ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 4, Issue 4, October 2014

Fatigue Failure of an Annular Plate Under the Action of Pulsating Moment and Pressure along the internal contour

LatifKh. T., Nagiyeva N. M.
National Academy of Sciences of Azerbaijan, Institute of Mathematics and Mechanics
Baku, AZ 1143, str. B. Vahabzadeh 9

Abstract: - We solve a problem on fatigue failure of an annular elastic plastic plate in the case when internal contour is loaded with pulsating moment and pressure. The damage condition that determines the amount of loadings before appearance of first damages and the cyclic strength condition determining the number of loadings to fatigue failure are used. Intensity of residual deformations was accepted as a determining parameter of fatigue failure. The relations of plastic flow theory were used at basic deformation. Possibility of appearance of secondary plastic deformations was taken into account at unloading.

***Keywords* annular plate, pulsating moments and pressures; ideal plasticity; secondary plastic deformations; damage time; failure time.**

I. INTRODUCTION

We consider an annular plate loaded by a pulsating moment and pressure along the internal contour. It is assumed that at the deformation a part of a plate goes into the plastic state. The material's properties are described by the equations of plastic flow theory [1], the material's strengthening is excluded. After some number of loading cycles in the plane there happens fatigue failure (disturbance of continuity) the goal of the paper is to determine the number of pulsatingacting loadings preceding the appearance in the plate the first damages and the number of loadings to failure. The intensity of residual deformations is accepted as a determining parameter of fatigue failure. The unloading process is studied with regard to appearance of secondary plastic deformations [3]. In order to determine the intensity of residual deformations the solution of the elastico-plastic problem is used at basic loading of the plate from the natural state that was obtained by Nordgren and Naghdy [2].

II. STATEMENT AND SOLUTION OF THE PROBLEM

Within the plastic flow theory [1] we give the statement of a mathematical problem on deformation from the natural (unreformed) state of an annular plate with internal radius a, the external radius b at loading along the internal contour by the pressure $p(t)$ and distributed moment$M(t)$, where t is the time : $0 \le t \le t_*$, t_* is the time to fatigue failure. According to [1]. The equation of plastic flow theory accept in the form

$$\dot{\varepsilon}_{ij}^{p} = \lambda \frac{\partial F}{\partial \sigma_{ij}} \quad . (1)$$

Here $\dot{\varepsilon}_{ij}^{p}$ are velocities of plastic deformations ε_{ij}^{p} $\lambda$is the loading parameter: σ_{ij} are current stresses,F is the loading function. We assume that the plate's material has no strengthening. Therefore, in the plastic area we have $F = 0, \quad \lambda \ge 0$ [2].

To relations (1) we should add the relations for elastic components of deformations

$$\varepsilon_{ij}^{e} = \frac{1+v}{E}\left(\sigma_{ij} - \frac{3v}{1+v}\sigma\delta_{ij}\right), (2)$$

whereϑ is the Possion ratio,E is the modulus of longitudinal elasticity: $\delta_{ij}$are the Kronecker symbols, $\sigma = \sigma_{ij}\delta_{ij}/3$ is the mean stress

We shall use the cylindrical system of coordinates (r, θ, z), whose origin coincides with the center of the annular plate.

Therewith$a \le r \le b; \quad 0 \le \theta \le 2\pi$.

The stress components $\sigma_r, \sigma_\theta, \sigma_{r\theta}$ should satisfy the following equilibrium equations

$$\frac{\partial \sigma_r}{\partial r} = \frac{\sigma_\theta - \sigma_r}{r} \quad (3)$$

$$\frac{\partial \sigma_{r\theta}}{\partial r} - \frac{2}{r}\sigma_{r\theta} = 0 \quad (4)$$

Between the components of displacements $u_r, u_\theta$and $\varepsilon_r, \varepsilon_\theta, \varepsilon_{r\theta}$deformations it holds the Cauchy kinematic relation

$$\varepsilon_r = \frac{\partial u_r}{\partial r}; \varepsilon_\theta = \frac{u_r}{r}, \quad \varepsilon_{r\theta} = \frac{1}{2}\left(\frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}\right). \quad (5)$$

