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Free vibration analysis of SWCNT using CDM in the presence of nonlocal effect

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Abstract*— this paper deals with the free vibration analysis of single-walled carbon nanotube (SWCNT) bounded at the ends, with translational and elastic constraints, and attached mass. The nanotube is modelled as a beam and the effect of small length scale based on the nonlocal elasticity theory is considered. The governing equations of motion are derived using a variational approach and the free frequencies of vibrations are obtained employing the cell discretization method (CDM) in which the nanotube is reduced to a set of rigid bars linked together by elastic cells. The resulting discrete system takes into account nonlocal effects, constraint elasticity's and added mass. The natural frequencies and corresponding shift frequencies are calculated and numerical results for different boundary conditions are illustrated. Comparisons of the present numerical results with those from the open literature show an excellent agreement.

***Index Terms*—single-walled carbon nanotube, frequency analysis, non-local effect, boundary conditions, Cell Discrimination Method.**

I. INTRODUCTION

Carbon nanotubes (CNTs) constitute a prominent example of nanomaterials and nanostructures which have inspired extensive research activities in science and engineering field. Their discovery, since the publication of Iijima's paper [1] in 1991, has stimulated intensive studies to fulfil their potential applications in a variety of fields of engineering due to their extraordinary mechanical, physical and electrical properties. Numerous investigations for determining their physical properties and unique electronic properties are available in the literature, see, for example, [2-4]. Their high stiffness and strength, coupled with low density, have lead to carbon nanotubes (CNTs) usage in the emerging field of applications in nanoelectronics, nanodevices, nanocomposites, bio-nano-composites and so on [5–9]. Several investigations have shown that CNTs possess extraordinary strength, which is measured up to 100 times that of steel at one-sixth of the weight [10], as well as superior electrical and thermal conductivities. Moreover, such outstanding properties make CNTs promising candidates for resolution mass sensor and several studies have investigated the use of CNTs as a mass sensor [11-14]. In the earlier studies, the investigations on carbon nano tubes have mainly focused on numerous experiments [15] although these texts, at nanoscale, are very cumbersome. In addition, several studies on the material properties and mechanical behaviours of CNTs have been conducted by using either atomistic modelling or continuum mechanics modelling. Among the methods of atomistic simulations, the classical molecular dynamic (MD) simulations are the most common method in investigating the behaviour of CNTs [16-17]. In molecular dynamics simulations, atoms are considered as particles interacting to each other by means of several types of potential fields. Although those simulations generate abundant results for understanding the behaviour of structures, the atomistic model involves complex computational processes and is still formidable and expensive, especially for large-sized atomic system, and this explain why, in recent years, the continuum models play an essential role in the study of CNTs. Several researchers implemented the elastic models of beams to study the dynamic problems, such as vibration and wave propagation, of carbon nanotubes [18-20]. Although the classical continuum methods are efficient in performing mechanical analysis of CNTs, their applicability to identify the small-scale effects on carbon nanotubes mechanical behaviours is questionable. The importance of size effect has been pointed out in a number of studies where the size dependence of the properties of nanotubes have been investigated. For example, Sun and Zhang, in [21], discussed the scarce applicability of continuous models to nanotechnology and proposed a semi-continuum model in studying nano-materials. At this point, the nonlocal elastic continuum models are more pertinent in predicting the structural behaviour of nanotubes because of being capable of taking into account the small-scale effects. It is well-known that the nonlocal elasticity theory assumes that the stress state, at a given reference point, is considered to be a function of the strain field at all points of the body. The origins of the nonlocal theory of elasticity go to pioneering works, published in early 80s, by Eringen [22]. In [23] Reddy reports a complete development of the classical and shear deformation beam theories using the nonlocal constitutive differential equations and derived the solutions for bending, buckling and natural frequencies problems of simply supported beams. In recent years, many researchers have applied the nonlocal elasticity concept to bending, buckling and vibration analysis of nanostructures. Although initiated by the work of Eringen, the possibility of using the nonlocal continuum theory in the field of nanotechnology was first reported by Peddieson et al. [24]: the Authors have used nonlocal Euler-Bernoulli model for static analysis of nano-beams and particular attention is paid to cantilever beams which are often used as actuators in small-scale systems. Further applications of the nonlocal elasticity theory have been employed in studying the buckling problem [25-26] and vibration problems, by applying Euler-Bernoulli beam and shell theories and Timoshenko beam theory, in CNTs [25-30]. Wang and Hu [31] presented a study on the flexural wave propagation in a single-walled carbon nanotube (SWCNT) through the use of the nonlocal continuum mechanics and the molecular dynamics simulation based on the Terroff–Brenner potential. Lu et al. [32] established a nonlocal Euler–Bernoulli beam model to obtain frequency equations and modal shape functions of simply supported, clamped and cantilever beams. Reddy and Pang [33] reformulated the equation of motion of the Euler-Bernoulli and Timoshenko beam theories, using the nonlocal differential constitutive relations of Eringen. Following this approach, the equations of motion are used to evaluate the static bending, vibration and buckling responses of beams with various boundary conditions. This paper focuses on the study of free vibration analysis of SWCNT, bounded at the ends, with translational and elastic constraints, and attached masses, by using Cell Discretization Method (CDM) and in the presence of nonlocal effect. The method has already been used by the authors [34] and by Raithel and Franciosi [35] for different structural problems. Recently, De Rosa and Lippiello [18] have employed the CDM to investigate the free vibration frequencies problem of coaxial double-walled carbon nanotubes (DWCNTs). In addition, in [36], the Authors dealt with the variational problem of the nanotube, bounded at the ends and attached mass, located in a generic position, and the closed-form nonlocal frequency expression has been derived by means of the Hamilton's principle; then the resonant frequency and corresponding shift frequency have been calculated. Finally, De Rosa et al. [37] have proposed three different approaches to calculate the free vibration frequencies of a cantilever nanotube with, and without, an attached distributed mass. It is shown that the size-effects must be taken into account, and the frequencies have to be calculated according to the nonlocal elasticity theory. The nanotube is reduced to a set of rigid bars, linked together by elastic cells, where masses and stiffness's are supposed to be concentrated. The resulting discrete system is simple enough to allow to take into account nonlocal effects, constraint elasticity's and attached mass. The natural frequencies are calculated and it is possible to derive the relative shift frequencies; then numerical results for different boundary conditions are performed in order to evaluate the effect of the nonlocal coefficient on the natural frequency value. Comparisons of the present numerical results with those from the open literature show an excellent agreement.

