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Geometrical/Computational Dynamics Approximations for Helicopter-Rotor Instantaneous Rotation Center in Turbulence with Numerical Reuleaux Method

F Casesnoves MSc (Physics) MD
Computational Bioengineering Researcher
IIIS (International Institute of Informatics and Systemics,Individual Researcher)
Denver, Colorado State, USA
casesnoves.research.emailbox@gmail.com

Abstract
Civil Helicopter (CH) rotor blades are usually manufactured/designed with deformable materials, and formed by mechanical elements (we deal with them as voxels).Their movement is governed mainly by four angles, rotor, flapping, lagging, and torsional deflection. In severe turbulence/windy conditions structural dynamics of blades could become deformed and/or damaged. Lagging/flapping bending moment and torsional deflection angles could appear and be significant. We tried to analize physical/computational changes in the IRC position of rotor in flapping and lagging under turbulence conditions, with appropiate simulations. The method was to study the dynamics for the IRC biased position, which creates a variation chain in the Aerodynamics conditions of CH. We carried out a primary approximation to analyze geometrically/numerically the IRC variation with the NRM. We obtained initial algorithms which are developed with numerical simulations. A primary framework for the dynamics and physical Aerodynamics consequences derived from this data was determined. These results show agreement with basic principles of IRC determination in literature. Numerical data constitutes a first approximation for the approach to this problem in Numerical Reuleaux Method.

Keywords: Instantaneous Rotation center (IRC), Numerical Reuleaux Method (NRM),flapping,lagging,rotor,bending momente abstract; it sets the footnote at the bottom of this column.

I. INTRODUCTION

In previous publications [refs 2-5], the Numerical Reuleaux Method was presented as an approximated tool to obtain the IRC of a deformable-solid or pseudo-rigid body during an arbitrary movement. Most of these contributions are focused on Computational Bioengineering applications of the NMR. But in this new publication we step forward towards an industrial-aerospace-mechanical paradigm, that is, the mathematical and dynamical consequences in Aerospace-Helicopter Research of the IRC variation of the blades of a CH. That is, when wind, turbulence, and other posible lagging/flapping bending, torsional deflections, and random damaging forces/phenomena can cause deflection,lagging, flapping, and/or twist significant magnitude angles.

The Integral and Partial Differential Equations (PDEs) that determine the aerodynamics of the lagging/flapping bending and torsional deflection displacements/angles are modifed by the difference between the Rotor IRC (Mechanical IRC) and the Instantaneous Rotation Center of each blade (and the total blade group). Which is defined by the change in position of every voxel mass center of the blade (2D, in 3D we use points so far). This variation could be caused by turbulence, a wind shock, or any other atmospheric phenomena that makes the blade change its normal shape or damages/modifies the material structure/mechanical settings of the blade (s). Extensive studies have been carried out in recent decades about the rotor blades lagging, flapping, and torsional deflection. Moire Interferometry techniques (laser based) and simulations have proven to be useful to quantify magnitudes and complementary data of blades deformations (structural dynamics).Other types of engineering techniques have been ACTOS system based on sensors and ultra-fast thermomenters (Siebert and collaborators, 2006) for small turbulences. They take the study related to cloud particles formation.Classical studies (Ormiston and Bousman 1972) show a primary determination of flapping frequency as threshold for stability limit, together with desestabilizing effects.

Aeroelastic conditions studies during shipboard engagement/disengangement are also useful for operations safety realted to CH in ships. Suction-type wind tunnel has also been used for experimental measurement work.

However, in these techniques/methods the IRC of the blades are referred mainly to the rotor position. We deal and demonstrate in this contribution that the application of the basic forward NRM algorithm yields to a parameters-variation in the aerodynamical forces acting on the rotor blades.

The mathematical development is not very complicated, and geometrical corrections to the classical Integral and Differential equations for structural dynamics of CH blades can be approximated and developed.In 2D, the mass centers of every blade can be used to obtaing the lagging common mass center for all the blade.In 3D, however [ref complementary 1] it is not possible at this stage to use the same technique.In the following, we presented the mathematical framework for a primary approximation to apply the NRM to CH dynamics in flapping and lagging conditions. Those conditions can be caused by turbulence, air sudden shocks/streams, exterior damage, or other unpredictable circumstances.Results are shown in Tables of simple simulations based on previous publicatons [ref 5].

2. MATHEMATICAL AND GEOMETRICAL METHOD(S) BASIC PRINCIPLES FOR LAGGING BENDING

We define the following

1.-There is a Dynamic IRC (DIRC) when shape of the blade changes,and it does not correspond to the Rotor spatial point

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(Rotor becomes a Virtual-enforced-IRC,namely, Rotor-IRC (RIRC)). In lagging, all mathematical and geometrical development is on rotor plane (2D).

2.-If the voxel of the blades rotates around the DIRC, its real velocity is tangent to the circle (2D) around the DIRC.

3.-Therefore, the velocity of the voxel has 2 components related to the RIRC, one tangential, and one radial, Vt and Vr .

4.- This implies variations of the parameters of Equations (PDEs), of the lagging bending moment and deflection torsion of the blades, and also in flapping.