The following boundary conditions should be satisfied:

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 4, Issue 4, October 2014

$$\sigma_r|_{r=a} = -p(t); \sigma_r|_{r=b} = 0 u_\theta|_{r=b} = 0$$

, (6)

$$\int_0^{2\pi} \sigma_{r\theta} r^2 d\theta = M(t). \quad (7)$$

Furthermore these hold the conjugation conditions u_r and σ_r for $r = r_s$, where r_s is the radius of a circle separating elastic and plastic areas. Elastico-plastic problem (1) - (7) was solvedin the paper of Nordgren and Naghdi [2]. Write this solution:

The loading function Fin chosen in the form:

$$F = \frac{1}{4}(\sigma_r - \sigma_\theta)^2 + \sigma_{r\theta}^2 - \tau_s^2,$$

where τ_s is the yield point at pure shear.

$$\sigma_r = \frac{M(t)}{2\pi r^2}, \text{ for } a \le r \le b \quad (9)$$

In the elastic domain $r_s \le r \le b$ we have:

$$\left.\begin{matrix}\sigma_\theta \\ \sigma_r\end{matrix}\right\} = \frac{\left[r_s^4 - \left(\frac{M(t)}{2\pi\tau_s}\right)^2\right]^{1/2}}{b^2}\left(1 \pm \frac{b^2}{r^2}\right)\tau_s ,$$

(10)

$$\left.\begin{matrix}\varepsilon_\theta \\ \varepsilon_r\end{matrix}\right\} = \frac{\tau_s}{E}\left[r_s^4 - \left(\frac{M(t)}{2\pi\tau_s}\right)^2\right]^{1/2}\left(\frac{1-v}{b^2} \pm \frac{1+v}{r^2}\right),$$

(11)

$$\varepsilon_{r\theta} = \frac{1+v}{E}\frac{M(t)}{2\pi r^2}, \quad (12)$$

$$u_r = \frac{\tau_s}{E}\left[r_s^4 - \left(\frac{M(t)}{2\pi\tau_s}\right)^2\right]^{1/2}\left(\frac{(1-v)r}{b^2} + \frac{1+v}{2}\right),$$

(13)

$$u_\theta = -\frac{(1+v)M(t)}{2\pi E}\left(\frac{1}{r} - \frac{r}{b^2}\right)$$

(14)

In the plastic domain $a \le r \le r_s$ we have

$$\sigma_r = \tau_s\left\{ln\frac{r^2 + \left[r^4 - \left(\frac{M(t)}{2\pi\tau_s}\right)^2\right]^{1/2}}{a^2 + \left[a^4 - \left(\frac{M(t)}{2\pi\tau_s}\right)^2\right]^{\frac{1}{2}}} + \frac{\left[a^4 - \left(\frac{M(t)}{2\pi\tau_s}\right)^2\right]^{\frac{1}{2}}}{a^2} - \frac{\left[r^4\left(\frac{M(t)}{2\pi\tau_s}\right)^2\right]^{1/2}}{r^2}\right\} - p(t),$$

(15)

$$\sigma_0 = \sigma_r + \frac{2\tau}{r^2}\left[r^4 - \left(\frac{M(t)}{2\pi\tau_s}\right)^2\right]^{1/2}, \quad (16)$$

$$\varepsilon_r = \frac{1-v}{E}\sigma_r + \frac{2\tau_s}{Er^2}\left\{\left[r^4 - \left(\frac{M(t)}{2\pi\tau_s}\right)^2\right]^{1/2}(1-v) - \left[r_s^4 - \left(\frac{M(t)}{2\pi\tau_s}\right)^2\right]^{1/2}\right\}$$

(17)

$$\varepsilon_\theta = \frac{1-v}{E}\sigma_r + \frac{2\tau_s}{Er^2}\left[r_s^4 - \left(\frac{M(t)}{2\pi\tau_s}\right)^2\right]^{1/2}$$

(18)

$$\varepsilon_{r\theta} = \frac{1+v}{2\pi r^2 E}M(t) + \frac{1}{\pi r^2}\int_0^t \dot{\lambda} M(\tau)d\tau,$$