II. THEORETICAL APPROACH

Let us consider the single-walled carbon nanotube, with span L, mass density ρ, Young's modulus E, cross-sectional area A and second moment of area I. The small-scale effect is taken into account by using the nonlocal theory for Euler-Bernoulli beam theory, so that the parameter $\mu = e_0 a$ is introduced, where e_0 is a material constant, which has to be determined through experimental results, and a is an internal characteristic length of the nanotube. In order to analyze the dynamic behavior of the structure under consideration, the governing equations of motion, by considering the small-scale effect, have been derived using a variational approach:

$$T = \frac{1}{2}\int_0^L \rho A \left(\frac{\partial v(z,t)}{\partial t}\right)^2 dz. \tag{1}$$

$$\begin{aligned} E = L_e - P = \frac{1}{2}\int_0^L EI\left(\frac{\partial^2 v(z,t)}{\partial z^2}\right)^2 dz + \frac{1}{2}k_{RL}\left(\frac{\partial v(0,t)}{\partial z}\right)^2 \\ + \frac{1}{2}k_{TL} v^2(0,t) + \frac{1}{2}k_{RR}\left(\frac{\partial v(L,t)}{\partial z}\right)^2 + \frac{1}{2}k_{TR} v^2(L,t) \\ - \int_0^L \mu^2 \rho A \frac{\partial^2 v(z,t)}{\partial t^2}\frac{\partial^2 v(z,t)}{\partial z^2} dz. \end{aligned} \tag{2}$$

Where k_{RL} and k_{TL} are rotational and translational stiffness respectively at z=0, while, analogously, k_{RR} and k_{TR} are rotational and translational stiffness at z=L, respectively. In the above equations, the abscissa z represents the spatial coordinate while t is the time; in (1) and (2) T denotes the kinetic energy, L_e is the strain energy and P is the potential energy due to nonlocal inertia forces.

III. DISCRETIZATION OF SWCNT BY MEANS OF CDM METHOD

In this section the so-called "Cell Discretization Method" (CDM), employed to analyze the dynamic behaviour of structure under consideration, is discussed. The nanotube is reduced to a set of *t* rigid bars with length *l* connected by *n* elastic cells, (see Fig. 1).

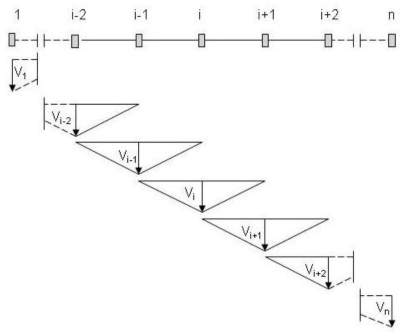

Fig 1: Structural system CDM.

The moment of inertia I and the cross-sectional area A will be evaluated at the cells abscissae, obtaining the concentrated stiffness k=EI/*l* and the concentrated masses $m=\rho A l$ for the nanotube. Both these quantities can be organized into the so-called unassembled stiffness diagonal matrix ***k*** and the unassembled mass diagonal matrix ***m***, with dimension $(n \times n)$. In this way, the structure is reduced to a classical holonomic system, with *n* degrees of freedom; in particular, *n* vertical displacements *v*, at the cells abscissae will be conveniently assumed as Lagrangian coordinates and will be organized into