In Fig 1, pictured, basic physical dynamic changes of velocity for one voxel of the blade located at medium zone of the blade .We drew a basic simulation, the blade voxel is at its normal shape, and then (2D approximation) it experiments a bending moment that changes the position/shape of the voxel.The velocity after that instantaneous rotation results divided into a tangent and a radial components. Because the total velocity is tangent to the circle around the IRC determined by NRM, and not to the rotor circle. As a result, the centrifugal force over the voxel is modified by the radial component. There is an instantaneous impulse F x Δt = m x Δv in radial direction. Δv is the magnitude of the radial velocity, since at initial position the radial velocity was null. Also the instantaneous velocity, and therefore the angular moment L= mv x r (all vectors, vectorial product) tangent to the voxel related to rotor decreases since the magnitude of the tangent velocity has decreased.

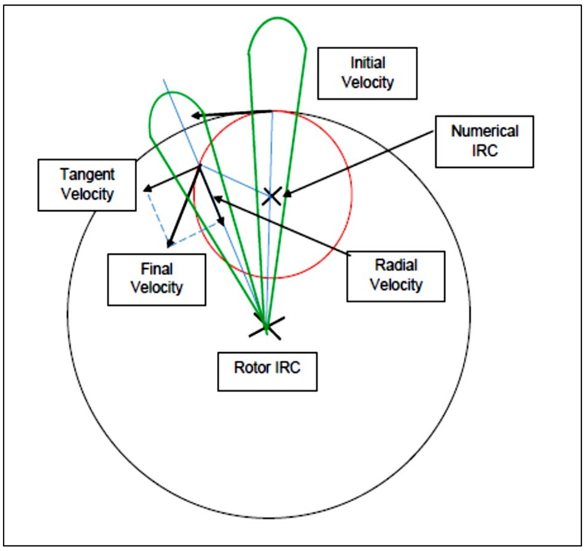
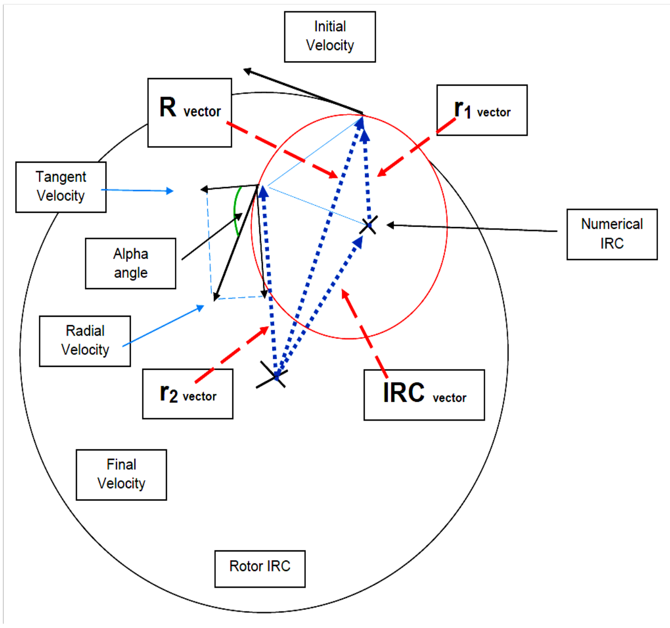


Fig 1.-Basic physical dynamic changes of velocity for one voxel

Fig 2 is an sketch of the calculations of the radial and tangent components of the final velocity for a voxel/element of a blade. What is intended geometrically, is to obtain a formula to determine the alpha angle (given in the sketch).In that way, we can calculate both radial and tangent velocities from the magnitude of the final velociity.



3.-APPLICATION OF NRM ON ROTOR BLADES, LAGGING DYNAMICAL DECOMPOSITION.

To carry out the analysis of trigonometric formula the cosine theorem is applied on the previous sketch of Fig 2 to determine the alpha angle. Once the alpha angle is got, we can guess the tangent and radial components of the velocity because the velocity components, tangent and radial, define an alpha angle and a π/2-alpha angle with the velocity vector, such as

$$\alpha = ar\cos\left[\frac{\left\|\vec{R}-\vec{IRC}\right\|^2+\left\|\vec{r}_2\right\|^2-\left\|\vec{IRC}\right\|^2}{2\times\left\|\vec{R}-\vec{IRC}\right\|\times\left\|\vec{r}_2\right\|}\right];$$

Equation [1]

We divided a blade in several voxels and simulate the determination of the IRC using the NRM algorithm, forward problem, as in previous publications [refs 2-5].The NRM algorithm formulas that were used are,Mathematical development from classical equations to NRM applications, first part,are,

$$f\left(\vec{x}\right)=\sum_{i=1}^{N}\frac{\int_{V_i}\rho_i\left(\vec{r}\right)dv}{\sum_{j=1}^{N}\int_{V_j}\rho_j\left(\vec{r}\right)dv}\times\left\|\vec{x}-\vec{x}_i\right\|$$

$$f(\vec{x})=\sum_{i=1}^{N}\frac{|\Delta V|}{\sum_{j=1}^{N}|\Delta V_j|}\times\|\vec{x}-x_i\|^2,$$

Eqs [2]

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Weight factors are considered equal (unity) in these simulations for simplicity.The innovation in 2D of this contribution in NRM calculations is that we deal now with the IRC corresponding to a group of mass centers, eachone belongs to a voxel/element of the blade.We assert the following,

Theorem 1.-In 2D NRM,the common/optimal mass center for several coincident points with each individual mass center of every pixel is located in the line/polygon that joints all the pixel rotation centers.