(19) (8)

$$u_r = \frac{1-v}{E}r\sigma_r + \frac{2\tau_s}{rE}\left[r_s^4 - \left(\frac{M(t)}{2\pi\tau_s}\right)^2\right]^{1/2},$$

(20)

$$u_\theta = \bar{\theta}\frac{(1-v)M(t)}{2\pi E}\left(\frac{1}{r} - \frac{r}{b^2}\right) - \frac{2}{\pi}\left(1 - \frac{r}{b}\right)\int_0^t \dot{\lambda} M(\tau)d\tau.$$

(21)

The variable σ_r, contained in (16) (18) is represented by formula (15). The loading parameter $\dot{\lambda}$ is determined by the formula:

$$\dot{\lambda} = \frac{2}{E}\left\{\frac{M\dot{M}/(4\pi^2\tau_s^2)}{\left[r^4 - \left(\frac{M(t)}{2\pi\tau_s}\right)^2\right]^{1/2}} + \frac{2r_s^3\dot{r}_s{\_}M\dot{M}/(4\pi^2\tau_s^2)}{\left[\left(r^4 - \left(\frac{M(t)}{2\pi\tau_s}\right)^2\right)\left(r_s^4 - \left(\frac{M(t)}{2\pi\tau_s}\right)^2\right)\right]^{1/2}}\right\}.$$

(22)

The unknown radius r_s is the solution of the following equation:

$$\frac{p(t)}{\tau_s} = ln\frac{r_s^2 + \left[r_s^4 - \left(\frac{M(t)}{2\pi\tau_s}\right)^2\right]^{\frac{1}{2}}}{a^2 + \left[a_s^4 - \left(\frac{M(t)}{2\pi\tau_s}\right)^2\right]^{1/2}} + \frac{\left[a^4 - \left(\frac{M(t)}{2\pi\tau_s}\right)^2\right]^{1/2}}{a^2} - \frac{\left[r_s^4 - \left(\frac{M(t)}{2\pi\tau_s}\right)^2\right]^{1/2}}{b^2}.$$

(23)

The represented solution holds for

$$\left(\frac{M(t)}{2\pi\tau_s}\right) \le a^2,$$

$$\frac{p(t)}{\tau_s} < +\left[1 - \left(\frac{M(t)}{2\pi\tau_s}\right)^2\right]^{1/2}.$$

The mentioned conditions are connected with the use of determining equations of plastic flow theory.

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 4, Issue 4, October 2014

The first plastic deformations will appear on the internal contour of the plate. Therewith $r_s = a$ and from (23) it follows the relation that holds between the internal pressure $P(t)$and distributed moments(t):

If $M(t) = 0$, we have $p_s = \frac{b^2 - a^2}{b^2}\tau_s$, but if$P(t)$=0 , then$M_s = 2\pi a^2 \tau_s$.

Suppose that external forces $P(t)$ and$M(t)$ are monotonically increasing in time interval$0 \leq t \leq \frac{t_1}{2}$. Let beginning with time $t = \frac{t_1}{2}$ these forces begin to decrease and for time t= t_1 become zero:$P(t_1)$=0, $M(t_1) = 0$. The unloading in the plate happens at time interval$\frac{t_1}{2} \leq t \leq t_1$.We shall assume that the unloading process is accompanied by appearance of secondary plastic deformations. Using the above represented solution, define the residual stresses, strains and displacements that remain the annular disk after removing the internal pressure $p(t)$ and distributed moment $M(t)$.And we'll assume that the forces $p(t)$ and $M(t)$ at loading from the natural state attained such a value that in the unloading process necessarily the secondary plastic deformations well appear. Now we must determine the conditions of appearance of secondary plastic deformations, and also find the residual stresses, strains and displacements. We'll use V.V. Moskvitin's theorem on secondary plastic deformations [3]. According to this theorem, the residual stresses$\sigma_r^0, \sigma_\theta^0, \sigma_{r\theta}^0$, residual strains $\varepsilon_r^0, \varepsilon_\theta^0, \varepsilon_{r\theta}^0$, residual displacements u_r^0, u_θ^0 may be determined by the following formulas:

