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Anisotropic Fluid Model of Neutron Star in Isotropic Coordinates

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***Abstract*— We present a spherically symmetric solution of the general relativistic field equations in isotropic coordinates for anisotropic neutral fluid, compatible with a super dense star modeling by considering a specific choice of anisotropy factor Δ which involves an anisotropy parameter α. The solution is well behaved for all the values of u lying in the range 0 < u ≤ 0.1690 and the value of anisotropy parameter α lying from 0.020711 to 0.048 .Further, we have constructed a super-dense star model with all degree of suitability. By assuming surface density $\rho_b = 2\times10^{14}\, g/cm^3$ the resulting well behaved solution has a maximum mass of neutron star, M = 1.48 M⊕ and radius R= 13.02 km for α = 0.0208. Maximum mass is decreasing with the increase of anisotropy (α). The robustness of our result is that it matches with the recent discoveries.**

Index Terms—**Fluid ball, General Relativity, Isotropic coordinates, Schwarzschild metric**

I. INTRODUCTION

Relativistic modeling of compact bodies with anisotropic fluid, in spherical symmetric space -time has continued to attract the attention of researchers since the pioneering work of Bowers and Liang [1]. Further, contemporarily The theoretical investigations of Ruderman [2] about more realistic stellar models show that the nuclear matter may be locally anisotropic at least in certain very high density ranges ($\rho > 10^{15}$ g/ cc), where the nuclear interactions in the stellar matter must be treated relativistically. According to these views, in such massive stellar objects the radial pressure may not be equal to the tangential pressure. Some of the interesting publications in this regard include the work of Dev and Gleiser, [3] ,Mak and Harko[4,5], Chaisi and Maharaj [6] ,Komathiraj and Maharaj [7,8], Thirukkanesh and Ragel [9] , Maurya and Gupta[10] .All these exact solutions of Einstein field equations are in curvature coordinates describing the interior gravitational field of anisotropic fluid spheres. These solutions can be used as models of compact fluid spheres. However, in this paper, we have obtained a new class of exact solutions Einstein field equations in isotropic coordinates describing the interior gravitational field of anisotropic fluid spheres and suitably explain the neutron star model.

II. CONDITIONS FOR WELL BEHAVED SOLUTION

For well behaved nature of the solution in isotropic coordinates, the following conditions should be satisfied:

.(i) The solution should be free from physical and geometrical singularities i.e. finite and positive values of central pressure , central density and non zero positive values of e^{λ} and e^{ω}.

(ii) The solution should have positive and monotonically decreasing expressions for radial pressure, transversal pressure and density with the increase of *r*.

(iii) The solution should have positive value of ratio of pressure-density and less than 1(weak energy condition) and less than 1/3(strong energy condition) throughout within the star, monotonically decreasing as well.

(iv) The casualty condition should be obeyed i.e. velocity of sound should be less than that of light throughout the model. In addition to the above the velocity of sound should be decreasing towards the surface i.e. $\frac{d}{dr}\left(\frac{dp}{d\rho}\right) < 0$ or $\left(\frac{d^2p}{d\rho^2}\right) > 0$ for $0 \le r \le r_b$ i.e. the

Velocity of sound is increasing with the increase of density .In this context it is worth mentioning that the equation of state at ultra-high distribution has the property that the sound speed is decreasing outwards, Canuto [11].

(v) $\frac{p}{\rho} \le \frac{dp}{d\rho}$, Everywhere within the ball.

$$\gamma = \frac{d\log_e P}{d\log_e \rho} = \frac{\rho}{p}\frac{\mathrm{d}p}{\mathrm{d}\rho} \Rightarrow \frac{\mathrm{d}p}{\mathrm{d}\rho} = \gamma\frac{\mathrm{p}}{\rho}$$, for realistic

Matter $\gamma \ge 1$. The stiffness parameter should increase from its lowest value 4/3 from the center to infinity.

(vi) The red shift z should be positive, finite and monotonically decreasing in nature with the increase of *r*.

(vii) The anisotropy factor should be zero at the center and be increasing towards the surface.