For 3D we use the classical algorithm of NRM [ref 5]. In 2D there are modifications of the NRM because it is physically more precise, and leads to further applications, to work with mass centers of the blades instead the classic NRM geometrical points.
We refer initially to the classic text [ref 1] of Bramwell's Helicopter Dynamics to set the basic PDEs for this primary approximation study.The first analysis corresponds to the PDE of the Lagging Bending Moment,as follows [picture from ref [1],Where r is radial coordinate and y corresponds to the 3D cartesian coordinate system of the rotor. Alpha is the bending angle, and this autor applies the angle approximation of the figure.The calculation of the torque exerted by the centrifugal force is modified by the radial force created in the deformation of the blade (Fig 3).

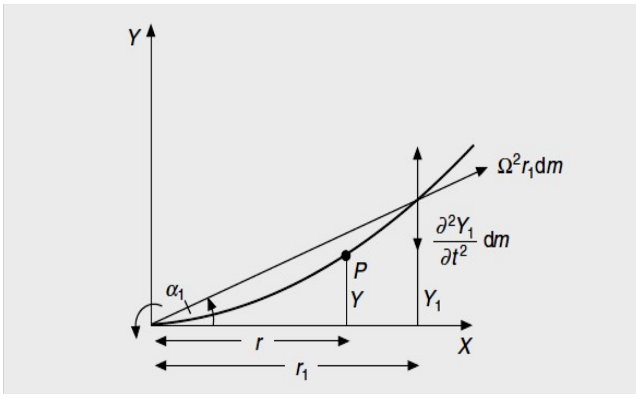

Fig 3.-Lagging torque determination.

4.-LAGGING PDE MATHEMATICAL DEVELOPMENT

In Appendix 1 and 2 we show the lagging partial differential equations development wit additional explanations. Given these equations, it is necessary to introduce the modifications caused by the variation of the IRC of the rotor.
Now, continuing Appendices 1 an 2, we develop the corrections based on the DIRC determination, taken directly from [ref 1]. These corrections imply the implementation of the Fr into the integral factor G, so we get G'.

$$G' = \int_{r_0}^{r_1} (m\omega^2 r - F_r)dr$$

Eq [3]

And we use this new value to modify the classical equations, the numerical methods that are used in [ref1]are also applicable for this modified equation.

Then,Eqs[4],

$$\frac{\partial^2 M}{\partial r^2} = \frac{\partial^2}{\partial r^2}\left[EI\frac{\partial^2 Y}{\partial r^2}\right] = \frac{\partial}{\partial r}\left[G'\frac{\partial Y}{\partial r}\right] - m\left[\frac{\partial^2 Y}{\partial r^2} - \omega^2 Y\right];$$

or

$$\frac{\partial^2}{\partial r^2}\left[EI\frac{\partial^2 Y}{\partial r^2}\right] - \frac{\partial}{\partial r}\left[G'\frac{\partial Y}{\partial r}\right] + m\left[\frac{\partial^2 Y}{\partial r^2} - \omega^2 Y\right] = 0;$$

So we have a PDE with r and t variables. The analytical resolution of this equation is made by variables separation [ref 1],

$$Y = RT(x)\chi(t)$$

$$\frac{d^2}{dx^2}\left[EI\frac{d^2 T}{dx^2}\right] - R^2\frac{d}{dx}\left(G\frac{dT}{dx}\right) - m(v^2+1)\Omega^2 R^4 T = 0$$

Frequency Equation,

$$d^2\chi/d\psi^2 + v^2\chi = 0$$

Eqs [5]

And after the NRM corrections of this contribution with the Frequency Equation Equal,

$$\frac{d^2}{dx^2}\left[EI\frac{d^2 T}{dx^2}\right] - R^2\frac{d}{dx}\left(G'\frac{dT}{dx}\right) - m\left(v^2+1\right)\Omega^2 R^4 T = 0$$

Eqs [6]

5.-APPLICATION OF NRM ON ROTOR BLADES IN FLAPPING

In this Section we develop the following,

BASIC PRINCIPLES FOR FLAPPING BENDING

1.-There is a Dynamic IRC (DIRC) when shape of the blade changes,and it does not correspond to the Rotor spatial point (Rotor becomes a Virtual-enforced-IRC,namely, Rotor-IRC (RIRC)). In general, flapping bending moment (3D) geometry is more complicated than lagging (2D). But partial differential equations result simpler.

2.-If the voxel of the blades rotates around the DIRC, its real velocity is tangent to the circle (2D) around the DIRC. As a principal difference with lagging, the DIRC circle in flapping is within a different plane of the rotor one (Figure 4).

3.-Therefore, the velocity of the voxel has 2 components related to the RIRC, one tangential, and one radial, Vt and Vr . And these components hold the same angle of the DIRC related to rotor plane. What is relevant is the radial velocity, that causes variations in the flapping equations.

4.-This implies variations of the parameters of Equations (PDEs), of the flapping bending moment and deflection torsion of the blades, and also some rather complicated geometrical determinations.

