b)\,G_2(z,t)-\frac{a}{k_1}S_0(\alpha,\beta,\mu_m a)G_1(z,t) \tag{14}$$

Applying Laplace transform defined in [5] to equation (13) one obtains

$$\frac{d^2\bar{T}^*}{dz^2}-p^2\bar{T}^*=\frac{-\bar{g}^*}{k}-\psi^* \tag{15}$$

where $p^2=\mu_m^2+\dfrac{s}{\alpha}$.

Solution of the differential equation (15) is given by

$$\bar{T}^*=Ae^{pz}+Be^{-pz}+\text{P.I.} \tag{16}$$

where $A$ and $B$ are arbitrary constants.

Using equation (9) and equation (10) in equation (16), We get

$$A+B+\psi(0)=\bar{f}_1^* \tag{17}$$

$$Ae^{p\xi}+Be^{-p\xi}+\psi(\xi)=\bar{f}_2^* \tag{18}$$

where $\psi(0)=P.I.|_{z=0}$ and $\psi(\xi)=P.I.|_{z=\xi}$

Solving (17) and (18) one obtains

$$A=\frac{\bar{f}_2^*-e^{-p\xi}\bar{f}_1^*+e^{-p\xi}\psi(0)-\psi(\xi)}{2\sinh p\xi}$$

$$B=\frac{\bar{f}_1^*e^{p\xi}-\bar{f}_2^*+\psi(\xi)-e^{p\xi}\psi(0)}{2\sinh p\xi}$$

Substituting the values of $A$ and $B$ in equation (16) one obtains

$$\bar{T}^*=[\bar{f}_2^*-\psi(\xi)]\frac{\sinh pz}{\sinh p\xi}-[\bar{f}_1^*-\psi(0)]\frac{\sinh p(z-\xi)}{\sinh p\xi} \tag{19}$$

Applying inversion of Laplace transform and Marchi – Zgrablich transform to the equation (19) and using condition (11), one obtain

$$T(r,z,t)=\frac{2\alpha\pi}{\xi^2}\sum_{m,n=1}^{\infty}\frac{S_0(k_1,k_2,\mu_m r)}{\mu_m}n(-1)^{n+1}e^{-\alpha t\left(\mu_m^2+\frac{m^2\pi^2}{\xi^2}\right)}\times\int_0^t\left\{\sin\left[\frac{n\pi z}{\xi}\right](\bar{f}_2(t-u)-\phi)-\sin\left[\frac{n\pi(z-\xi)}{\xi}\right](\bar{f}_1(t-u)-\psi)\right\}du \tag{20}$$

$$F(r,t)=\frac{2\alpha\pi}{\xi^2}\sum_{m,n=1}^{\infty}\frac{S_0(k_1,k_2,\mu_m r)}{\mu_m}n(-1)^{n+1}e^{-\alpha t\left(\mu_m^2+\frac{m^2\pi^2}{\xi^2}\right)}\int_0^t\left\{\sin\left[\frac{n\pi}{\xi}h\right](\bar{f}_2(t-u)-\phi)-\sin\left[\frac{n\pi}{\xi}(h-\xi)\right](\bar{f}_1(t-u)-\psi)\right\}du \tag{21}$$

## IV. DETERMINATION OF THERMAL DEFLECTION

Using equation (20) in equation (2) one obtains

$$M_T=a_TE\frac{2\alpha\pi}{\xi^2}\int_0^h\sum_{m,n=1}^{\infty}\left\{z\frac{S_0(k_1,k_2,\mu_m r)}{\mu_m}n(-1)^{n+1}e^{-\alpha t\left(\mu_m^2+\frac{m^2\pi^2}{\xi^2}\right)}\int_0^t\left[\sin\left[\frac{n\pi}{\xi}z\right](\bar{f}_2(t-u)-\phi)-\sin\left[\frac{n\pi}{\xi}(z-\xi)\right](\bar{f}_1(t-u)-\psi)\right]du\right\}dz \tag{22}$$

We assume the solution of equation (1) satisfying condition (5) as

$$\omega(r,t)=\sum_{m=1}^{\infty}C_m(t)[S_0(k_1,k_2,\mu_m r)-S_0(k_1,k_2,\mu_m b)] \tag{23}$$

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where $\mu_m$ are the positive roots of the transcendental equation

$$S_0(k_1, k_2, \mu_m a) - S_0(k_1, k_2, \mu_m b) = 0 \qquad (24)$$

It can be easily seen that

$$\omega = \frac{\partial \omega}{\partial r} = 0 \quad \text{at} \quad r = a, b \qquad (25)$$