$$\sigma_r^0 = \sigma_r - \sigma_r^*; \quad \sigma_\theta^0 = \sigma_\theta - \sigma_\theta^*; \quad \sigma_{r\theta}^0 = \sigma_{r\theta} - \sigma_{r\theta}^*;$$

$$\varepsilon_r^0 = \varepsilon_r - \varepsilon_r^*; \quad \varepsilon_\theta^0 = \varepsilon_\theta - \varepsilon_0^*; \quad \varepsilon_{r\theta}^0 = \varepsilon_{r\theta} - \varepsilon_{r\theta}^*;$$

(24)

$$u_r^0 = u_r - u_r^*; \quad u_\theta^0 = u_\theta - u_\theta^*.$$

Here

$\sigma_r, \sigma_\theta, \sigma_{r\theta}, \varepsilon_r, \varepsilon_\theta, \varepsilon_{r\theta}, u_r, u_\theta$ are stresses, strains and displacements before the beginning of unloading and are determined by the formulas (10)-(14) with regard to (22). Therewith, in these formulas,$p(t)$ and $M(t)$should be replaced by $p(t_1/2)$и$M(t_1/2)$.Unlike the considered plate, the yield point at the shear of the fictitious plate material is$2\sigma_s$. The secondary plastic deformations will appear in the zone adjoining to the interior contour of the plate. Let r_0 be a radius of a circle that separates the area of secondary plastic deformations:$a \leq r \leq r_0$.For composing the equation plastic determiningr_0,we should use equation (23) having substituted in it$2\sigma_s$ forσ_s:

$$\frac{P(t_1/2)}{2\tau_s} = ln\frac{r_0^2 + \left[r_0^4 - \left(\frac{M(t_1/2)}{4\pi\tau_s}\right)^2\right]^{1/2}}{a^2 + \left[a^4 - \left(\frac{M(t_1/2)}{4\pi\tau_s}\right)^2\right]^{1/2}} + \frac{\left[a^4 - \left(\frac{M(t_1/2)}{4\pi\tau_s}\right)^2\right]^{1/2}}{a^2} - \frac{\left[r_0^4 - \left(\frac{M(t_1/2)}{4\pi\tau_s}\right)^2\right]^{1/2}}{b^2}.$$

(25)

According to (24)

$$\left(\frac{M(t_1/2)}{4\pi\tau_s}\right) \leq a^2; \ \frac{P(t_1/2)}{2\tau_s} < 1 + \left[1 - \left(\frac{M(t_1/2)}{4\pi\tau_s a^2}\right)^2\right]^{1/2}$$

(26)

will be a condition of appearance of secondary plastic deformations.

If$M(t) \equiv 0$, there should be$P > \frac{2(b^2 - a^2)}{a^2 b^2}\tau_s$, but if$P(t) \equiv 0$, then$M > 4\pi a^2 \tau_s$.

In the area of secondary plastic deformations $a \leq r \leq r_0$ the residual stresses, strains and displacements will be:

$$\sigma_r^0 = \tau_s\left\{ln\frac{\left[r^2 + \left(r^4 - \left(M(t_1/2)/(2\pi\tau_s)\right)^2\right)^{1/2}\right]\left[a^2 + \left(a^4\left(M(t_1/2)/(4\pi\tau_3)\right)^2\right)^{1/2}\right]^2}{\left[r^2 + \left(r^4 - \left(M(t_1/2)/(4\pi\tau_s)\right)^2\right)^{1/2}\right]^2\left[a^2 + \left(a^4 - \left(M(t_1/2)/(2\pi\tau_s)\right)^2\right)^{1/2}\right]}\right.$$