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5.-In flapping, the mass center method cannot be used because the mass centers of the elements are in a straight line. This does not agree to basic theorems of [complementary ref 1].

On Fig 4, pictured, the flapping variation in the spatial position of a mass element/voxel of a blade. It is the same sketch tan the lagging (upper view). But the final velocity is within a different plane and not within the rotor plane. Note that the DIRC (red) circle is in a different plane of the rotor plane, the only point in common with rotor plane is the initial velocity. This has consequences for the geometrical calculations.

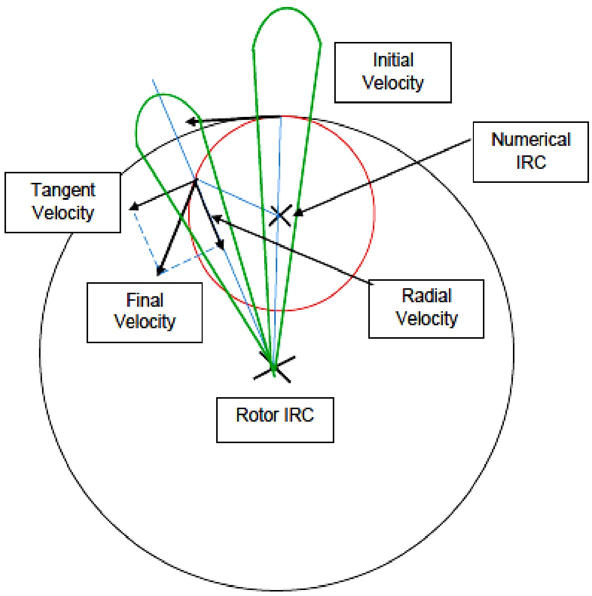

Fig 4.-Flapping planes spatial 2D setting

In Fig 5, pictured, the geometrical curves of the element mass center around rotor and DIRC during a flapping discretized instant. The theoretical flapping calculations involve the following steps,

ALPHA ANGLE DETERMINATION FOR FLAPPING

1.-Determine Equation (1) of line that joins the DIRC with final velocity point.

2.-Determine Equation (2) of perpendicular line to (1) at final velocity point.

3.-Determine Equation (3) of line that joins rotor and final velocity point.

4.-Determine angle between (2) and (3). Angle formed between two straight lines, classical formula.That is alpha, Equation 1.

5.-Radial Component of final velocity is V_f x cosα .Tangent velocity is V_f x sinα. Formula for alpha, in Equation 1.

The mathematical development is similar to the lagging one. Figures here show the main mechanical equilibrium equations [taken from ref 1 directly].

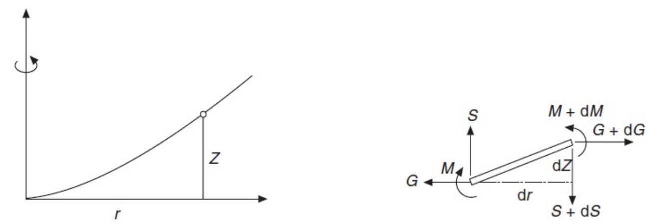

Fig 5.-Theoretical flapping calculations.

Then, the Equations result [ref 1] for equilibrium,

$$dG + m\Omega^2 r\, dr = 0$$

$$dS + m\, dr\, \frac{\partial^2 Z}{\partial t^2} = 0$$

$$G\, dZ + S\, dr - dM = 0$$

Eqs [7]

We differentiate in the same way,

$$\frac{\partial^2}{\partial r^2}\left(EI\frac{\partial^2 Z}{\partial r^2}\right) - \frac{\partial}{\partial r}\left(G\frac{\partial Z}{\partial r}\right) + m\frac{\partial^2 Z}{\partial t^2} = 0$$

By using,

$$M = EI\frac{\partial^2 Z}{\partial r^2}$$

Eqs [8]

So we get the same function G to be modified by the application of Numerical Reuleaux Method and after the final variables separation method we obtain [ref 1],

$$\frac{d^2}{dx^2}\left(EI\frac{d^2 S}{dx^2}\right) - R^2\frac{d}{dx}\left(G\frac{dS}{dx}\right) - m\lambda^2\Omega^2 R^4 S = 0$$

$$\frac{d^2\phi}{d\psi^2} + \lambda^2\phi = 0$$

Eqs [9]

Then we substitute the function G by G' as a result of the theoretical calculations,numerical methods of [ref 1] are also applicable, with Frequency Equation equal to,

$$\frac{d^2}{dx^2}\left[EI\frac{d^2 S}{dx^2}\right] - R^2\frac{d}{dx}\left(G'\frac{dS}{dx}\right) - m\lambda^2\Omega^2 R^4 S = 0$$

Eq [10]

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6.-RESULTS, CALCULATIONS AND COMPUTATIONAL SOFWARE

We detail 2D simulations results for lagging in Table 1 as follows (Data from Ref 5) We detail 3D simulations (points, no mass centers) for flapping in Table 2 as follows [taken from ref 5 computational files]. In Table 3, we show 3D points Matrices details. Numerical data, however, constitutes a primary approximation in this rather complicated aerodynamics problem. Computational software was carried out with Freemat 4.2 (Samit Basu, General Plublic License),and other Free Numerical Software).Programs are based on previous publications [ref 5] Special matrices of points adjustments were carried out to speed up the calculations. We worked in simple precision. The 2D and 3D software is simple since we worked with small voxel numbers.