Hence solution (22) satisfies condition (5).
Now

$$\nabla^4 \omega = \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)^2 \sum_{m=1}^{\infty} C_m [S_0(k_1, k_2, \mu_m r) - S_0(k_1, k_2, \mu_m b)] = 0 \qquad (26)$$

We use the well known result

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right) S_0(k_1, k_2, \mu_m r) = -\mu_m^2\, S_0(k_1, k_2, \mu_m r)] \qquad (27)$$

in equation (26) to obtain

$$\nabla^4 \omega = \sum_{m=1}^{\infty} C_m \mu_m^4\, S_0(k_1, k_2, \mu_m r) \qquad (28)$$

And

$$\nabla^2 M_T = a_T E \frac{2\alpha\pi}{\xi^2} \int_0^h \sum_{m,n=1}^{\infty} \Big\{ z\mu_m S_0(k_1, k_2, \mu_m r)\, n(-1)^{n+2} \times e^{-\alpha t\left(\mu_n^2 + \frac{n^2\pi^2}{\xi^2}\right)} \int_0^t \begin{bmatrix} \sin\left[\frac{n\pi}{\xi} z\right](\bar{f}_2(t-u) - \phi) \\ -\sin\left[\frac{n\pi}{\xi}(z-\xi)\right](\bar{f}_1(t-u) - \psi) \end{bmatrix} du \Big\} dz \qquad (29)$$

Using equation (28) and equation (29) in the equation (1) one obtains

$$\sum_{m=1}^{\infty} C_m \mu_m^4 S_0(k_1, k_2, \mu_m r) = a_T E \frac{2\alpha\pi}{\xi^2 D(1-v)} \times \int_0^h \sum_{m,n=1}^{\infty} \Big\{ z\mu_m S_0(k_1, k_2, \mu_m r)\, n(-1)^{n+2} e^{-\alpha t\left(\mu_n^2 + \frac{n^2\pi^2}{\xi^2}\right)} \times \int_0^t \begin{bmatrix} \sin\left[\frac{n\pi}{\xi} z\right](\bar{f}_2(t-u) - \phi) \\ -\sin\left[\frac{n\pi}{\xi}(z-\xi)\right](\bar{f}_1(t-u) - \psi) \end{bmatrix} du \Big\} dz \qquad (30)$$

On solving equation (30) one obtains

$$C_m(t) = a_T E \frac{2\alpha\pi}{\xi^2 D(1-v)} \times \int_0^h \sum_{n=1}^{\infty} \Big\{ \frac{z}{\mu_m^3} n(-1)^{n+2} e^{-\alpha t\left(\mu_n^2 + \frac{n^2\pi^2}{\xi^2}\right)} \times \int_0^t \begin{bmatrix} \sin\left[\frac{n\pi}{\xi} z\right](\bar{f}_2(t-u) - \phi) \\ -\sin\left[\frac{n\pi}{\xi}(z-\xi)\right](\bar{f}_1(t-u) - \psi) \end{bmatrix} du \Big\} dz \qquad (31)$$

Using equation (31) in equation (23) We get

$$\omega(r,t) = \frac{2\alpha\pi a_t E}{\xi^2 D(1-v)} \int_0^h \sum_{m,n=1}^{\infty} \Big\{ \frac{z}{\mu_m^3} n(-1)^{n+2} e^{-\alpha t\left(\mu_n^2 + \frac{n^2\pi^2}{\xi^2}\right)} \times \int_0^t \begin{bmatrix} \sin\left[\frac{n\pi}{\xi} z\right](\bar{f}_2(t-u) - \phi) \\ -\sin\left[\frac{n\pi}{\xi}(z-\xi)\right](\bar{f}_1(t-u) - \psi) \end{bmatrix} du \Big\} dz \times [S_0(k_1, k_2, \mu_m r) - S_0(k_1, k_2, \mu_m b)] \qquad (32)$$