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the *n*-dimensional vector **v.** Moreover, for the single-walled carbon nanotube, the *n*-1 rotations of the rigid bars can be calculated as a function of the Lagrangian coordinates as follows:

$$\phi_i = \frac{v_{i+1} - v_i}{l}, \quad i = 1, \ldots n-1. \tag{3}$$

or, in matrix form: $\boldsymbol{\phi} = \mathbf{V}\mathbf{v}$, where **V** is a rectangular *transfer matrix* with *n*-1 rows and *n* columns.
The *relative* rotations between the two faces of the elastic cells are given by:

$$\psi_1 = \phi_1, \qquad \psi_i = \phi_i - \phi_{i-1}, \qquad \psi_n = -\phi_{n-1}. \tag{4}$$

or in matrix form $\boldsymbol{\psi} = \boldsymbol{\Delta}\,\boldsymbol{\phi}$, where **Δ** is a rectangular *transfer matrix* with *n* rows and *n*-1 columns.
The strain energy L_e, (the first term of (2)), is concentrated at the cells of the nanotube, and is given by:

$$L_e = \frac{1}{2}\sum_{i=1}^{n} k_{ii}\,\psi_i^2. \tag{5}$$

The strain energy should be expressed as functions of the Lagrangian coordinates, by using (4) and (5), as follows:

$$L_e = \frac{1}{2}\boldsymbol{\psi}^T\mathbf{k}\boldsymbol{\psi} = \frac{1}{2}\phi^T\boldsymbol{\Delta}^T\mathbf{k}\,\boldsymbol{\Delta}\phi = \frac{1}{2}\mathbf{v}^T\left(\mathbf{V}^T\boldsymbol{\Delta}^T\mathbf{k}\,\boldsymbol{\Delta}\mathbf{V}\right)\mathbf{v}. \tag{6}$$

so that, the total strain energy can be expressed as:

$$L_e = \frac{1}{2}\mathbf{v}^T\mathbf{K}\,\mathbf{v}. \tag{7}$$

where $\mathbf{K} = (\mathbf{V}^T\boldsymbol{\Delta}^T\,\mathbf{k}\,\boldsymbol{\Delta}\,\mathbf{V})$ is the assembled stiffness matrix.
The last term of (2), as function of the Lagrangian coordinates, assumes the following form:

$$P = \sum_{i=1}^{n} \mu^2 \rho A\, \ddot{v}_i \psi_i. \tag{8}$$

or:

$$P = \mu^2 \ddot{\mathbf{v}}^T\left(\mathbf{m}_{n1}\boldsymbol{\Delta}\mathbf{V}\right)\mathbf{v}. \tag{9}$$

by defining the assembled nonlocal effects matrix $\mathbf{M}_{n1} = (\eta^2\, \mathbf{m}_{n1}\,\boldsymbol{\Delta}\,\mathbf{V})$, the equation (9) assumes the following form:

$$P = \ddot{\mathbf{v}}^T\mathbf{M}_{n1}\mathbf{v}. \tag{10}$$

The kinetic energy, Eq. (1), is simply expressed as:

$$T = \frac{1}{2}\sum_{i=1}^{n} m_{ii}\,\dot{v}_i^2. \tag{11}$$

and it can be re-written as:

$$T = \frac{1}{2}\dot{v}^T\mathbf{m}\dot{v}. \tag{12}$$

Finally, the equation of motion will have the following form:

$$\mathbf{M}_t\ddot{\mathbf{v}} + \mathbf{K}\mathbf{v} = \mathbf{0}. \tag{13}$$

where $\mathbf{M}_t$ is the global assembled stiffness matrix:

$$\mathbf{M_t} = -\mathbf{M}_{nl} + \mathbf{M}. \tag{14}$$

and **0** is a null vector.
The problem of vibration analysis consists of determining the conditions under which the equilibrium equation (13) can be satisfied. One assumes that the free motion is a simple harmonic motion of the form:

$$\mathbf{v}(t) = \mathbf{D}\cos(\omega_i\, t - \theta). \tag{15}$$

where **D** is the mode shape that is time-independent. By deriving twice the equation (15) respect to t, one gets:

$$\ddot{\mathbf{v}}(t) = -\omega_i^2\mathbf{D}\cos(\omega_i\, t - \theta). \tag{16}$$

substitution of this expression into (13) gives

$$\left(\mathbf{K} - \omega_i^2\mathbf{M}_t\right)\mathbf{D} = \mathbf{0}, \quad i = 1, \ldots n. \tag{17}$$

where ω_i^2 are the frequencies of natural vibration, or eigenvalues, and, as already said, **D** is the mode shape or eigenvector. A solution to this homogeneous system of equation exists only if the determinant of the coefficient's matrix set equal to zero:

$$\det\left(\mathbf{K} - \omega_i^2\mathbf{M}_t\right) = \mathbf{0}. \tag{18}$$

The boundary conditions are:

$$-EIv'''(z) - \omega_i^2\mu^2\rho A v'(z) - k_{TL}v(z) = 0, \quad z = 0. \tag{19}$$

$$EIv''(z) + \omega_i^2\mu^2\rho A v(z) - k_{RL}v'(z) = 0, \quad z = 0. \tag{20}$$

$$EIv'''(z) + \omega_i^2\mu^2\rho A v'(z) - k_{TR}v(z) = 0, \quad z = L. \tag{21}$$

$$-EIv''(z) - \omega_i^2\mu^2\rho A v(z) - k_{RR}v'(z) = 0, \quad z = L. \tag{22}$$

and they represent added terms of mass and flexibilities matrix of the motion equation. The strain energy of the vertically flexible constraints, of (2), as a function of the Lagrangian coordinates, is given by:

