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# Thermoelastic problem of a thin circular plate due to Heat generation

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***ABSTRACT- This Paper is concerned with transient thermoelastic problem of a thin circular plate occupying the space $D: 0 \le r \le a,\ -h \le z \le h$, with radiation type boundary conditions. In this paper an attempt has been made to determine the temperature distribution, displacement function and thermal stresses of a thin circular plate with the help of integral transform techniques. The result are obtained in terms Bessel's function in the form of infinite series and depicted graphically.***

***Key words*-Thermoelastic Response, thin Circular Plate, Thermal Stresses, integral transform.**

## I. INTRODUCTION

The direct and inverse problem of thermo elasticity of thin circular plate have been considered by Nowacki. W [3]. Roy Choudhari [6] has succeeded in determining the quasi-static thermal stresses in a circular plate subjected to transient temperature along the circumference of circular upper face with lower face at zero temperature and the fixed circular edge thermally insulated. Khobragade et al. [7-16] have studied the thermoelastic behavior of a thick and circular plate. In all aforementioned investigations they have not considered any thermoelastic problems with boundary conditions of radiation type. This Paper is concerned with transient thermoelastic problem of a thin circular plate occupying the space $D: 0 \le r \le a,\ -h \le z \le h$, with radiation type boundary conditions.

## II. STATEMENT OF THE PROBLEM

Consider a thin circular plate of thickness 2h, occupying the space $D: 0 \le r \le a,\ -h \le z \le h$, the material is homogeneous and isotropic. The differential equation governing the displacement function $U(r,z,t)$ as Ozisik [5] is

$$\frac{\partial^2 U}{\partial r^2} + \frac{1}{r}\frac{\partial U}{\partial r} = (1+\upsilon)a_t T \qquad (1)$$

where $\upsilon$ and $a_t$ are Poison's ratio and the linear coefficient of the material of the plate and T is the temperature distribution of the plate satisfying the differential equation as Noda et al.[4] is

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} + \frac{\theta(r,z,t)}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad (2)$$

Subject to the initial condition

$M_t(T,1,0,0) = F(r,z)$ for all

$$0 \le r \le a,\ -h \le z \le h \qquad (3)$$

The boundary conditions are

$$M_r(T,1,0,a) = 0 \text{ for all } -h \le z \le h,\ t > 0 \qquad (4)$$

$$M_z(T,1,k_1,h) = f_1(r,t) \qquad (5)$$

$$M_z(T,1,k_2,-h) = \left(-\frac{\theta_0}{\lambda}\right) f_2(r,t) \text{ for all } 0 \le r \le a, t > o \qquad (6)$$

The most general expression for these condition can be given by

$$M_v(f,k,\bar{\bar{k}},\$) = (\bar{k} f + \bar{\bar{k}}\hat{f})_{v=\$}$$

where the prime (^) denotes the differentiation with respect to v.

The stress function $\sigma_{rr}$ and $\sigma_{\theta\theta}$ as Ozisik [5] are given by

$$\sigma_{rr} = -2\mu \frac{1}{r}\frac{\partial U}{\partial r} \qquad (7)$$

$$\sigma_{\theta\theta} = -2\mu \frac{\partial^2 U}{\partial r^2} \qquad (8)$$

Where $\mu$ is the Lame's constant.

The stress functions $\sigma_{r\theta}, \sigma_{\theta z}$ and $\sigma_{zz}$ are zero within the plate in the plane state of stress. The equation (1) to (8) constitute the mathematical formulation of the problem under consideration.

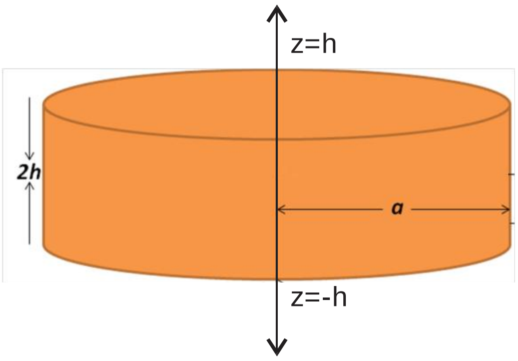


**Fig. 1: The geometry of the problem**

## III. SOLUTION OF THE PROBLEM

Applying the finite Hankel transform defined in [5] and finite Marchi-Fasulo transform defined in [2] to the equation (2), using boundary conditions (3) to (6) and then taking inversion of them, we get

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$$T(r,z,t)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left(\frac{q}{p^2}+\left(\overline{F}^*-\frac{q}{p^2}\right)e^{-p^2t}\right)\frac{P_m(z)}{\lambda_m}f_0(\xi_n,r) \quad (9)$$

Where $f_0(\xi_n,r)=-\frac{\sqrt{2}}{b}\left(\frac{J_0(\xi_n r)}{\xi_n J_0(\xi_n b)}\right)$,

$$\lambda_m=\int_{-h}^{h}P_m^{\,2}(z)\,dz=h\left[Q_m^{\,2}+W_m^{\,2}\right]+\frac{\sin(2a_m h)}{2a_m}\left[Q_m^{\,2}-W_m^{\,2}\right]$$

The Eigen values $a_m$ are the solutions of the equation

$$\left[\alpha_1 a\cos(ah)+\beta_1\sin(ah)\right]\times\left[\beta_2\cos(ah)+\alpha_2 a\sin(ah)\right]$$
$$=$$
$$\left[\alpha_2 a\cos(ah)-\beta_2\sin(ah)\right]\times\left[\beta_1\cos(ah)-\alpha_1 a\sin(ah)\right]$$

$\alpha_1$, $\alpha_2$, $\beta_1$ and $\beta_2$ are constants.

$$P_m(z)=Q_m\cos(a_m z)-W_m\sin(a_m z)$$
$$Q_m=a_m(\alpha_1+\alpha_2)\cos(a_m h)+(\beta_1-\beta_2)\sin(a_m h)$$
$$W_m=(\beta_1+\beta_2)\cos(a_m h)+(\alpha_2-\alpha_1)a_m\sin(a_m h)$$

Equation (9) is the desired solution of the given problem with $\beta_1=\beta_2=1$ and $\alpha_1=k_1$, $\alpha_2=k_2$.