$+$

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 4, Issue 4, October 2014

$$+\frac{1}{a^2}\left[\left(a^4-\left(\frac{M(t_1/2)}{2\pi\tau_s}\right)^2\right)^{1/2}-2\left(a^4-\left(\frac{M(t_1/2)}{4\pi\tau_s}\right)^2\right)^{1/2}\right]$$

$$-\frac{1}{r^2}\left[\left(r^4-\left(\frac{M(t_1/2)}{2\pi\tau_s}\right)^2\right)^{1/2}-2\left(r^4-\left(\frac{M(t_1/2)}{4\pi\tau_s}\right)^2\right)^{1/2}\right],$$

(27)

$$\sigma_\theta^0=\sigma_r^0+\frac{2\tau_s}{r^2}\left\{\left[r^4-\left(\frac{M(t_1/2)}{2\pi\tau_s}\right)^2\right]^{1/2}-\left[r^4-\left(\frac{M(t_1/2)}{4\pi\tau_s}\right)^2\right]^{1/2}\right\}$$

(28)

$$\sigma_{r\theta}^0=0.$$

(29)

$$\varepsilon_r^0=\frac{1-v}{E}\sigma_r^0+\frac{2\tau_s}{Er^2}\left\{(1-v)\left[\left(r^4-\left(\frac{M(t_1/2)}{2\pi\tau_s}\right)^2\right)^{1/2}-2\left(r^4-\left(\frac{M(t_1/2)}{4\pi\tau_s}\right)^2\right)^{1/2}\right]-\right.$$

$$\left.-\left(r_s^4-\left(\frac{M\left(\frac{t_1}{2}\right)}{2\pi\tau_s}\right)^2\right)^{\frac{1}{2}}+2\left(r_0^4-\left(\frac{M(t_1/2)}{2\pi\tau_s}\right)^2\right)^{1/2}\right\}$$

(30)

$$\varepsilon_\theta^0=\frac{1-v}{E}\sigma_r^0+\frac{2\tau_s}{Er^2}\left\{\left[r_s^4-\left(\frac{M(t_1/2)}{2\pi\tau_s}\right)^2\right]^{1/2}-2\left[r_0^4-\left(\frac{M(t_1/2)}{4\pi\tau_s}\right)^2\right]^{1/2}\right\}$$

, (31)

$$\varepsilon_{r\theta}^0=\frac{1}{\pi r^2}\int_0^{t_1/2}(\dot\lambda-\dot\lambda_*)M(\tau)d\tau,$$ (32)

$$u_r^0=\frac{1-v}{E}r\sigma_r^0+\frac{2\tau_s}{rE}\left\{\left[r_s^4-\left(\frac{M(t_1/2)}{2\pi\tau_s}\right)^2\right]^{1/2}-2\left[r_0^4-\left(\frac{M(t_1/2)}{4\pi\tau_s}\right)^2\right]^{1/2}\right\}$$

.(33)

$$u_\theta^0=-\frac{2}{\pi}\left(1-\frac{r}{b}\right)\int_0^{t_1/2}(\dot\lambda-\dot\lambda_*)M(\tau)d\tau.$$

(34)

The quantity σ_r^0 that is contained in formulas (28), (30), (31) and (33) is determined by formula (27). The formula for the quantity $\dot\lambda$, contained in (32) and (34) is written similar to formula (22) having substituted at the last one $2\tau_s$ for τ_s, r_0 for r_s, $t_1/2$ for t.

The residual stresses, strains and displacements in the area $r_0\le r\le r_s$, where the unloading process happens elastically, may be found also by formulas (24). Therewith the formulas σ_r, σ_θ, $\sigma_{r\theta}$, ε_r, ε_θ, $\varepsilon_{r\theta}$, u_r, u_θ are determined as in the previous case of loading. The quantities $\sigma_r^*, \sigma_\theta^*, \sigma_{r\theta}^*, \varepsilon_r^*, \varepsilon_\theta^*, \varepsilon_{r\theta}^*, u_r^*, u_\theta^*$ are the stresses, strains and displacements that appear in the fictitious annular plate at its elastic deformation by the forces $(t_1/2)$, $M(t_1/2)$. For finding them, we'll use independent solutions of Lame's elastic problem on action of the internal pressure $P(t_1/2)$ in the plate under consideration and the distributed moment $M(t_1/2)$ along the internal contour of this plate. Proceeding from this, the quantities $\sigma_r^*, \sigma_\theta^*, \sigma_{r\theta}^*, \varepsilon_r^*, \varepsilon_\theta^*, \varepsilon_{r\theta}^*, u_r^*, u_\theta^*$ are determined by the following formulas

$$\left.\begin{matrix}\sigma_\theta^*\\ \sigma_r^*\end{matrix}\right\}=\frac{a^2p(t_1/2)}{b^2-a^2}\left(1\pm\frac{b^2}{r^2}\right),$$