III. FIELD EQUATIONS IN ISOTROPIC COORDINATES

We consider the static and spherically symmetric metric in isotropic co-ordinates

$$ds^2 = -e^{\omega}\{dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\} + c^2e^{\upsilon}dt^2 \quad (1)$$

Where, ω and υ are functions of r .Einstein's field equations of gravitation for a non empty space- time are

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$$R^i_j - \frac{1}{2} R\delta^i_j = -\frac{8\pi G}{c^4} T^i_j$$
$$= -\frac{8\pi G}{c^4}[(p_\perp + \rho c^2)v^i v_j - p_\perp \delta^i_j + (p_r - p_\perp)\chi_j \chi^i] \quad (2)$$

Where R^i_j is Ricci tensor, T^i_j is energy- momentum tensor , R is the scalar curvature, p_r denotes the radial pressure, $p_\perp$ is the transversal pressure, ρ the density distribution, χ^i is the unit space like vector in the radial direction and v_i the velocity vector, satisfying the relation

$$g_{ij} v^i v^j = 1 \quad (3)$$

Since the field is static, we have

$$v^1 = v^2 = v^3 = 0 \text{ and } v^4 = \frac{1}{\sqrt{g_{44}}} \quad (4)$$

Thus we find that for the metric (1) under these conditions and for matter distributions with anisotropic pressure the field equation (2) reduces to the following.

$$\frac{8\pi G}{c^4} p_r = e^{-\omega}[\frac{(\omega')^2}{4} + \frac{\omega'}{r} + \frac{\omega'\upsilon'}{2} + \frac{\upsilon'}{r}] \quad (5)$$

$$\frac{8\pi G}{c^4} p_\perp = e^{-\omega}[\frac{\omega''}{2} + \frac{\upsilon''}{2} + \frac{(\upsilon')^2}{4} + \frac{\omega'}{2r} + \frac{\upsilon'}{2r}] \quad (6)$$

$$\frac{8\pi G}{c^2}\rho = -e^{-\omega}[\omega'' + \frac{(\omega')^2}{4} + \frac{2\omega'}{r}] \quad (7)$$

Where, prime (′) denotes differentiation with respect to r . From equations (5) and (6) we obtain following differential equation in ω and υ .

$$e^{-\omega}(\frac{\omega''}{2} + \frac{\nu''}{2} + \frac{(\upsilon')^2}{4} - \frac{(\omega')^2}{4} - \frac{\omega'.\nu'}{2} - \frac{\omega'}{2r} - \frac{\upsilon'}{2r}) = \frac{8.\pi.G}{c^4}(p_\perp - p_r) \quad (8)$$

Our task is to explore the solutions of equation (8) and obtain the fluid parameters $p_r, p_\perp$ and ρ from equation (5), (6) and (7) respectively. To solve the above equation we consider a seed solution as Pant et al[19].

We also take,

$$\frac{8.\pi.G}{c^4}(p_\perp - p_r) = \Delta = \frac{2.\alpha.C^2 r^2 (1 + Cr^2)^{\frac{=22}{13}}}{B^2} \quad (9)$$

Where Δ is the anisotropy factor whose value is zero at the center and increases towards the boundary and α is a positive constant defined as anisotropy parameter?

IV. BOUNDARY CONDITIONS IN ISOTROPIC COORDINATES

(2)

For exploring the boundary conditions, we use the principle that the metric coefficients g_{ij} and their first derivatives $g_{ij,k}$ in interior solution (I) as well as in exterior solution (E) are continuous upto and on the boundary B. The continuity of metric coefficients g_{ij} of I and B on the boundary is known first fundamental form. The continuity of derivatives of metric coefficients g_{ij} of I and B on the boundary is known second fundamental form. The exterior field of a spherically symmetric static charged fluid distribution is described by Schwarzschild metric as

$$ds^2 = \left(1 - \frac{2GM}{c^2 R}\right)c^2 dt^2 - \left(1 - \frac{2GM}{c^2 R}\right)^{-1} dR^2 - R^2 d\theta^2 - R^2 \sin^2\theta \, d\phi^2 \quad (10)$$

Where M is the mass of the ball as determined by the external observer and R is the radial coordinate of the exterior region. Since Schwarzschild metric (10) is considered as the exterior solution, thus we shall arrive at the following conclusions by matching first and second fundamental forms:

(6)

$$e^{\upsilon_b} = \left[1 - 2\frac{GM}{c^2 R_b}\right] \quad (11)$$

$$R_b = r_b . e^{\frac{\omega_b}{2}} \text{ and } p_r(r = r_b) = 0 \quad (12)$$

$$\frac{1}{2}\left(\omega' + \frac{2}{r}\right)_b r_b = \left(1 - 2\frac{GM}{c^2 R_b}\right)^{1/2} \quad (13)$$

$$\frac{1}{2}(\nu')_b r_b = \frac{GM}{c^2 R_b}\left(1 - 2\frac{GM}{c^2 R_b}\right)^{-1/2} \quad (14)$$

Equations (11) to (14) are four conditions, known as boundary conditions in isotropic coordinates. Moreover, (12) and (14) are equivalent to zero pressure of the interior solution on the boundary.