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$$L_{TL} = \frac{1}{2} k_{TL} v_1^2 \qquad (23)$$
$$L_{TR} = \frac{1}{2} k_{TR} v_n^2.$$

so that the assembled stiffness matrix must be modified as follows:

$$K[1,1] = K[1,1] + k_{TL} \qquad (24)$$
$$K[n,n] = K[n,n] + k_{TR}.$$

The presence of vertically flexible intermediate supports can be similarly dealt with. If the constraint is placed at given abscissa (see Fig. 2) its strain energy is equal to:

$$L_{TL} = \frac{1}{2} k_{hv} \left(\frac{l_v}{l} v_{i+1} + \left(\frac{l - l_v}{l} \right) v_i \right)^2. \qquad (25)$$

Consequently the following terms are added at the stiffness matrix:

$$K[i,i] = \left(\frac{l - l_v}{l} \right)^2 k_{hv}$$
$$K[i+1,i+1] = \left(\frac{l_v}{l} \right)^2 k_{hv} \qquad (26)$$
$$K[i+1,i] = \frac{l_v}{l} \left(\frac{l - l_v}{l} \right) k_{hv}$$
$$K[i,i+1] = \frac{l_v}{l} \left(\frac{l - l_v}{l} \right) k_{hv}.$$

where l_v is the length of rigid bar among the flexible constraint and elastic cell *i*, and k_{hv} is the intermediate support stiffness.

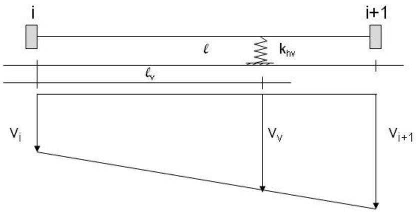

Fig 2: Structural system CDM with intermediate support stiffness.

The rotational stiffness of the four end constraints can be taken into account by summing up the corresponding flexibilities with the flexibilities of the end elastic cells. For example, for the end constraints, one gets:

$$k[1,1] = \frac{k[1,1] k_{RL}}{k_{RL} + k[1,1]} \qquad (27)$$
$$k[n,n] = \frac{k[n,n] k_{RR}}{k_{RR} + k[n,n]}.$$

These terms will be included in the matrix **k** of (6). In order to take into account the boundary conditions (19-22) the global mass matrix must be modified as follows:

$$M[1,1] = M[1,1] - \frac{\mu^2 \rho A}{l}$$
$$M[1,2] = M[1,2] + \frac{\mu^2 \rho A}{l} \qquad (28)$$
$$M[n,n] = M[n,n] - \frac{\mu^2 \rho A}{l}$$
$$M[n\text{-}1,n] = M[n\text{-}1,n] + \frac{\mu^2 \rho A}{l}.$$

The presence of added concentrated masses can be similarly dealt with. If the mass is placed at given abscissa (see Fig. 3) its strain energy is equal to:

$$T_M = \frac{1}{2} M \left(\frac{l_m}{l} v_{i+1} + \left(\frac{l - l_m}{l} \right) v_i \right)^2. \qquad (29)$$

Consequently the terms of the mass matrix must be added to the following terms:

$$M[i,i] = \left(\frac{l - l_m}{l} \right)^2 M$$
$$M[i+1,i+1] = \left(\frac{l_m}{l} \right)^2 M \qquad (30)$$
$$M[i+1,i] = \frac{l_m}{l} \left(\frac{l - l_m}{l} \right) M$$
$$M[i,i+1] = \frac{l_m}{l} \left(\frac{l - l_m}{l} \right) M.$$

where l_m is the length of rigid bar among the concentrated mass *M* and elastic cell *i*.

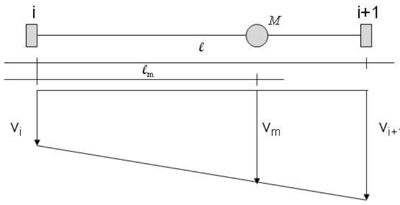

Fig 3: Structural system CDM and added mass.

IV. NUMERICAL COMPARISONS

In order to show the potentialities of the proposed approach (CDM), several numerical examples have been performed; using a general code developed in *Mathematica* [38], and the obtained results are compared with those of available works in literature and listed in bibliography.

A. Numerical comparison between CDM and analytical approach of a SWCNT with various boundary conditions and in presence of nonlocal parameter.

A first numerical comparison has been performed with reference to the paper by Reddy and Pang [33], in which the equations of motion of the Euler- Bernoulli and Timoshenko beam theories, reformulated using the non local differential constitutive relation of Eringen [22], are used to evaluate the static bending, vibration and buckling responses of beams with various boundary conditions.

Table 1 shows properties of a single-walled carbon nanotube (SWCNT) which will be used to the numerical comparison.

SWCNT properties density	Symbol	Value	Unit
Inner diameter	D_1	$0.68*10^{-9}$	m
Outer diameter	D_2	$0.8*10^{-9}$	m
Length	L	$10*10^{-9}$	m
Density	ρ	2240	Kg/m^3
Young's modulus	E	$1*10^{12}$	Pa

Table 1: Single-walled carbon nanotube properties.

Assuming the nonlocal parameter value $\eta=e_0a$ /L = 0.4, the first three circular frequencies have been calculated by using CDM method, with a lower number cells i.e. n = 500, and exact approach, so as deduced to [33]. The SWCNT is modelled as an Euler-Bernoulli beam with various boundary conditions at two ends which are of a variety of combinations, namely supported-supported (S-S), clamped-clamped (C-C), clamped-free (C-F) and clamped-supported (C-S). The numerical comparison, between the results given by the proposed method (CDM) and the results given by Reddy and Pang [33], is illustrated in Table 2. As it can be seen, the circular frequencies values show an excellent agreement among the results obtained by employing the two procedures.