## V. SPECIAL CASE AND NUMERICAL RESULTS

Setting $f_1(r,t) = \delta(r - r_0) \times (1 - e^{-t})$

$$f_2(r,t) = \delta(r - r_0) \times (1 - e^{-t})\, e^{\xi}, \qquad (33)$$

$a$ =2, b =3 , h =1, $k_1$ = 0.25, $k_2$ =0.25, k =0. 86, $r_0$ =0.75, t = 1 sec. $\xi = 0.5$, $\rho = \frac{2\alpha\pi}{\xi^2}$ in equation (21) one obtains the unknown temperature gradient as

$$\frac{F(r,t)}{\rho} = \sum_{m,n=1}^{\infty} \frac{S_0(0.25, 0.25, \mu_m r)}{\mu_m} n(-1)^{n+1}\, e^{-\alpha\left(\mu_m^2 + (39.44)m^2\right)} \times \int_0^1 \begin{Bmatrix} \sin[(6.28)n](\bar{f}_2(t-u) - \phi) \\ -\sin[(3.14)n](\bar{f}_1(t-u) - \psi) \end{Bmatrix} du \qquad (34)$$

## VI. CONCLUSION

In this paper, the temperature distribution and thermal deflection of a finite length hollow cylinder have been determined in series form in terms of Bessel's functions by applying finite Marchi Zgrablich transform and Laplace transform techniques. The researchers have plotted the graphs taking the material properties of aluminium, and the numerical computation has been inferred accordingly.

Graph 1. In this graph the temperature distribution T(r,z,t) tends to decrease along the radius between 1.5 to 3, 3 to 4.5 and 4.5 to 5.5, which shows a reduction in the rate of heat propagation in a sinusoidal form; while it tends to increase with heating time from t=0.5 to t=3.

Graph 2. In this graph the temperature distribution T(r,z,t) alternates at different points of radius.

Graph 3. The thermal deflection W(r,t) decreases at different intervals of radius, and tends to decrease with heating time from t=0.5 to t=3. The graph shows a sinusoidal nature.

Graph 4. The thermal deflection W(r,t) alternates at different points of radius.

Graph 5. In this graph the unknown temperature gradient F(r,t) tends to decrease along the radius between 1.5 to 3, 3 to 4.5 and 4.5 to 5.5, which shows a reduction in the rate of heat propagation in a sinusoidal form; while it tends to increase with heating time from t=0.5 to t=3.

Graph 6. (F(r,t) versus t for different values of r): In this graph the unknown temperature gradient F(r,t) alternates at different points of radius.

The results presented here may be useful in solving engineering problems, particularly for aerospace engineering for stations of a missile body not influenced by nose tapering.

## VII. ACKNOWLEDGEMENT

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## REFERENCES

[1] Marchi, E. and Zgrablich, A.: Heat conduction in sector of hollow cylinder with radiation, Atti, della Acc. Sci. di. Torino, 1 (1967), 373-382.

[2] Nowacki, W.: The state of stress in thick circular plate due to temperature field. Ball. Sci. Acad. Polon Sci. Tech. 5 (1957), 227.

[3] Roy Choudhary, S.K.: A note on quasi-static thermal deflection of a thin clamped circular plate due to ramp-type heating on a concentric circular region of the upper face, J. of the Franklin. Institute, 206 (1973), 213-219.

[4] Sneddon, I. N: The use of integral transform, Mc Graw Hill book co. (1974), chap.3.

[5] Noda, N; Hetnarski, R.B; Tanigawa, y: Thermal Stresses, second edition Taylor & Francis, New York (2003), 260.

[6] Gahane, T.T, Varghese, V. and Khobragade, N. W (2009): "Transient Thermoelastic Problem of a cylinder with heat sources", Int. J Latest Trend Math, Vol. 2 No. 1, pp. 25-36, 2012, UK.

[7] Raut, G.N., Varghese, V. and Khobragade, N. W. (2009): "On the plane strain and plane stress solutions of uniformly heated functionally graded solid cylinder or disc problems", Advances in Math. Sci. and Appl., vol. 19, No.1, 403-413, Japan.

[8] Kamdi, D. B, Khobragade, N. W, and Durge, M. H (2009): "Transient Thermoelastic Problem for a Circular Solid Cylinder with Radiation", Int. Journal of Pure and Applied Maths, vol. 54, No. 3, 387-406, Academic Publication.

[9] Kulkarni, Ashwini A and Khobragade, N. W (2012): "Thermal stresses of a finite length hollow cylinder", Canadian Journal on Science and Engg. Mathematics Research, Vol. 3 No. 2, pp. 52-55, Canada.

[10] Warbhe, M. S and Khobragade, N.W (2012): "Numerical Study Of Transient Thermoelastic Problem Of A Finite Length Hollow Cylinder", Int. Journal of Latest Trends in Maths, Vol. 2, No. 1, pp. 4-9, UK.