(35)

$$\sigma_{r\theta}^*=\frac{M(t_1/2)}{2\pi r^2},$$

(36)

$$\left.\begin{matrix}\varepsilon_\theta^*\\ \varepsilon_r^*\end{matrix}\right\}=\frac{a^2b^2p(t_1/2)}{(b^2-a^2)E}\left(\frac{1-v}{b^2}\pm\frac{1+v}{r^2}\right),$$

(37)

$$\varepsilon_{r\theta}^*=\frac{1+v}{E}\frac{M(t_1/2)}{2\pi r^2},$$

(38)

$$u_r^*=\frac{a^2b^2p(t_1/2)}{(b^2-a^2)E}\left((1-v)\frac{r}{b^2}+\frac{1+v}{r}\right),$$

(39)

$$u_\theta^*=-\frac{(1+v)M(t_1/2)}{2\pi E}\left(\frac{1}{r}-\frac{r}{b^2}\right).$$

(40)

Consequently, the residual stresses, strains and displacements in the area $r_0\le r\le r_s$ are determined as the differences of appropriate formulas (15)-(21) subject to with regard to (22) and substitution of $t_1/2$ for . It remains to define the residual sought-for quantities in the area $r_s\le r\le b$. These quantities are also found on the base of formula (24). Therewith the quantities σ_θ, σ_r, $\sigma_{r\theta}$, ε_r, ε_θ, $\varepsilon_{r\theta}$, u_r, u_θ and $\sigma_{r\theta}$ are expressed by the formulas (15)-(21), where r_s is determined by formula (23).

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 4, Issue 4, October 2014

The quantities $\sigma_\theta^*, \sigma_r^*, \sigma_{r\theta}^*, \varepsilon_r^*, \varepsilon_\theta^*, \varepsilon_{r\theta}^*, u_r^*, u_\theta^*$ are represented by formulas (35)-(40). Now consider an issue on fatigue failure of the plate under consideration. Let the plate under consideration after total unloading be again loaded by monotonically increasing forces $P(t)$and$M(t)$,and let the loading process continue to time $t = \frac{3}{2}t_1$ and in the sequel the total loading process happen to time$\left(\frac{3}{2}t_1,\ 2t_1\right)$.Suppose that such loading and unloading process of the plate continues up to fatigue failure. Therewith the time of each loading - unloading cycle equalst_1. The number of cycles to fatigue failureN_0 will be$N_0 = t_*/t_1$. The number of the cycles before appearance of first damagesN_f is expressed by the formula: $N_f = t_f/t_1$, wheret_f is the time to appearance of first damages. From physical reasoning it is clear that the first damages in the plate will appear on the internal contour$r = a$. The plate will fail just from this contour. Determine the number of loading cycles before the appearance of first damages and the number of the cycles to failure of the plate along the radius r. To determine them we use the following known formulas [3]:

$$N_0 = A_0\left(\frac{\varepsilon_t^0}{\varepsilon_s}\right)^{\alpha_0};\ N_f = A_1\left(\frac{\varepsilon_t^0}{\varepsilon_s}\right)^{\alpha_1},$$

(41)

where$A_0, \alpha_0,\ \ A_1, \alpha_1$are universal constants of the material , ε_t^0 is the intensity of residual strains, ε_s is some reduction strain,

In conformity to the problem under consideration the intensity of residual strains are determined by the formula:

$$\varepsilon_t^0 = \frac{2}{3}\left[(\varepsilon_r^0)^2 + \left(\varepsilon_\theta^0\right)^2 - \varepsilon_r^0\varepsilon_\theta^0 + 3\left(\varepsilon_{r\theta}^0\right)^2\right]^{\frac{1}{2}}$$

. (42)

Since the plates material is cyclic -ideal, then the residual stresses and displacement in any k-th loading cycle will be the same as in the first unloading. We take into account this fact in calculating the intensity of residual strains, and in defining the numbersN_0 andN_f.