7.-AEROSPACE ENGINEERING APPLICATIONS

We can assert that the approximated dynamical analysis of the helicopter rotor rotation center agrees to the formal Numerical Reuleaux Method principles. It is the mathematical link between the rotor IRC and the IRC determined by NRM what can be used as starting point to develop all the new approximations. What is also dynamically significant is the radial velocity component that appears after deformation in 2D and 3D. This is the primary first change within the classical dynamics of the Helicopter Theory, due to the application of the NRM.

8.- DISCUSSION AND CONCLUSIONS

The approximations that have been carried out show mathematically/physically that the IRC of the rotor in helicopter varies its position during flapping and lagging under turbulence conditions. This 3D position-variation yields to a series of dynamical changes related to the new IRC spatial location, e.g., moments, impulse, etc. In consequence, it has been analyzed the variations/decomposition of the blades velocity for this turbulence situation.

The velocity of the voxel has 2 components related to the RIRC, in lagging bending moment, one tangential, and one radial, Vt and Vr . The lagging moment calculations are the most simple ones, since the geometry facilitates the modification factors for integral and differential equations. These calculations can be implemented both in the Integral and PD Equations an din the Numerical and Analytical methods used to resolve these equations. We have developed primary approximations for the classic CH Integral and PD equations of Lagging and Flapping bending moments.

The main innovation in NRM Algorithm is the substitution of geometrical points for mass center points correponding to elements/voxels (2D only). This yields to a Instantaneous Rotation Center of the mass center of the blade, which is an approximated IRC. This implies that during lagging the approximated total mass center of the blade rotates around a DIRC and not around the rotor. In 3D, calculation of mass centers rotation can be carried out after determining the DIRC by NRM usual method. Simulations of the IRC of Mass Centers (2D), and in 3D (points) can be considered acceptable as a primary approximation at this stage.

To summarize, a new dynamics perspective for the lagging and flapping phenomena during turbulence in helicopter flight has been shown with primary approximations. New applications for helicopter-rotor IRC are derived for this research with Numerical Reuleaux Method.. More rigurous dynamical analysis will be developed in future contributions with corresponding computational simulations.

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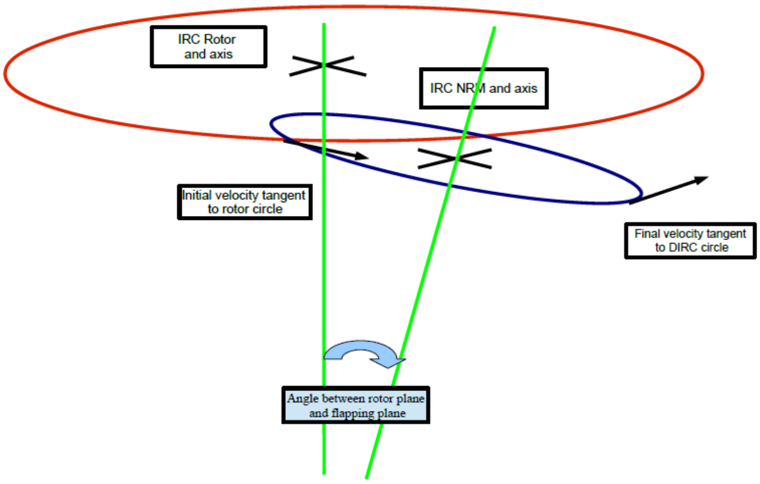

Fig 6.- The theoretical flapping calculations in 3D.This figure is also enhanced at Appendix Section.

The unmodified/classical Integral Equation is,

$$M = EI\frac{\partial^2 Y}{\partial r^2} = \int_r^R m\,\omega^2\, r_1\,(r_1 - r) sin\alpha_1 dr_1 - \int_r^R m\,\omega^2\, r_1\,(Y_1 - -Y) cos\alpha_1 dr_1 - \int_r^R m\,\frac{\partial^2 Y}{\partial t^2}(r_1 - r) dr_1$$

And the modified equation becomes,

$$M = EI\,\frac{\partial^2 Y}{\partial r^2} = \int_{r_0}^{r_1}(m\omega^2 r_1 - F_r)(r_1 - r) sin\alpha_1 dr_1 - \int_{r_0}^{r_1}(m\omega^2 r_1 - F_r)(Y_1 - Y) cos\alpha_1 dr_1 - \int_{r_0}^{r_1} m\frac{\partial^2 Y}{\partial t^2}\,(r_1 - r) dr_1$$

Where we have added the Force (F_r) in radial direction r_1 and the component that is opposed to the aerodynamic Force, created by the tangential velocity. Note that the limits of the integral correspond to the voxel dimensions.

And we have implemented the magnitude of the F_r into the centrifugal force integrals (direction r_1 for every voxel). The first two integrals correspons to the integrand vectorial product between the radial centrigugal force and the position vector of the blade or voxel, that is the reason because of which we get trigonometric factors.

Approximations carried out by [ref] to cancel trigonometric terms are based on the difference of magnitude between radial values and Y values along the blade [see Eq 2.1] .