[11] Lamba, N.K; and Khobragade, N.W (2011): "Analysis of Coupled Thermal Stresses in an Axisymmetric Hollow Cylinder", Int. Journal of Latest Trends in Maths, Vol. 1, No. 2, UK.

[12] Lamba, N.K, Walde, R. T and Khobragade, N.W (2012): "Stress functions in a hollow cylinder under heating and cooling processes", Journal of Statistics and Mathematics. Vol. 3, Issue 3, pp. 118-124, BIO INFO Publication (Impact Factor 4.47).

[13] Gahane, T. T and Khobragade, N.W (2012): "Transient Thermoelastic Problem Of A Semi-infinite Cylinder With Heat Sources", Journal of Statistics and Mathematics, Vol. 3, Issue 2, pp. 87-93, BIO INFO Publication. (Impact Factor 4.47).

[14] Hiranwar Payal C and Khobragade, N.W (2012): "Thermoelastic Problem Of A Cylinder With Internal Heat Sources", Journal of Statistics and Mathematics, Vol. 3, Issue 2, pp. 87-93, BIO INFO Publication. (Impact Factor 4.47).

[15] Dange, W. K; Khobragade, N.W, and Durge, M. H (2012): "Thermal Stresses of a finite length hollow cylinder due to heat generation", Int. J. of Pure and Appl. Maths, (accepted).

[16] N. W. Khobragade, Lalsingh Khalsa and Mrs Ashwini Kulkarni: Thermal Deflection of a Finite Length Hollow Cylinder due to Heat Generation, Int. J. of Engg. And Information Technology, vol. 3, Issue 1, pp. 372-375, (2013).

[17] Anjali C. Pathak, Payal Hiranwar, Lalsingh Khalsa and N. W. Khobragade (2013): Thermoelastic Problem Of A Semi Infinite Cylinder With Internal Heat Sources, Int. J. of Engg. and Information Technology, vol. 3, Issue 1, USA (Impact Factor 1.895).

[18] R.T. Walde, Anjali C. Pathak and N.W. Khobragade (2013): Thermal Stresses of a Solid Cylinder With Internal Heat Source, Int. J. of Engg. and Information Technology, vol. 3, Issue 1, pp. 407-410, USA (Impact Factor 1.895).

[19] Jadhav, C.M; and Khobragade, N.W (2013): "An Inverse Thermoelastic Problem of finite length thick hollow cylinder with internal heat sources", Advances in Applied Science Research, 4(3); 302-314.

[20] Khobragade, N.W (2013): Thermal stresses of a hollow cylinder with radiation type conditions, Int. J. of Engg. And Information Technology, vol. 3, Issue 5, pp.25-32, USA (Impact Factor 1.895).

[21] Khobragade, N.W (2013): Thermoelastic analysis of a solid circular cylinder, Int. J. of Engg. and Information

Technology, vol. 3, Issue 5, pp. 155-162, USA (Impact Factor 1.895).

[22] Khobragade, N.W (2013): Thermoelastic analysis of a thick hollow cylinder with radiation conditions, Int. J. of Engg. And Information Technology, vol. 3, Issue 4, pp.380-387, USA (Impact Factor 1.895).

## APPENDIX

The finite Marchi-Zgrablich integral transform is defined as

$$\bar{f}_p(n) = \int_a^b x f(x) \cdot S_p(k_1, k_2, \mu_n x)\, dx \qquad (35)$$

and its inversion is given by

$$f(x) = \sum_{n=1}^{\infty} \frac{\bar{f}_p(n)\, S_p(k_1, k_2, \mu_n x)}{C_n} \qquad (36)$$

where

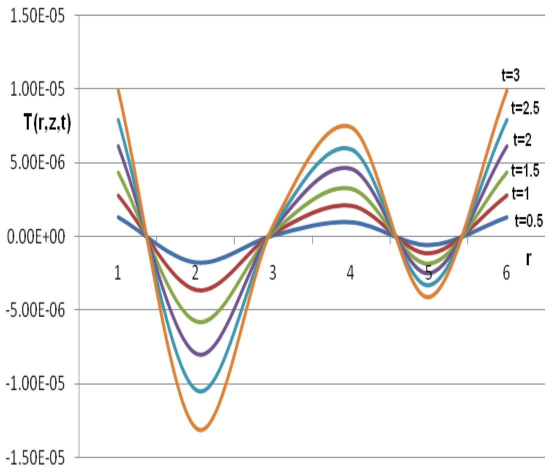
$$C_n = \frac{b^2}{2}\{S_p^2(k_1, k_2, \mu_n b) - S_{p-1}(k_1, k_2, \mu_n b) \cdot S_{p+1}(k_1, k_2, \mu_n b)\}$$

