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Numerical covariance analysis of MDoF hysteretic non- linear mechanical system subjected to modulated stochastic inputs

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Abstract*— Structural systems subjected to random dynamic actions, response evaluation must be based on a stochastic approach. If nonlinear mechanical behaviour is considered, only in very few cases complete exact solutions exist. In this work, a numerical procedure is proposed, providing an expression of response in the state space that in our best knowledge has not yet been presented in literature, by using a formal justification with reference to seismic input processes modelled as linear filtered white noise with time varying parameters. Structural nonlinearities are modelled by the hysteretic differential Bouc-Wen model (BWM). The non-linear differential equation of the response covariance matrix is then solved by using a numerical algorithm, which updates the equivalent linear system coefficients at each step. Numerical computational effort is minimized by taking into account symmetry characteristic of state space covariance matrix. As application of the proposed method, both a single degree of freedom system and multi-storey shear type buildings are analysed to obtain the response in terms of standard deviation of displacements and its sensitivity. Finally, the mean value of dissipated hysteretic energy and time variation of integration steps are evaluated.

***Index Terms*— Lyapunov Equation, Covariance analysis, stochastic dynamics, non stationary processes, Bouc-Wen hysteretic non linear model, equivalent linearization.**

I. INTRODUCTION

An extensive categories of engineering situations deals with structural response to dynamic non deterministic actions, like earthquake accelerations; in this case, structures often present a non linear behavior, so a random dynamic analysis is needed due to the intrinsic random nature of inputs. The principal aim of studying such a system is the knowledge of the structural response and the auto-covariance matrix in the space state. With reference to a non stationary, multi correlated input, there are numerical difficulties, due to the fact that input should be defined by using processes modulated both in amplitude and in frequency; these troubles, in addition to system non linearity's, make the analytical solution of motion differential equations quite difficult.

Some mathematical approaches aimed to this issue, use modal analysis, while some others utilize approximate numerical implementation, by mean of constant time step algorithms and without taking into account advantages related to covariance matrix symmetry properties (see for instance [15]).

In this work, a numerical approach for complete space state evolutive covariance evaluation in nodal state space is proposed; the non linear behavior is overcome implementing the equivalent statistical linearization technique. Thus, the Lyapunov matrix differential equation is converted into a vectorial one by a simple formal rule; therefore, for a generic linear m-dof, the number of independent equations is $\frac{3}{2}\left(3m^2+m\right)$, rather than $9m^2$.

The algorithm proposed for the numerical solution is applied to a non-stationary Gaussian filtered input, modulated in amplitude and frequency, acting on a general non linear MDoF base excited system. Its effectiveness is evaluated in determining the response and its sensitivity.

II. NON LINEAR HYSTERETIC MECHANICAL MODEL

To assess the structural response of buildings subjected to earthquakes an enough accurate modelling of their non linear behaviour is needed, characterized by hysteresis. Thus, the non linear differential Bouc-Wen model [3] [22] has been widely adopted in field of structural dynamic.

For a one degree of freedom system (fig.1) having a mass m subjected to a generic base acceleration a(t) and characterized by a hysteretic constitutive law described by means of the Bouc-Wen model (fig. 2) the motion equation is:

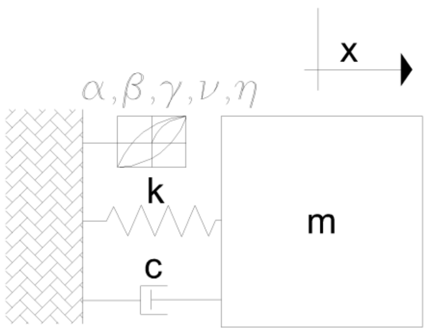

Fig. 1 SDoF non-linear system Bouc-Wen model

$$m\ddot{x}(t)+F(x,\ \dot{x},\ z,\ t)=ma(t) \qquad (1)$$

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Where the first term is the inertia force whereas the restoring force is $F(x,\ \dot{x},\ z)$. This term can be divided into two parts:

$$F(x,\ \dot{x},\ z;\ t) = L(x,\ \dot{x};\ t) + H(x,\ \dot{x},\ z;\ t) \qquad (2)$$