Boundary conditions	Method	ω□	ω□	ω□
S-S	[33]	3.40838 $\times10^{11}$	8.09454 $\times10^{11}$	1.26308 $\times10^{12}$
	CDM	3.40837 $\times10^{11}$	8.09448 $\times10^{11}$	1.26306 $\times10^{12}$
C-C	[33]	7.15706 $\times10^{11}$	1.17243 $\times10^{12}$	6.07502 $\times10^{12}$
	CDM	7.15699 $\times10^{11}$	1.17241 $\times10^{12}$	1.66169 $\times10^{12}$
C-F	[33]	2.11816 $\times10^{11}$	6.20757 $\times10^{11}$	1.2976 $\times10^{12}$
	CDM	2.11816 $\times10^{11}$	6.20751 $\times10^{11}$	1.2976 $\times10^{12}$
C-S	[33]	5.05024 $\times10^{11}$	9.92921 $\times10^{11}$	1.45459 $\times10^{12}$
	CDM	5.05022 $\times10^{11}$	9.92909 $\times10^{11}$	1.45456 $\times10^{12}$

Table 2- Numerical comparison among Reddy-Pang [33] and CDM of SWCNT: the first three circular frequencies associated to four type boundary conditions are reported.

B. Effect of the nonlocal parameter and added mass on the natural frequency of a fixed –free SWCNT.

In the following numerical example, the fixed-free single-walled nanotube, with attached mass at the free end, is considered. In recent years, fixed-free SWCNTs have attracted a lot of interest due to their suitability for a wide range of applications, such as vacuum microelectronic devices, nanosensors and nanoactuators. In addition, a particular attention has been devoted to a fixed-free SWCNTs based mass –sensor; in fact, the essence of mass sensing in a resonator is based on the fact that it is natural frequencies is sensitive to the added mass. In this numerical example the effects of the added mass and the nonlocal parameter on the natural frequency shift are considered and discussed widely. Based on a cantilever beam model with a rigid mass at the free end, an analytical solution and approximate numerical procedure have been employed. In Table 3, the geometrical and material properties of single-walled carbon nanotube, so as deduced from the paper of Mehdipour et al. [13], are reported.

SWCNT properties	Symbol	Value	Unit
Inner diameter	D_1	$18.8\ 10^{-9}$	m
Outer diameter	D_2	$33\ 10^{-9}$	m
Density	ρ	1330	Kg/m^3
Length	L	$5.5\ 10^{-6}$	m
Young’s modulus	E	$32\ 10^{9}$	Pa

Table 3 - Geometrical and material properties of the single-walled carbon nanotube, [13].

In this numerical example, the comparison with the results given by the exact procedure, so as deduced in [37], and those obtained by Mehdipour et al. [13], in the absence of the nonlocal effects (see Table 2 of [13]), is performed. Table 4 summarizes the results of the natural frequencies of the fixed-free SWCNT for various added mass and nonlocal effect values η = [0,0.1,0.5].

Attached mass (fg)	[13]	Exact value □ □ □	CDM □ □ □
0		861556.099	861553.410
20	190401.785	190401.785	190401.630
22	181934.726	181934.727	181934.547
24	174505.207	174505.207	174504.928
26	167917.297	167917.297	167917.161
28	162023.235	162023.236	162023.048
30	156709.208	156709.202	156709.003
35	145419.280	145419.280	145419.165
40	136263.504	136263.504	136263.382
50	122175.371	122175.371	122175.239

a)

Attached mass (fg)	Exact value η=0.1	CDM η=0.1
0	865297.483	865293.769
20	190458.580	190458.232
22	181984.329	181983.981
24	174549.017	174548.672
26	167956.359	167956.027
28	162058.350	162058.033
30	156740.992	156740.701
35	145444.711	145444.457
40	136284.446	136284.165
50	122190.484	122190.264

b)

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Attached mass (fg)	Exact value η=0.5	CDM η=0.5
0	1001753.90	1001749.17
20	191850.755	191850.376
22	183197.961	183197.595
24	175619.250	175618.892
26	168909.364	168909.001
28	162914.065	162913.736
30	157514.913	157514.595
35	146062.595	146062.310
40	136792.486	136792.225
50	122556.341	122556.102

c)

Table 4 - Comparison of natural frequency for fixed-free SWCNT with different mass additions and in absence and presence of nonlocal effect.

In the first column of Table 4, the values of the concentrated mass, at the free end, are reported; the second and third column give the natural frequency values, $f_1 = \omega_1/2\pi$, in the absence of nonlocal effects and for increasing values of the attached concentrated mass. As can be observed, the numerical comparison shows that the two procedures lead to the same results, as obtained by solving the boundary value problem. The fourth column of the Table 4 gives, instead, the values of the first natural frequency, by using the Cell Discretization Method (CDM), for varying values of the attached mass *M*. The results have been obtained with zero non-dimensional stiffness coefficients k_{TR} and k_{RR} while k_{TL} and k_{RL} are large enough. As can be observed, the results provide an excellent agreement. Finally, in the other columns of Table 4 the first natural frequency values have been reported introducing the non-dimensional nonlocal parameter η = [0.1,0.5]. The results have been obtained by solving the equations of motion, so as deduced to [39], and by means of CDM method. The natural frequency values show that the exact results are in good agreement with the CDM simulation results and the correctness of the proposed numerical method. From Table 4, the following considerations apply:

- If the concentrated mass value increases, the natural frequency values decrease;
- If the nonlocal effect increases, the natural frequency values increase, as observed in [33] applying the Equation 163.