Processing of experimental data with regard to cyclic durability for the steel of the mark EH-826, that are contained in the paper [4], for $\varepsilon_s = 0{,}01$ gave the following results:

$$\alpha_0 \approx \alpha_1 \approx 1{,}6;\ \frac{A_1}{A_0} \approx 0{,}56;\ A_0 \approx 1{,}8\cdot 10^6$$

cycles. By using these data and after calculating the intensity of residual strains by formula (42) allowing for relations (27),(30)-(32), the number of loading cycles to appearance of first damages and to fatigue failure of the internal contour of the plate were found. Therewith formula (41) was used. The graphs of dependences $\frac{N_0}{A_0} \sim \frac{M(t_1/2)}{2\pi a^2 \tau_s}, \frac{N_f}{A_0} \sim \frac{M(t_1/2)}{2\pi a^2 \tau_s}$ on the interval contour $(r = a)$ of the plate are depicted in fig.1.

Besides the abovementioned digital data, the following values of parameters were used::$p(t_1/2)/\tau_s = 1;\ b/a = 4$;
$v = 0{,}3; E = 2\cdot 10^4 MPa$.

Kinetics of plate failure along the radius $r$for$M(t_1/2)/(2\pi a^2\tau_s = 0{,}4)$was determined (fig 2).After onset of failure of the internal contour, the amplified fatigue failure of the remaining part the plate is observed.

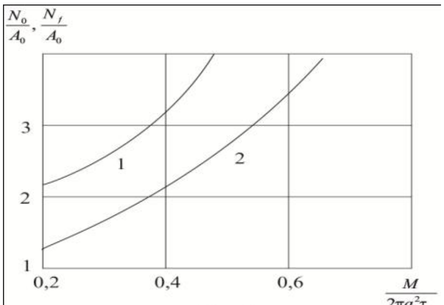

Fig. 1.

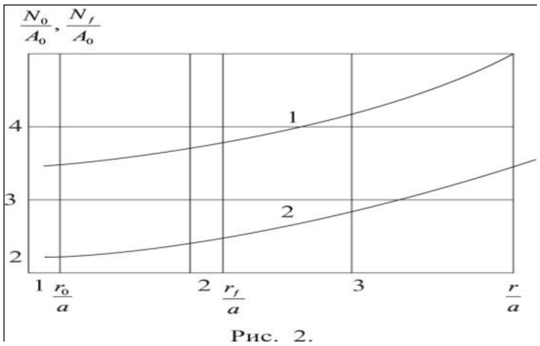

Рис. 2.

III. CONCLUSIONS

1. The conditions of appearance of secondary plastic deformations at total unloading after prior elastic-plastic deformation of an annular plate under the action of pressure and distributed moment along the internal contour were determined
2. The number of loadings to fatigue failure of an annular plate at its cyclic elastico-plastic deformation under

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 4, Issue 4, October 2014

pressure and moment pulsatingacting along the internal contour was determined.

REFERENCES

[1] Kachanov L.M. Bases of theory of elasticity. Moscow, Nauka, 1969, 420 p. (Russian)

[2] Nordgren R.P., Naghdi P.M. Loading and unloading solutions for an elastic-plastic annular plate in the state of plane stress under combined pressure and couple International Journal of Engineering Science, 1963, No1, p.33-70.

[3] Moskvitin V.V.Plasticity at variable loadings. Moscow, MGU, 1965, 264 p. (Russian)

[4] IvanovaV.S. Ragozin Yu.I., Vorobev N.A. Regularity of failure of metals at static and cyclic loadings. In: Thermoplastics of materials and structural elements.

[5] Issue 4., Кiev, Naukovadumka, 1967, pp. 277-285 (Russian)