Where the first one $L(x,\ \dot{x};\ t)$ is due to the linear viscous-elastic contribution and the second one $H(x,\ \dot{x},\ z;\ t)$ is due to the hysteretic one:

$$L(x,\ \dot{x};\ t) = c\dot{x}(t) + \alpha k x(t) \qquad (3)$$

$$H(x,\ \dot{x},\ z;\ t) = (1-\alpha)\ kz(t) \qquad (4)$$

c is the damping and k is the initial elastic stiffness. The new variable $z(t)$ is an additional internal one which satisfies the following nonlinear equation:

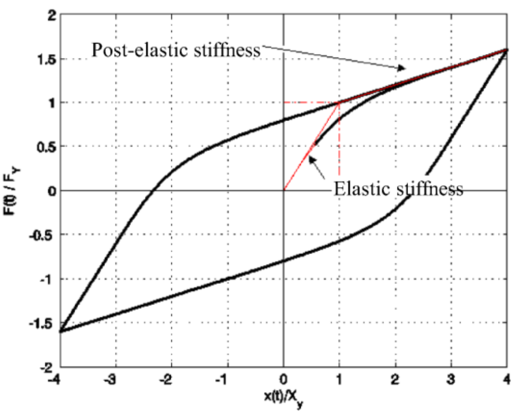

Fig. 2: Bouc Wen Hysteretic Model

$$\dot{z}(t) = G(z,\dot{x}) = \dot{x}(t)\left[\lambda - |z(t)|^{\eta}\left(\beta + \gamma \cdot sig\{z(t)\} \cdot sig\{\dot{x}(t)\}\right)\right]$$
(5)

The five parameters β, γ, η, α and λ, which appear in (5) are the shape factors of the hysteretic cycle [2]. Engineering mechanical quantities can be related to the analytical parameters previous described in order to model real structural elements. For softening laws modelling, able to describe structural elements subjected to strong earthquakes, it is possible to define the initial and the post-elastic stiffness, k_i and k_f, as $k_i = \left(\frac{\partial G}{\partial x}\right)_{Z=0} = \alpha k + (1-\alpha)k\lambda$ and $k_f = \left(\frac{\partial G}{\partial x}\right)_{z_{\max}} = \alpha k$. Moreover, if the unloading stiffness is equal to the elastic one (as it happened in many cases) then $\beta = \gamma$. In order to describe the mechanical constitutive law, only three mechanical parameters are needed: the initial elastic stiffness k_i, the post-elastic one k_f and finally the maximum hysteretic restoring force F_Y (or maximum elastic displacement x_Y).

Considering now a seismic excitation $F(t) = -m\ddot{x}_g$, the motion equations are:

$$\begin{cases} m\ddot{x}(t) + c\dot{x}(t) + \alpha k x(t) + (1-\alpha)\ kz(t) = -m\ddot{x}_g(t) \\ \dot{z}(t) = \dot{x}(t)\left[1 - \frac{1}{2} z(t)\beta\left(1 + sig\{z(t)\} \cdot sig\{\dot{x}(t)\}\right)\right] \end{cases} \qquad (6)$$

where the parameter η can be properly evaluated according to the relative smoothness in transition between elastic and post-elastic phases.

Introducing $\omega_0 = \frac{k}{m}$, $f_Y = \frac{F_y}{m}$, $f(t) = \frac{F(t)}{m}$ and $2\xi_0\omega_0 = \frac{c}{m}$, equation (6) can be rewritten as:

$$\begin{cases} \ddot{x}(t) + 2\xi\omega_0\dot{x}(t) + \alpha\omega_0^2 x(t) + (1-\alpha)\, z(t)\omega_0^2 = -\ddot{x}_g(t) \\ \dot{z}(t) = \dot{x}(t)\left[1 - \frac{1}{2}\left(\frac{z(t)}{X_Y}\right)^{\eta}\left(1 + sig\{z(t)\} \cdot sig\{\dot{x}(t)\}\right)\right] \end{cases} \qquad (7)$$