C. Effect of nonlocal parameter and added mass on the natural frequency on SWCNT with various boundary conditions.

In this numerical examples, one evaluates the variation of natural frequency for a SWCNT with an added mass and with various boundary conditions. The added mass is located at the midspan of the nanotube and the numerical calculations have been performed for clamped- clamped (Table 5), supported-supported (Table 6) and clamped-supported (Table 7) single-walled carbon nanotube and setting the small-scale parameter equal to η =0,0.1,0.5. For all numerical examples, the same material and geometrical data, given in Table 3, are adopted.

Attached mass (fg)	Exact value η=0	CDM η=0
0	5482297.14	5481206.68
20	1502749.43	1502724.97
22	1437598.15	1437575.50
24	1380245.33	1380223.93
26	1329250.06	1329228.63
28	1283518.54	1283498.00
30	1242203.48	1242183.34
35	1154174.73	1154156.16
40	1082546.72	1082529.60
50	971943.84	971928.10

a)

Attached mass (fg)	Exact value η=0.1	CDM η=0.1
0	5172508.06	5172429.74
20	1495729.82	1495706.22
22	1431449.12	1431426.48
24	1374800.74	1374779.02
26	1324385.00	1324364.03
28	1279137.03	1279116.76
30	1238230.41	1238210.79
35	1150985.96	1150967.68
40	1079914.34	1079897.20
50	970037.451	970022.04

b)

Attached mass (fg)	Exact value η=0.5	CDM η=0.5
0	2693298.79	2693276.24
20	1351852.69	1351834.13
22	1303856.38	1303838.27
24	1260630.78	1260613.10
26	1221435.04	1221417.71
28	1185679.89	1185663.00
30	1152889.98	1152873.45
35	1081502.99	1081487.28
40	1021917.89	1021902.80

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50	927361.577	927347.74

c)

Table 5 - Comparison of natural frequency for clamped-clamped SWCNT with different mass additions and in absence and in presence of nonlocal effect.

Attached mass (fg)	Exact value η=0	CDM η=0
0	2418424.61	2418416.77
20	743117.78	743114.97
22	711558.90	711556.85
24	683706.48	683702.99
26	658887.61	658884.75
28	636588.84	636585.67
30	616410.71	616408.56
35	573318.15	573315.46
40	538159.17	538157.47
50	483713.87	483711.17

a)

Attached mass (fg)	Exact value η=0.1	CDM η=0.1
0	2307245.31	2307237.82
20	739715.77	739715.77
22	708571.04	708568.23
24	681055.07	681052.37
26	656513.93	656511.35
28	634447.60	634445.11
30	614466.29	614463.81
35	571753.02	571750.79
40	536864.29	536862.07
50	482773.15	482771.27

b)

Attached mass (fg)	Exact value η=0.5	CDM η=0.5
0	1298764.81	1298761.88
20	669803.40	669801.04
22	646434.21	646431.91
24	625349.70	625347.46
26	606200.62	606198.49
28	588708.12	588705.97
30	572646.50	572644.40
35	537615.74	537613.74
40	508313.42	508311.51
50	461703.77	461702.07

c)

Table 6 – Comparison of natural frequency for supported-supported SWCNT with different mass additions and in absence and in presence of nonlocal effect.

Attached mass (fg)	Exact value η=0	CDM η=0
0	3778040.81	3778006.45
20	1127770.95	1127766.06
22	1079539.32	1079525.21
24	1037008.30	1036998.08
26	999136.95	999122.23
28	965132.36	965122.78
30	934378.43	934369.17
35	868751.33	868742.66
40	815255.60	815247.18
50	732495.53	732484.72

a)

Attached mass (fg)	Exact value η=0.1	CDM η=0.1
0	3577356.99	3577326.52
20	1122233.41	1122221.99
22	1074681.64	1074670.67
24	1032701.94	1032691.40
26	995285.01	995274.81
28	961660.19	961650.34
30	931227.50	931217.95
35	866218.43	866209.53
40	813162.16	813153.79
50	730976.88	730969.35

b)

Attached mass (fg)	Exact value η=0.5	CDM η=0.5
0	1907298.47	1907288.62
20	1008947.14	1008938.29
22	974062.50	974053.84
24	942550.26	942541.80
26	913901.73	913893.44
28	887709.16	887701.04
30	863641.48	863633.55
35	811095.54	811087.94
40	767091.05	767083.79
50	697012.23	697005.51

c)

Table 7 - Comparison of natural frequency for clamped-supported SWCNT with different mass additions and in absence and in presence of nonlocal effect.