All these calculations yield to a PDE [refs] which is simple and can be resolved by variables separation method, or by Lagrangian, Rayleight-Ritz, or Garlekin Method.

Once approximations are carried out by [ref], we follow the [ref 1] to obtain a modified differential equation by differentiation,

Appendix 1.-Mathematical development from classical equations to NRM applications, first part.

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We detail one trigonometric approximation for the integral equation,

$$m\,\omega^2\, r_1\, sin\alpha_1 = m\,\omega^2\, r_1 \frac{Y_1}{\sqrt{Y_1^2 \ + r_1^2}} \cong m\,\omega^2\, Y_1 \frac{r_1}{\sqrt{Y_1^2 \ + r_1^2}} \cong m\,\omega^2\, Y_1 \times 1 = m\,\omega^2\, Y_1$$

The double differentiation respect to variable r results [ref 1],

$$\frac{\partial M}{\partial r} = -\int_r^R m\Omega^2 Y_1 \mathrm{d}r_1 + \frac{\partial Y}{\partial r}\int_r^R m\Omega^2 r_1\, \mathrm{d}r_1 + \int_r^R m\,\frac{\partial^2 Y_1}{\partial t^2}\,\mathrm{d}r_1$$

$$= -\int_r^R m\Omega^2 Y_1 \mathrm{d}r_1 + \frac{\partial Y}{\partial r} G(r) + \int_r^R m\,\frac{\partial^2 Y_1}{\partial t^2}\,\mathrm{d}r_1$$

and

$$\frac{\partial^2 M}{\partial r^2} = \frac{\partial^2}{\partial r^2}\left[EI\frac{\partial^2 Y}{\partial r^2}\right] = \frac{\partial}{\partial r}\left[G\frac{\partial Y}{\partial r}\right] - m\left(\frac{\partial^2 Y}{\partial r^2} - \Omega^2 Y\right)$$

$$\frac{\partial^2}{\partial r^2}\left[EI\frac{\partial^2 Y}{\partial r^2}\right] - \frac{\partial}{\partial r}\left[G\frac{\partial Y}{\partial r}\right] + m\left(\frac{\partial^2 Y}{\partial r^2} - \Omega^2 Y\right) = 0$$

Appendix 2.-Theoretical lagging calculations. Mathematical development from classical equations to NRM applications, second part.

Table 1.-Lagging 2D mass center simulations. The innovation is to use the mass centers of each blade element instead the geometrical voxel points.

2D SIMULATIONS MASS CENTERS LAGGING							
ANGLE	IRC ROTOR	LAGGING VARIATION OF MASS CENTER IN COORDINATES (1) FIRST ELEMENT ε_1 (2) SECOND ELEMENT ε_2	IRC LAGGING	IRC RMS ERROR	INITIAL AND FINAL POINTS OF MASS CENTERS (1) 1ST ELEMENT (2) 2ND ELEMENT		COMMENTS
20	(0,0)T	(-0.7773,-0.2767)	(0.5949,0.4624)	10^{-3}	(-1,3) (-1,1)	(0.0863,3.1611) (-0.5977,1.2817)	
20	(0,0)T	(-0.6018,-0.1562)	(0.1894,0.2000)	10^{-5}	(-1,3) (-1,1)	(0.0863,3.1611) (-0.5977,1.2817)	
50	(0,0)T	(-0.0273,-0.2034) for 2 we vary coordinate y	(-0.1999,-0.2001)	10^{-4}	(-1,3) (-1,1)	(1.6552,2.6944) (0.1232,1.4088)	ALMOST EXACT DATA GOT
50	(0,0)T	(0.4467,-0.7982)	(0.2999,0.3000)	10^{-5}	(-1,3) (-1,1)	(1.6552,2.6944) (0.1232,1.4088)	
50	(0,0)T	(-0.4034,-0.2943)	(-0.2997,-0.3012)	10^{-4}	(-1,3) (-1,1)	(1.6552,2.6944) (0.1232,1.4088)	
AVERAGE	N/A	N/A	N/A	10^{-4}			