A. *The equivalent linearization method*

Various mathematical methods can be used in order to solve the differential stochastic problem in eq. (7). In this work, the equivalent stochastic linearization is adopted. The basic idea of the scheme, is that the equation describing the non linear system can be replaced by an appropriate linear one considering that the hypothesis of a Gaussian response process still holds. The approximate linearized form of the original non linear equation is then achieved minimizing the difference between the non linear equation and the linearized one [18]. Then, the equation governing the internal variable $z(t)$ is replaced with the following:

$$\dot{z}(t) = G(z,\dot{x}) = -c^{eq}(t)\dot{x}(t) - k^{eq}(t)\dot{z}(t) \qquad (8)$$

where the linearized coefficients $c^{eq}(t)$ and $k^{eq}(t)$ are nonlinear functions of covariance response elements. Atalik and Utku [1] provided these equivalent coefficients, which appear in equation (8), for the most common case (used by many authors) of $\eta = 1$, and using the above stated conditions ($\beta = \gamma$ and $\lambda = 1$) in the hypothesis of processes z and $\dot{x}$ jointly Gaussian:

$$c^{eq}(t) = \sqrt{\frac{2}{\pi}}\beta\left[\sigma_z(t) + \frac{\langle \dot{x}(t)z(t)\rangle}{\sigma_{\dot{x}}(t)}\right] - 1 \qquad (9)$$

$$k^{eq}(t) = \sqrt{\frac{2}{\pi}}\beta\left[\sigma_{\dot{x}}(t) + \frac{\langle \dot{x}(t)z(t)\rangle}{\sigma_z(t)}\right] \qquad (10)$$

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where terms $\sigma_z(t)$ and $\sigma_{\dot{x}}(t)$ are, the standard deviations of variables z and $\dot{x}$ respectively and also $\langle \dot{x}(t)z(t)\rangle$ [1] is the cross covariance of mentioned variables.

With reference to energetic balance, it is noteworthy to underline that it plays a key role in evaluating the performance of a structure. In literature, usually, the "relative energy balance" is considered; at the time t it can be written as:

$$E_k(t)+E_D(t)+E_s(t)+E_H(t)=E_I(t) \tag{11}$$

where: E_k , E_D , E_S , E_I are respectively the kinetic, dissipated by damping, elastic and input energy. The hysteretic dissipated energy contribution can be expressed as:

$$E_H=\int_0^t k(1-\alpha)z\dot{x}dt \tag{12}$$

During the seismic event part of the absorbed energy is stored into the structure in the form both of kinetic and strain energy; the residue fraction is dissipated by the damping and by structural hysteretic behavior. Finally, at the end of the motion the energy transferred to the structure is completely dissipated; therefore if t_q is the quiet time, the energy balance can be written as:

$$E_D(t_q)+E_H(t_q)=E_I(t_q) \tag{13}$$

III. PRE FILTERS APPROACH

The time domain stochastic analysis is usually performed considering input processes modelled by solving filter differential equations, whose input is a white noise process. It is known as *pre filter* technique.

This methodology is able to give a representation of physical phenomena whose frequency characterization has a serious variation during time and that could have structural resonant consequences, keeping on all the advantages offered by shot noise inputs [16].

In particular, in pre filter approach, the filter response is described by the $2m_f$ filter space state vector $\overline{Y}_f(t)=\left(\overline{X}_f(t),\dot{\overline{X}}_f(t)\right)^T$, solution of the $2m_f$ set of differential equations

$$\dot{\overline{Y}}_f(t)=\mathbf{A}_f(t)\overline{Y}_f(t)+\mathbf{G}_f\overline{W}_f(t) \tag{14}$$

that generally could have a time dependent form, when not only the frequency but also the amplitude of loads have an intrinsic evolutive nature. In (14) W_f is a vector of n_f white noise processes (stationary or no stationary), $\mathbf{G}_f$ is a $2m_f\times n_f$ matrix that couples the excitation components of forcing vector to the filter degree of freedom and finally $\mathbf{A}_f(t)$ is the $2m_f$ x $2m_f$ filter system matrix, whose generic form is

$$\mathbf{A}_f(t)=\begin{pmatrix}\mathbf{0} & \mathbf{I}\\ \mathbf{H}_f^1(t) & \mathbf{H}_f^2(t)\end{pmatrix} \tag{15}$$