For all three numerical examples, the natural frequency values show that the CDM simulation results are in excellent agreement with the exact results. As can be seen in Table 5-7, the natural frequency parameter of SWCNT decreases with increasing values of the attached mass and of the nonlocal effect parameter. Known the natural frequency value, it is possible to derive the non-dimensional frequency shift Δf by the following expression: $\Delta f = (f_0 - f_1)/f_0$, where f_0 is the non-dimensional frequency of the nanotube without added mass and neglecting the non local effect, and f_1 is the non-dimensional frequency with added mass and non local effect. In this way, it is possible to evaluate the nonlocal parameter influence on the relative frequency shift of the sensor with added mass.

D. Influence of nonlocal parameter, added mass and non-dimensional stiffness coefficient K_{TR} = 10 on the natural frequency of fixed at left SWCNT.

Let us consider a nanotube clamped at left and at the right side, constrained by an elastically flexible spring, with

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non-dimensional stiffness coefficient $K_{TR} = \frac{k_{TR} L^3}{EI} = 10$, and having a concentrated mass *M*, at the right end. For this numerical example the material and geometrical properties, given in Table 3, are adopted, and the aim is to evaluate the behaviour of natural frequency of the nanotube bounded at the left end with translational constraint. In Table 8 the natural frequency values are reported and for varying values of the attached mass between 0 and 50, whereas the nonlocal effect values η vary between 0,0.1 and 0.5.

Attached mass (fg)	Exact value η=0	CDM η=0
0	1706423.42	1706419.0 7
20	396309.47	396309.27
22	378692.76	378692.66
24	363233.51	363233.39
26	349524.61	349524.48
28	337258.94	337258.79
30	326199.88	326199.75
35	326199.88	302703.06
40	283646.99	283646.90
50	254323.62	254323.52

a)

Attached mass (fg)	Exact value η=0.1	CDM η=0.1
0	1701968.39	1701963.6 2
20	396422.62	396422.40
22	378791.99	378791.79
24	363321.45	363321.26
26	349603.24	349603.07
28	337329.79	337329.62
30	326264.15	326263.99
35	302754.78	302754.62
40	283689.65	283689.51
50	254354.54	254354.42

b)

Attached mass (fg)	Exact value η=0.5	CDM η=0.5
0	1663006.59	1663003.2 3
20	399291.61	399291.38
22	381295.57	381295.36
24	365531.06	365530.85
26	351572.18	351571.97
28	339098.77	339098.58
30	327864.84	327864.67
35	304034.00	304033.84
40	284742.22	284742.09
50	255113.29	255113.16

c)

Table 8 - A clamped nanotube constrained by an elastically flexible spring, with non-dimensional stiffness coefficient K_{TR} = 10, and having a concentrated mass.

Table 8 contains the exact values, so as obtained by solving the boundary value problem, and the results of the approximate method CDM, in absence and in presence of the nonlocal effect. As one can see, in absence of the added mass, the natural frequency decreases if the nonlocal coefficient increases, whereas if one considers the added mass, the natural frequency increases when the nonlocal effect increases.

E. Influence of nonlocal parameter, with different mass additions and with various boundary conditions on the natural frequency of a SWCNT.

Let us considers a single-walled carbon nanotube with various boundary conditions and with different added masses distributed along the length of nanotube. For all subsequent examples, the same material and geometrical data, given in Table 3, are assumed and the first three natural frequency values have been calculated, putting the small-scale parameter equal to η=[0,0.1,0.2,0.3,0.4,0.5].

a) Considering a clamped-free SWCNT with four added mass, the first numerical example is performed. The added mass are located at different abscissae of the nanotube: M_1 = 50 fg at L_1 = 0.25 L, M_2 = 40 fg at L_2 = 0.5 L, M_3 = 30 fg at L_3 = 0.75 L and M_4 = 20 fg at L_4 = L, respectively. In Table 9 the natural frequency values are reported.

η	f_1	f_2	f_3
0	141502.03	782366.97	2089881.15
0.1	141522.56	781216.34	2079450.05
0.2	141575.30	777797.12	2049031.96
0.3	141667.04	772172.21	2001084.50
0.4	141796.24	764456.96	1939129.74
0.5	141963.51	754798.11	1867144.09

Table 9 – Clamped-free SWCNT with four masses: (0.25 L , 50), (0.5 L , 40), (0.75 L , 30), (L , 20).

The results show that the first natural frequency value increases while the second and third natural frequency values decrease when the non-dimensional nonlocal effect parameter increases.

b) In this case, a simply-supported SWCNT with three added masses is considered. The added mass are located at different point of length of the nanotube: M_1 = 50 fg at L_1 = 0.25 L, M_2 = 40 fg at L_2 = 0.5 L and M_3 = 30 fg at L_3 = 0.75 L, respectively. In Table 10 the natural frequency values are listed.