## IV. THERMOELASTIC DISPLACEMENT FUNCTION

Substituting value of temperature distribution $T(r,z,t)$ from equation (9) in equation (1), one obtains the displacement function $U(r,z,t)$ as

$$U(r,z,t)=-(1+\upsilon)a_t\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left(\frac{q}{p^2}+\left(\overline{F}^*-\frac{q}{p^2}\right)e^{-p^2t}\right)\times\frac{P_m(z)}{\lambda_m}f_0(\xi_n,r) \quad (10)$$

## V. STRESS FUNCTIONS

Substituting the value of thermoelastic displacement function $U(r,z,t)$ from equation (10) in equations (7) and (8) one obtain the stress functions as

$$\sigma_{rr}=(1+\upsilon)a_t\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left(\frac{q}{p^2}+\left(\overline{F}^*-\frac{q}{p^2}\right)e^{-p^2t}\right)\frac{P_m(z)}{\lambda_m}\times\frac{2\mu}{r}\left[\frac{\sqrt{2}}{a}\frac{J_1(\xi_n r)}{J_1(\xi_n a)}\right] \quad (11)$$

$$\sigma_{\theta\theta}=(1+\upsilon)a_t\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left(\frac{q}{p^2}+\left(\overline{F}^*-\frac{q}{p^2}\right)e^{-p^2t}\right)\frac{P_m(z)}{\lambda_m}\times\xi_n\left[\frac{J_0(\xi_n r)}{J_1(\xi_n a)}-\frac{J_1(\xi_n r)}{(\xi_n r)J_1(\xi_n a)}\right] \quad (12)$$

## VI. SPECIAL CASE AND NUMERICAL RESULTS

Setting $F(r,z)=\delta(r-r_0)\times(z-h)^2\times(z+h)^2$ (13)

Applying finite Marchi-Fasulo transform and Hankel transform to the equation (13) we get

$$\overline{F}^*=3(k_3+k_4)J_0(\xi_n r_0)\times\left[\frac{a_m h\cos^2(a_m h)-\cos(a_m h)\sin(a_m h)}{a_m^{\,2}}\right] \quad (14)$$

Set

$k_3=0.2, k_4=0.2, h=0.25\,ft, \pi=3.14, a=2\,ft, r_0=1\,ft,$

$t=1\sec$, $k=0.86$

Using the equation (14) and the above values in equation (9), one obtains

$$T=\sum_{n=1}^{\infty}\frac{1}{\xi_n}J_0(\xi_n r)\left[\sum_{m=1}^{\infty}\frac{1}{\lambda_m}P_m(z)\left(\overline{F}^*+\int_0^1\overline{\Psi}^*\,e^{\alpha p^2t'}dt'\right)\right] \quad (15)$$

## VII. CONCLUSION

In this paper, the temperature distribution, displacement function and thermal stresses of a thin circular plate have been investigated. The expressions are obtained in terms of Bessel's function in the form of infinite series. The finite Hankel transform and finite Marchi-Fasulo transform techniques have been used to obtain the solution of the problem. The expressions are represented graphically. Any particular case of special interest can be derived by assigning suitable values to the parameters and functions in engineering applications.

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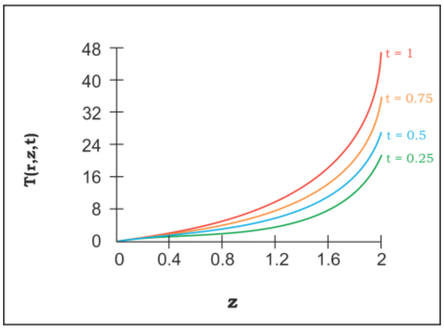

**Fig. 1 : Graph of temperature distribution vs. z**

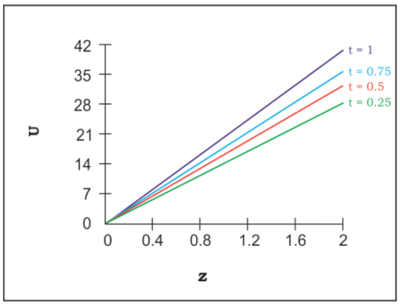

**Fig. 2 : Graph of displacement function vs. z**

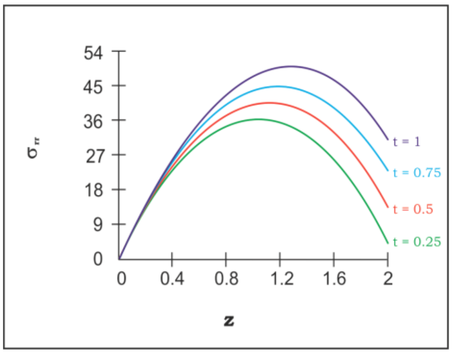

**Fig. 3: Graph of radial stress vs. z**

## AUTHOR BIOGRAPHY

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