Then, adopting the pre filter technique, the motion differential equations, written in the space state, $\overline{Y}_s(t)=\left(\overline{X}_s(t)\ \overline{Z}(t)\ \dot{\overline{X}}_s(t)\right)^T$ are:

$$\dot{\overline{Y}}_s(t)=\mathbf{A}_{\mathbf{S_{eq}}}(\mathbf{R},t)\overline{Y}_s(t)+\boldsymbol{\alpha}(t)\overline{Y}_f(t) \tag{16a}$$

$$\dot{\overline{Y}}_f=\mathbf{A}_f(t)\overline{Y}_f+\mathbf{G}_f\overline{W}_f(t) \tag{16b}$$

w-here

$$\mathbf{A}_{\mathbf{S_{eq}}}=\begin{pmatrix}\mathbf{0} & \mathbf{0} & \mathbf{I}\\ \mathbf{0} & \mathbf{K}^{eq} & \mathbf{C}^{eq}\\ \mathbf{H}_s^1 & \mathbf{H}_s^2 & \mathbf{H}_s^3\end{pmatrix} \tag{17}$$

is the structural equivalent system matrix and

$\mathbf{H}_S^1=-\mathbf{M}^{-1}\mathbf{K}_L$, $\mathbf{H}_S^2=-\mathbf{M}^{-1}\mathbf{K}_{Hy}$, $\mathbf{H}_S^1=-\mathbf{M}^{-1}\mathbf{C}$,

being $\mathbf{M},\mathbf{C},\mathbf{K_L}$ and $\mathbf{K_{Hy}}$ respectively the mass, damping, linear and hysteretic stiffness matrices, and

$$\boldsymbol{\alpha}(t)=\begin{pmatrix}\mathbf{0} & \mathbf{0}\\ \boldsymbol{\alpha}_1(t) & \boldsymbol{\alpha}_2(t)\end{pmatrix} \tag{18}$$

is a $3m_s x 2m_f$ time dependent matrix.

Equations space state structure can be summarized as:

$$\begin{cases}\dot{\overline{Y}}(t)=\mathbf{A_{eq}}(\mathbf{R},t)\overline{Y}(t)+\mathbf{G}(t)\overline{W}_f(t)\\ \dot{\overline{Z}}(t)=\mathbf{C}^{eq}\dot{\overline{X}}(t)+\mathbf{K}^{eq}\overline{Z}(t)\end{cases} \tag{19}$$

where:

$$\overline{Y}^T=\left(\overline{X}_s\quad \overline{X}_f\ \overline{Z}\ \dot{\overline{X}}_S\quad \dot{\overline{X}}_f\right)^T \tag{20}$$

is a new global $3m_s+m_f$ space state vector (structure plus filter and internal variable) while $\mathbf{G}(t)=\begin{pmatrix}0\\ G_f\end{pmatrix}$. As a consequence, the equivalent system matrix in its general form has the following expression:

[1] where $\langle\bullet\rangle$ denotes the mathematical expectation

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$$\mathbf{A}_{eq}(\mathbf{R},t)=\begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} \\ \mathbf{0} & \mathbf{0} & \mathbf{K}_{eq}(\mathbf{R},t) & \mathbf{C}_{eq}(\mathbf{R},t) & 0 \\ \mathbf{H}_S^1(t) & \boldsymbol{\alpha}_1(t) & \mathbf{H}_S^2(t) & \mathbf{H}_S^3(t) & \boldsymbol{\alpha}_2(t) \\ \mathbf{0} & \mathbf{H}_f^1(t) & \mathbf{0} & \mathbf{0} & \mathbf{H}_f^2(t) \end{pmatrix} \quad (21)$$