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FLAPPING 3D SIMULATIONS .LEAST SQUARES.POINTS METHOD.						
EULER ROTATION ANGLES θ= Around Z α= Around Y (Applied in this order)	IRC ROTOR FLAPPING DIRECTION IS Z AXIS	IRC DIRC LINEAR SYSTEM (Approximated Solution)	DEFORMATIONS BLADE VOXEL/ELEMENT 1 (1) in X (2) in Y (3) in Z ; ε_1 First Eq, ε_2 Second Eq ε_3 Third Eq	DEFORMATIONS BLADE VOXEL/ELEMENT 2 (1) in X (2) in Y (3) in Z	ERROR MAGNITUDE ORDER FORWARD PROBLEM	COMMENTS FOR FP
θ= 30 α = 40	$(0,0,0)^T$	X= 0.4965 Y=0.5060 Z=0.5040	ε_1 =-0.2531 ε_2 =+0.1492 ε_3 =+0.3984	ε_1 =+1.0183 ε_2 =-0.2940 ε_3 =+0.3839	10^{-2}	All matrices in any rotation were found regular
θ= 30 α = 40	$(0,0,0)^T$	X= 0.3979 Y=0.4040 Z=0.4028	ε_1 =-0.2265 ε_2 =+0.1147 ε_3 =+0.3092	ε_1 =-0.0236 ε_2 =-0.2054 ε_3 =+0.2936	10^{-2}	
θ= 40 α = 30	$(0,0,0)^T$	X= 0.9473 Y=0.8772 Z=0.9380	ε_1 =-0.8464 ε_2 =+0.6989 ε_3 =+1.3238	ε_1 =-0.9231 ε_2 =1(*) ε_3 =+0.6328	10^{-2}	Here we get a difficult second equation, we vary all coordinates
θ= 40 α =30	$(0,0,0)^T$	X= 0.4941 Y=0.5071 Z=0.5117	ε_1 =-0.6824 ε_2 =+0.2721 ε_3 =+0.5859	ε_1 =+0.5331 ε_2 =2(*) ε_3 =+0.2470	10^{-2}	Just the same
AVERAGE ERROR MAGNITUDE	N/A	N/A	ABOUT 10^{-1}	ABOUT 10^{-1}	10^{-2}	
COMMENTS & ADDITIONAL DATA		DIRC IS NOT VERY FAR FROM ROTOR	1(*) Defs in all coordinates ε_1 =+0.2798;1 ε_2 =+0.2192;2 ε_3=-0.1426;3	2(*) Defs in all coordinates ε_1 =-0.7202;1 ε_2 =-0.1232;2 ε_3=-1.5704;3		Errors approximately of [-2] order.

Table 2.-Flapping 3D simulations.

3D SIMULATIONS BLADE ELEMENT/VOXEL DATA		
VOXEL/ELEMENT	**INITIALPOINTS**	**FINAL POINTS**
VOXEL/ELEMENT 1	α=30 θ =40 (-1,2,1) (1,3,1) (1,3,0)	α=30 θ =40 (-0.0500,2.1748,1.1258) (1.8335,1.655,2.2132) (2.3335,1.6552,1.3472)
	α=40 θ =30 (-1,2,1) (1,3,1) (1,3,0)	α=40 θ =30 (-0.5222,2.2320,0.8521) (1.1696,2.0980,2.2869) (1.8124,2.0980,1.5209)
VOXEL/ELEMENT 2	α=30 θ =40 (-1,1,1) (1,1,1) (1,2,0)	α=30 θ =40 (-0.5067,2.9408,1.4472) (0.7201,0.1232,1.5704) (1.7768,0.8892,1.0258)
	α=40 θ =30 (-1,1,1) (1,1,1) (1,2,0)	α=40 θ =30 (-0.9232,1.3660,0.5307) (0.4036,0.3660,1.6441) (1.4294,1.2320,1.1995)

Table 3.-3D points Matrices table details.

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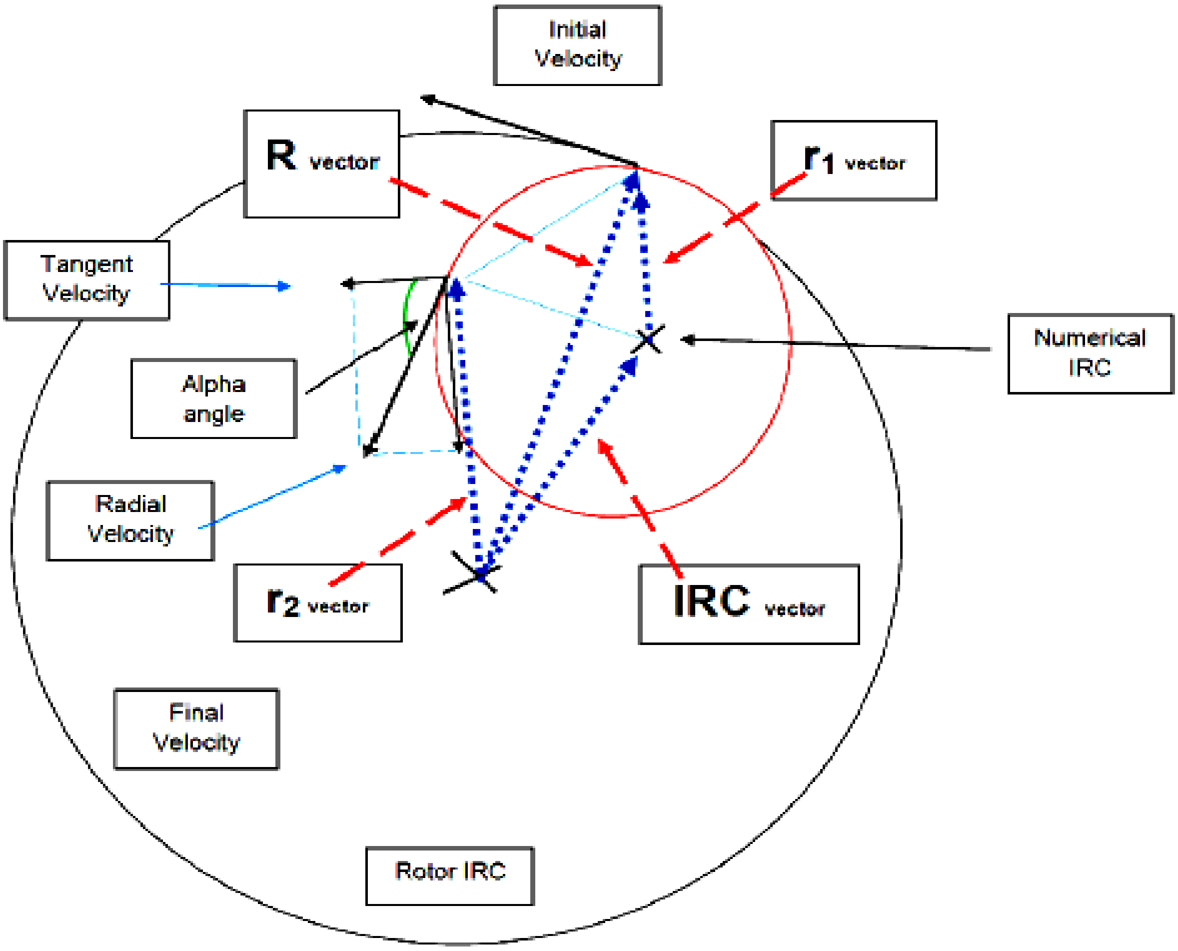