η	f_1	f_2	f_3
0	387444.07	1559037.98	3346383.90
0.1	386947.34	1551205.66	3313911.17
0.2	385501.47	1528379.19	3221593.31
0.3	383112.53	1492441.08	3082705.64
0.4	379840.62	1446074.07	2914140.11
0.5	375754.37	1392276.99	2731871.46

Table 10 – Simply-supported SWCNT with three added masses: (0.25 L , 50), (0.5 L , 40), (0.75 L , 30).

c) Let us consider a clamped-simply supported SWCNT with three added masses which are located at different point of length of the nanotube: M_1 = 50 fg at L_1 = 0.25 L, M_2 = 40 fg

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at L_2 = 0.5 L and M_3 = 30 fg at L_3 = 0.75 L, respectively. In Table 11 the natural frequency values are reported:

η	f_1	f_2	f_3
0	625170.90	1949149.37	3569855.07
0.1	624204.50	1938787.79	3537741.30
0.2	621333.54	1908620.04	3446344.19
0.3	616628.11	1861214.85	3308560.05
0.4	610210.78	1800226.62	3140771.74
0.5	602235.81	1729737.23	2958473.63

Table 11 – Clamped-simply supported SWCNT with three added masses: (0.25 L , 50), (0.5 L , 40), (0.75 L , 30).

d) Let us consider a clamped-clamped SWCNT with three added masses: M_1 = 50 fg at L_1 = 0.25 L, M_2 = 40 fg at L_2 = 0.5 L and M_3 = 30 fg at L_3 = 0.75 L, respectively. In Table 12 the natural frequency values are listed:

η	f_1	f_2	f_3
0	875152.01	2336526.54	3968107.05
0.1	873794.36	2324304.83	3932606.76
0.2	869757.50	2288696.47	3831553.57
0.3	863145.54	2232631.64	3679180.88
0.4	854128.34	2160306.80	3493590.45
0.5	842924.27	2076436.50	3291943.24

Table 12 – Clamped-clamped SWCNT with three added masses: (0.25 L , 50), (0.5 L , 40), (0.75 L , 30).

e) In Table 13 the natural frequency values of a free SWCNT at the ends, with intermediate constraints, located at 0.25 L and 0.75 L, and three added masses: M_1 = 20 fg at L_1 = 0, M_2 = 50 fg at L_2 = 0.5 L and M_3 = 20 fg at L_3 = L, respectively, have been obtained.

η	f_1	f_2	f_3
0	706641.32	1095322.11	2292122.60
0.1	706407.61	1096077.35	2285430.29
0.2	706158.07	1098722.93	2266027.40
0.3	709305.71	1106177.88	2236032.58
0.4	703253.33	1108098.86	2197106.04
0.5	696274.05	1111490.78	2153108.21

Table 13 – **Free-supported at 0.25 L, supported at 0.75 L and free at L, with three added masses: (0. L , 20), (0.5 L , 50), (1 L , 20).**

f) Let us consider a pinned SWCNT at the left, two supports at 0.5 L and L and two added masses: M_1 = 50 fg at L_1 = 0.25 L and M_2 = 50 fg at L_2 = 0.75 L respectively. In Table 14 the natural frequency are listed.

η	f_1	f_2	f_3
0	760458.36	1843525.70	1582691.17
0.1	779002.65	1837874.01	1316227.25
0.2	753393.85	1813603.17	9406144.55
0.3	751788.24	1778723.21	7003529.11
0.4	736167.08	1731610.48	5498307.41
0.5	719310.86	1675970.77	4508276.84

Table 14 – Pinned-supported SWCNT at the left, two supports at 0.5 L and L, with two added masses at (0.25 L , 50), (0.75 L , 50).

g) Let us consider a nanotube clamped at left and with a support at 0.5 L, with non-dimensional stiffness coefficient $K_T = \frac{k_{hv} L^3}{EI} = 1$, a support at L, with non-dimensional stiffness coefficient K_{TR} = 10, and with two added masses: M_1 = 50 fg at L_1 = 0.25 L and M_2 = 50 fg at L_2 = 0.75 L respectively. In Table 15 the natural frequency are listed.

η	f_1	f_2	f_3
0	354221.52	1555751.53	7655760.22
0.1	354145.59	1549634.82	7432602.14
0.2	353910.62	1531604.38	6626713.55
0.3	353524.41	1502557.89	5488324.91
0.4	352988.59	1463880.86	4513625.17
0.5	352307.88	1417288.43	3789187.79

Table 15 - Nanotube clamped at the left and with a support at 0.5 L, with non-dimensional stiffness coefficient K_T = 1, a support at L, with non-dimensional stiffness coefficient K_{TR} = 10, and with two added masses at (0.25 L , 50), (0.75 L , 50).

The Tables 9-15 give the three natural frequency values. The results show that the natural frequency decrease when the nonlocal effect increases.

VII. CONCLUSION

The free vibration frequencies of a SWCNT, bounded at the ends, with translational and elastic constraints, and attached masses, have been investigated by using the Cell Discretization Method (CDM) and employing the non-local Euler-Bernoulli beam theory. The system, under consideration, has been modelled as a set of rigid bars linked together by elastic cells, where masses and stiffnesses are supposed to be concentrated. Several numerical examples have been treated in detail, comparing exact method and the proposed approach which has excellent results. More particularly, emphasis has been given to the influence of the small-scale parameter, of the added mass, of the various boundary conditions on the free vibration frequency behaviour of single-walled carbon nanotube.

The proposed method leads to the following considerations:

- The clamped free nanotube represents a peculiar numerical case because it is the single numerical example in which the natural frequency increases when the nonlocal effect increases;
- The natural frequency decreases when the added mass value increases;
- A lot of the results listed in the Tables can be employed to know the relative frequency shift values.

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