Both structural matrix and response vectors evidently depend on a design parameter vector $\bar{b}$, whose elements may be mechanical parameters, as cross sections, Young modulus, boundary conditions etc. Also filter parameters and input intensity could be considered as design parameters. Thus, matrix equivalent space state structure plus filter and equation space state structure added to filter equation can be explicitly re-written as:

$$\mathbf{A}_{eq}(\mathbf{R},\bar{b},t)=\begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} \\ \mathbf{0} & \mathbf{0} & \mathbf{K}_{eq}(\mathbf{R},\bar{b},t) & \mathbf{C}_{eq}(\mathbf{R},\bar{b},t) & 0 \\ \mathbf{H}_S^1(\bar{b},t) & \boldsymbol{\alpha}_1(\bar{b},t) & \mathbf{H}_S^2(\bar{b},t) & \mathbf{H}_S^3(\bar{b},t) & \boldsymbol{\alpha}_2(\bar{b},t) \\ \mathbf{0} & \mathbf{H}_f^1(\bar{b},t) & \mathbf{0} & \mathbf{0} & \mathbf{H}_f^2(\bar{b},t) \end{pmatrix} \quad (22)$$

$$\dot{\bar{Y}}(b,t)=\mathbf{A}_{eq}(\mathbf{R},\bar{b},t)\bar{Y}(\bar{b},t)+\mathbf{G}(\bar{b},t)\overline{W}_f(t)$$

IV. SPACE STATE COVARIANCE AND SENSITIVITY EVALUATION

As pointed out in previous section, the motion equation in the space state is:

$$\dot{\bar{Y}}(t)=\mathbf{A}_{eq}(\mathbf{R},t)\bar{Y}(t)+\mathbf{G}(t)\overline{W}_f(t) \qquad (23)$$

and the Lyapunov Matrix Differential equation has the following expression:

$$\dot{\mathbf{R}}_{YY}(t)=\mathbf{A}_{eq}(\mathbf{R},t)\mathbf{R}_{YY}(t)+\mathbf{R}_{YY}(t)\mathbf{A}_{eq}(\mathbf{R},t)^T+\mathbf{B}(t) \qquad (24)$$

where the matrix $\mathbf{B}(t)$ can be expressed as:

$$\mathbf{B}(t)=\left\langle \mathbf{G}\overline{W}_f\bar{Y}^T\right\rangle+\left\langle \bar{Y}W_f{}^T\mathbf{G}^T\right\rangle \qquad (25)$$

The auto-covariance matrix $\mathbf{R}_{YY}$, because of its symmetry, is described by $\frac{1}{2}\left[\left(3m_s+m_f\right)^2+\left(3m_s+m_f\right)\right]$ independent elements.

Now, let $\bar{Y}^*(t)$ be the solution of (23); consequently we can state that system matrix $\mathbf{A}^*(t)=\mathbf{A}\left(\mathbf{R}^*(t),t\right)$. Afterwards, a matrix $\boldsymbol{\Phi}^*$ such that:

$$\begin{cases} \dot{\boldsymbol{\Phi}}^*=\mathbf{A}^*(t)\boldsymbol{\Phi}^* \\ \boldsymbol{\Phi}^*(0)=\mathbf{I} \end{cases} \qquad (26)$$

will exist.

Considering zero initial condition the solution of space state structure plus filter equation has the general expression:

$$\bar{Y}^*(t)=\int_0^t \boldsymbol{\Phi}^*(t-\tau)\mathbf{G}\overline{W}_f(\tau)d\tau \qquad (27)$$

Introducing (27) into (25) we obtain the complete expression of $\mathbf{B}(t)$:

$$\mathbf{B}(t)=\mathbf{P}(t)+\mathbf{P}^T(t) \qquad (28)$$

Where $\mathbf{P}(t)$ can be rewritten as:

$$\mathbf{P}(t)=\int_0^t \boldsymbol{\Phi}^*(t-\tau)\mathbf{G}\left\langle \overline{W}(\tau)\overline{W}^T(t)\right\rangle\mathbf{G}^T d\tau = \\ =\int_0^t \boldsymbol{\Phi}^*(t-\tau)\mathbf{N}(t,\tau)d\tau \qquad (29)$$

The matrix $\boldsymbol{\Phi}^*$, from a formal point of view, plays the same role of the so called transition matrix for a linear system $\boldsymbol{\Phi}$ [22]; in fact the latter, in the case of non-linear structural behaviour, cannot be exactly determined.