Fig 2 (Enhanced).-Radial and tangent components calculation.Blades contour are the same than Fig 1, but are not sketched for better vector components sharpness.

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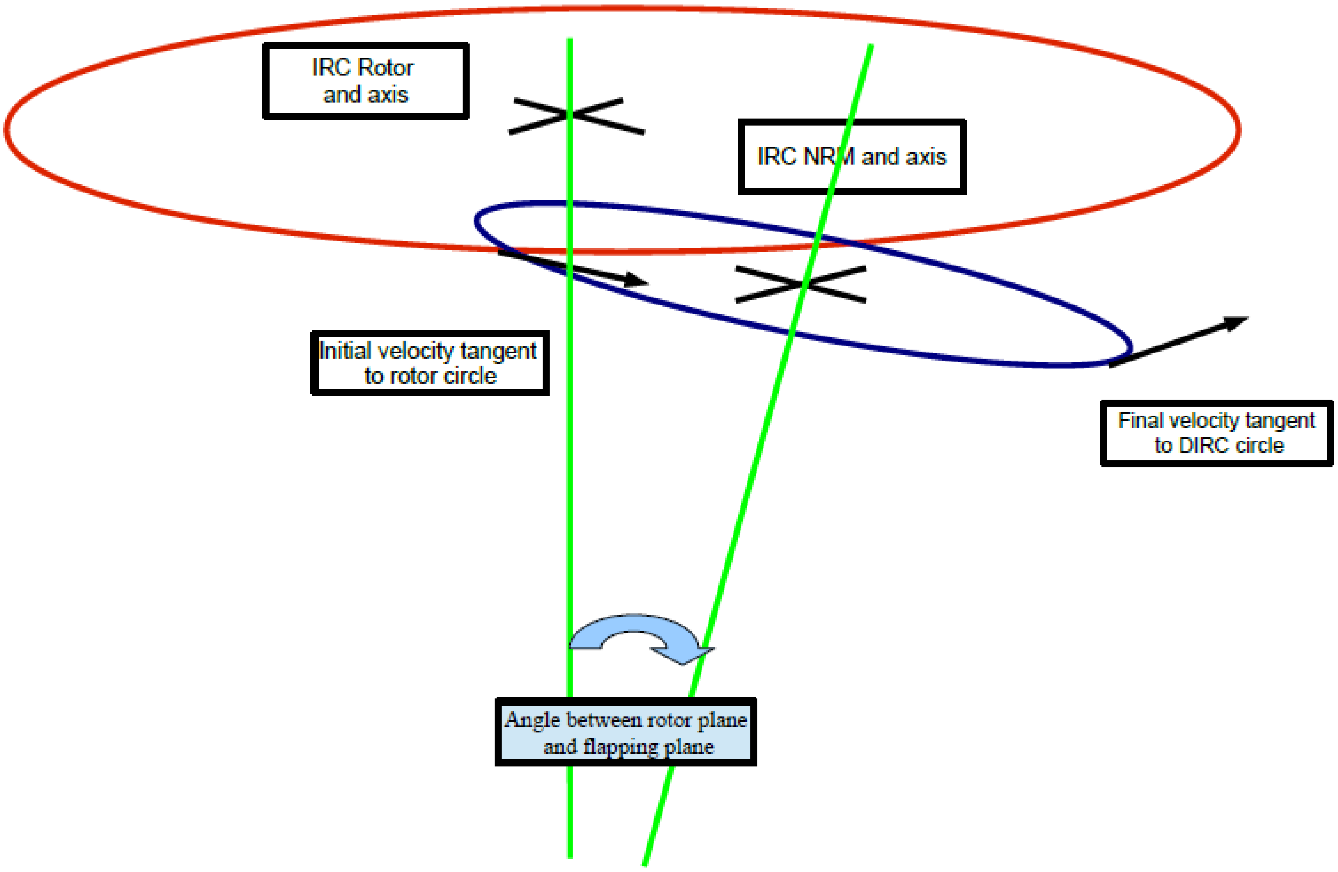

Fig 6 (Enhanced).-The theoretical flapping calculations in 3D.