V. A NEW CLASS OF SOLUTION

The equation (8) is solved by assuming the seed solution as Hajj-Boutros [12]. Thus we have,

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$$e^{\omega/2} = \mathbf{B}\left(1+Cr^2\right)^{-\frac{2}{13}}, x = Cr^2, y = \frac{d\upsilon}{dx} \, and$$

$$\frac{8.\pi.G}{c^4}(p_\perp - p_r) = \Delta = \frac{2.\alpha.C^2 r^2 (1+Cr^2)^{\frac{=22}{13}}}{B^2} \qquad (15)$$

$$\frac{dy}{dx} + \frac{4}{13}\frac{1}{(1+x)}y + \frac{1}{2}y^2 = -\frac{44}{169}\frac{1}{(1+x)^2} + \frac{\alpha}{(1+x)^2} \qquad (16$$

Which yields a following solution

$$e^{\frac{\nu}{2}} = \frac{\left\{1 + A\left(1+Cr^2\right)^{\frac{2S}{26}}\right\}\left(1+Cr^2\right)^{\frac{9-S}{26}}}{\mathbf{B}^2} \qquad (17)$$

(17)

$$2S = \sqrt{338\alpha - 7} \qquad (18)$$

The expressions for density and pressure are given by

$$\frac{8\pi G\rho}{c^2} = \frac{C}{169B^2 f^{44}}\left(312+88Cr^2\right) \qquad (19)$$

$$\frac{8\pi G p_r}{c^4} = \frac{C}{169B^2 f^{44}\{1+Af^{2S}\}}\left[\begin{matrix} Cr^2[f^{2S}A(74+18S)+(74+18S)] \\ +26 f^{2S}A(5+S)+(5-S)\end{matrix}\right]$$

(20)

$$\frac{8\pi G p_\perp}{c^4} = \frac{C}{169B^2 f^{44}\{1+Af^{2S}\}}\left[\begin{matrix} Cr^2[f^{2S}A(338\alpha+74+18S)+(338\alpha+74-18S)] \\ +26 f^{2S}A(5+S)+(5-S)\end{matrix}\right]$$

(20a) (19

$$Where, \; f = (1+Cr^2)^{\frac{1}{26}} \qquad (21)$$

VI. PROPERTIES OF THE NEW SOLUTION

The central values of pressure and density are given by

$$\left(\frac{8\pi G p_r}{c^4}\right)_{r=0} = \left(\frac{8\pi G p_\perp}{c^4}\right)_{r=0} = \frac{2C}{13B^2(1+A)}\left[5(1+A)+S(A-1)\right] (22)$$

(22)

$$\left(\frac{8\pi G\rho}{c^2}\right)_{r=0} = \frac{24C}{13B^2} \qquad (23)$$

(23)

The central values of pressure and density will be non zero positive definite, if the following conditions will be satisfied.

A > (S-5)/(S+5), C > 0. (24)

Subjecting the condition that positive value of ratio of pressure-density and less than 1 at the centre i.e. $\frac{p_0}{\rho_0 c^2} \le 1$ which leads to the following inequality,

$$\{\frac{p_r}{\rho c^2}\}_{r=0} = (\frac{p_\perp}{\rho c^2})_{r=0} = \frac{5(1+A)+SA-S}{12(1+A)} = \frac{5}{12} - \frac{S(1-A)}{12(1+A)} \le 1$$

(25)

All the values of A which satisfy equation (24), will also lead to the condition $\frac{p_0}{\rho_0 c^2} \le 1$.