However, a significant simplification in $\mathbf{P}(t)$ takes place when the forcing vector is a white noise process. In this case, the knowledge of $\boldsymbol{\Phi}^*$ is not needed: due to the Dirac function properties both integrals are equal to $\boldsymbol{\Phi}^*(0)\mathbf{G}\mathbf{R}_{ww}(t,t)\mathbf{G}^T$; by using the second of (26) we obtain:

$$\mathbf{B}(t)=2\mathbf{G}\mathbf{R}_{ww}(t,t)\mathbf{G}^T=\begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{L}(t) \end{bmatrix}_{[(3m_s+m_f)x(3m_s+m_f)]} \qquad (30)$$

where the sub matrix $\mathbf{L}(t)$ has the generic diagonal element equal to:

$$\left[\mathbf{L}(t)\right]_{i,i}=\frac{2\pi S_{0i}}{m_i^2}\varphi_i^2(t) \qquad (31)$$

while the extra diagonal elements hold:

$$\left[\mathbf{L}(t)\right]_{i,j}=\frac{2\pi S_{0i}}{m_i^2}\varphi_i^2(t)=\frac{2\pi S_{0j}}{m_j^2}\varphi_j^2(t) \qquad (32)$$

V. APPLICATION: SEISMIC BASE EXCITED BUILDINGS - MODELLING OF GROUND MOTION

A typical non-stationary stochastic load is represented by seismic loads acting on base of multi-storey buildings. Ground accelerograms are often modelled as a zero mean stochastic no stationary process. This assumption is able to consider in the real way intrinsic probabilistic nature of earthquakes. Moreover, a suitable description with a no constant contents both in amplitude and in frequency could be considered. A wide used stochastic approach is those proposed by Kanai and Tajimi [14][23], based on linear second order filter applied to a stationary white noise. Therefore, ground acceleration $\ddot{\mathrm{X}}_g(t)$ is given by:

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$$\begin{cases} \ddot{X}_g(t) = \ddot{X}_f(t) + \varphi(t)W(t) \\ \ddot{X}_f(t) + 2\xi_g\omega_g\dot{X}_f(t) + \omega^2{}_g X_f(t) = -\varphi(t)W(t), \end{cases} \quad (33)$$

where $X_f(t)$ is the response of the Kanai-Tajimi filter, having frequency ω_g and damping coefficient ξ_g ,eventually time depending, and $W(t)$ is the white noise, whose constant bilateral Power Spectral Density (PSD) function is S_0. It is related to the Peak Ground Acceleration PGA $\ddot{X}_g^{max}$ by means of relation:

$$S_0 = \frac{0.074\xi_g(\ddot{X}_g^{max})^2}{\omega_g(1+4\xi_g{}^2)}. \quad (34)$$

No stationary characteristic is introduced by the deterministic temporal modulation function $\varphi(t)$, which controls intensity variation, without change earthquake frequency contents. Moreover, different modulation functions have been proposed in literature with the aim of best accordance with real accelerograms time evolution. The most used are the Sinozuka - Sato [20], Hsu - Bernard [10], Iwan -Hou [11] and the Jennings et al. [12] modulation functions [16], reported in appendix A.

A. *Building – ground motion equations*

The building model is performed on a generic shear type structure; the constitutive behaviour of the structure, characterized by stiffness depending on deformation and on hysteretic dissipation, will be described, as earlier pointed out, by adopting the differential BW explained before. Then, for the m-DoF shear-type structure shown in fig. 3, the non linear restoring forces $\overline{Z}(t)$, the ground and interstorey relative displacement vectors $\overline{Y}$ and $\overline{X}$ are introduced, where the generic $i-th$ relation between the two is: $\overline{X}_i = \overline{Y}_i - \overline{Y}_{i-1}$ being $i = 1...m$ and assuming that $Y_0 = 0$ so we can affirm that:

$x_1 = y_1$, $x_2 = y_2 - y_1$, $x_3 = y_3 - y_2$, $x_m = y_m - y_{m-1}$ and then $y_i = \sum_{k=1}^{i} x_k$.