$$-\frac{a^2}{2}\{S_p^2(k_1, k_2, \mu_n a) - S_{p-1}(k_1, k_2, \mu_n a) \cdot S_{p+1}(k_1, k_2, \mu_n a)$$

$$S_p(k_1, k_2, \mu_n x) = J_p(\mu_n x)\{Y_p(k_1, \mu_n a) + Y_p(k_2, \mu_n b)\} - Y_p(\mu_n x)\{J_p(k_1, \mu_n a) + J_p(k_2, \mu_n b)\}$$

where $J_p(\mu x)$ and $Y_p(\mu x)$ are Bessel's functions of first and second kind respectively of order $p$.

Graph 1: T(r,z,t) versus t for different values of t

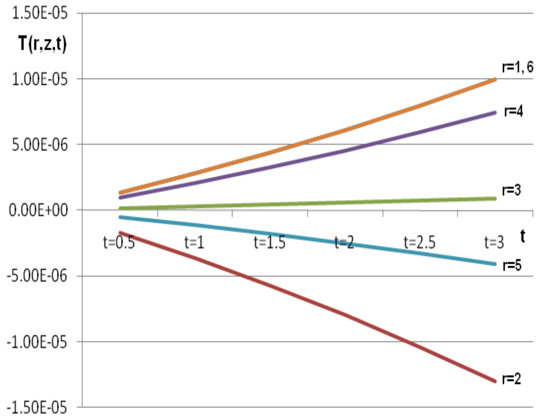



OPERATIONAL PROPERTY:

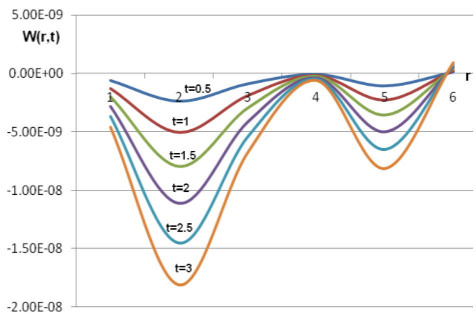
$$\int_a^b x\left\{\frac{\partial^2 f}{\partial x^2} + \frac{1}{x}\frac{\partial f}{\partial x} - \frac{p^2}{x^2} f\right\} S_p(k_1, k_2, \mu_n x) dx =$$

$$\frac{b}{k_2} S_p(k_1, k_2, \mu_n b)\left\{f + k_2 \frac{\partial f}{\partial x}\right\}_{x=b}$$

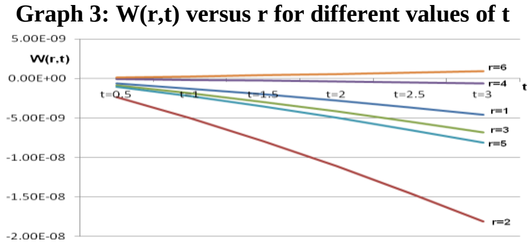
$$-\frac{a}{k_1} S_p(k_1, k_2, \mu_n a)\left\{f + k_1 \frac{\partial f}{\partial x}\right\}_{x=a} - \mu_n^2\ \bar{f}_p(n)$$



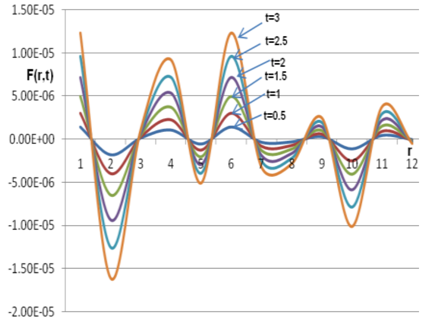

Graph 2: T(r,z,t) versus t for different values of r



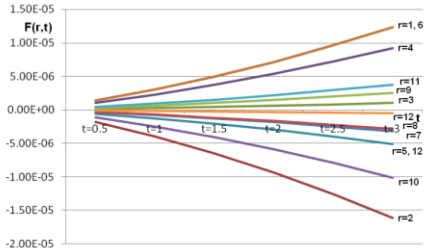

Graph 3: W(r,t) versus r for different values of t




Graph 4: W(r,t) versus t for different values of r




**Graph 5: F(r,t) versus r for different values of t**




**Graph 6: F(r,t) versus t for different values of r**

## AUTHOR BIOGRAPHY

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