Applying the boundary conditions from (11) to (14), we get the values of the arbitrary constants in terms of Schwarzschild parameters $u = \frac{GM}{c^2 R_b}$ and radius of the star R_b.

$$A = \frac{13.u.(1+Cr_b^2)^{\frac{9-S}{26}} - (9-S).Cr_b^2.d.(1+Cr_b^2)^{\frac{-17-S}{26}}}{(9+S).d.Cr_b^2.(1+Cr_b^2)^{\frac{-17+S}{26}} - 13.u.(1+Cr_b^2)^{\frac{9+S}{26}}}$$

$$B = \sqrt{\frac{(1+Cr_b^2)^{\frac{9-S}{26}} + A(1+Cr_b^2)^{\frac{9+S}{26}}}{d}} \quad (32)$$

$$C = \frac{13(1-d)}{(13d-9)r_b^2}$$

(34)

Where we'd' given by

$$d = (1-2u)^{1/2}$$

Whose

Surface density is given by

$$\frac{8\pi G}{c^2}\rho_b R_b^2 = \frac{Cr_b^2[312+88Cr_b^2]}{169(1+Cr_b^2)^2}$$

Central red- shift is given by (36)

$$Z_0 = \left[\frac{B^2}{1+A} - 1\right]$$

(37)

The surface red shift is given by

$$Z_b = [e^{-\frac{\nu_b}{2}} - 1] = d^{-1} - 1$$

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Table1: The effect of anisotropy on maximum radius and maximum mass of a star.

$\frac{r}{r_b}$	$\frac{8\pi G}{c^4} p_r r_b^2$	$\frac{8\pi G}{c^4} p_\perp r_b^2$	$\frac{8\pi G}{c^2} \rho r_b^2$	$\frac{p_r}{\rho c^2}$	$\frac{p_\perp}{\rho c^2}$	$\frac{1}{c^2}(\frac{dp_r}{d\rho})$	$\frac{1}{c^2}(\frac{dp_\perp}{d\rho})$	γ	Δ
0	2.059444	2.059444	22.679109	0.090808	0.09080	0.135437	0.119459	1.491	0.000000
0.1	1.994553	2.002166	22.199630	0.089846	0.09018	0.135234	0.119458	1.505	0.007613
0.2	1.814585	1.842844	20.865910	0.086964	0.08831	0.134629	0.119456	1.548	0.028258
0.3	1.556536	1.613058	18.942257	0.082173	0.08515	0.133634	0.119448	1.626	0.056522
0.4	1.263941	1.350245	16.741837	0.075496	0.08065	0.132262	0.119424	1.752	0.086304
0.5	0.973111	1.086038	14.528994	0.066977	0.07475	0.130523	0.119362	1.949	0.112927
0.6	0.707178	0.841081	12.475787	0.056684	0.06741	0.128422	0.119231	2.266	0.133903
0.7	0.476895	0.625505	10.666128	0.044711	0.05864	0.125962	0.118991	2.817	0.148610
0.8	0.284389	0.441968	9.121407	0.031178	0.04845	0.123143	0.118600	3.949	0.157579
0.9	0.126991	0.288837	7.827358	0.016224	0.03690	0.119964	0.118015	7.394	0.161846
1	0.000000	0.162536	6.753732	0.000000	0.02406	0.116431	0.117198	infinity	0.162536

Table 2: The march of pressure, density, pressure-density ratio, square of adiabatic speed of sound within the ball for α= 0.0208 for which u_{max}=0.1687

α	u_{max}	For $\rho_b = 2\text{x}10^{14}$g/cc $R_b(km)$	$\frac{M}{M_\Theta}$
0.020711	0.1690	13.03	1.4856
0.021	0.1679	13.02	1.474
0.025	0.1539	12.83	1.33
0.03	0.1344	12.43	1.13
0.035	0.11	11.71	0.87
0.040	0.08	10.38	0.557
0.045	0.041	7.81	0.216
0.048	0.004	2.53	0.0068

VII. DISCUSSIONS AND CONCLUSIONS

From Table 2, it has been observed that the physical quantities

$p_r, p_\perp, \frac{p_r}{\rho c^2}, \frac{p_\perp}{\rho . c^2}, \frac{dp_r}{c^2 d\rho}, \frac{dp_\perp}{c^2 d\rho}, z$) are positive at the centre and within the limit of realistic state equation and monotonically decreasing while the quantities γ and Δ are increasing for all values of u satisfying the inequality 0.1690 > u >0. Thus, the solution is well behaved for all values $0 < u < 0.1690$ for α lying in the range 0.0207 to 0.048·

Table 1 shows that with the increase in the value of anisotropy in the fluid ball the mass decreases. This is because of diversion of pressure away from radial direction. we present a model of super dense neutron star based on the particular solution discussed above by assuming surface density $\rho_b = 2\times10^{14} g/cm^3$ the resulting well behaved solution has a maximum mass M = 1.48 M⊕ and radius R= 13.02 km.

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