Inter-storey restoring forces are expressed as $h_i(x_i,\ \dot{x}_i,\ z_i)$:

$$m_i\ddot{u}_i + h_i(x_i,\ \dot{x}_i,\ z_i) - h_{i+1}(x_{i+1},\ \dot{x}_{i+1},\ z_{i+1}) = 0 \quad (35)$$

where the absolute displacement is

$$u_i = y_i + x_g \quad (36)$$

$$m_i\ddot{y}_i + h_i(x_i, \dot{x}_i, z_i) - h_{i+1}(x_{i+1}, \dot{x}_{i+1}, z_{i+1}) = -m_i\ddot{x}_g \quad (37)$$

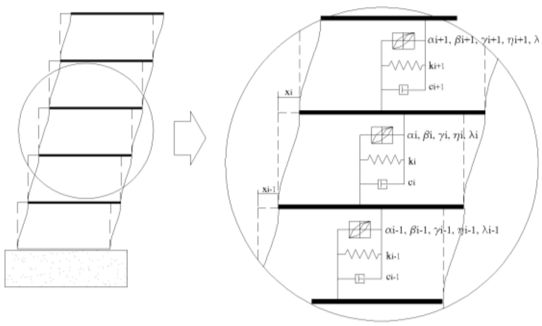

Fig. 3 MDoF structure

The restoring force has the following expression:

$$\begin{cases} h_i(x_i, \dot{x}_i, z_i) = c_i\dot{x}_i + \alpha_i k_i x_i + (1-\alpha_i)k_i z_i \\ \dot{z}_i = G(\dot{x}_i, z_i) \end{cases} \quad (38)$$

Then for each storey we obtain:

$$\begin{cases} \ddot{y}_i + \hat{h}_i(x_i, \dot{x}_i, z_i) - \hat{h}_{i+1}(x_{i+1}, \dot{x}_{i+1}, z_{i+1}) = -\ddot{x}_g \\ \dot{z}_i = G(\dot{x}_i, z_i) \end{cases} \quad (39)$$

where

$$\begin{aligned} &\hat{h}_i(x_i, \dot{x}_i, z_i) = 2\xi_i\omega_i\dot{x}_i + \alpha_i\omega_i^2 x_i + (1-\alpha_i)\omega_i^2 z_i \\ &\hat{h}_{i+1}(x_{i+1}, \dot{x}_{i+1}, z_{i+1}) = \mu_{i+1}\left(2\xi_{i+1}\omega_{i+1}\dot{x}_{i+1} + \alpha_{i+1}\omega_{i+1}^2 x_{i+1} + (1-\alpha_{i+1})\omega_{i+1}^2 z_{i+1}\right) \end{aligned} \quad (40)$$

and $\mu_{i+1} = \frac{m_{i+1}}{m_i}$

As stated in previous section ground motion could be modelled as:

$$\ddot{X}_g = \ddot{X}_f + \varphi(t)W(t) \quad (41)$$

Furthermore, to obtain the motion equations for the complete non linear structural system of a *m* floors is convenient to express the equations introducing the inter storey drift vector:

$$\overline{\Delta}_x = [X_1, X_2 - X_1, ..., X_m - X_{m-1}]^T = [\Delta_1, \Delta_2, ..., \Delta_m]^T \quad (42)$$

and the internal variable vector $\overline{Z}$ related to the inter storey system.

A linear equation associates the inter storey drift $\overline{\Delta}_x(t)$ and displacement $\overline{X}(t)$ vectors:

$$\overline{\Delta}_x(t) = \mathbf{T}\overline{X}(t) \Leftrightarrow \overline{X}(t) = \mathbf{T}^{-1}\overline{\Delta}_x(t),$$

Where $\mathbf{T}$ is called transformation matrix and has the following expression:

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$$\mathbf{T} = \begin{pmatrix} 1 & 0 & 0 & 0 & & & & \\ -1 & 1 & 0 & 0 & & & & \\ 0 & -1 & 1 & 0 & & & 0 & \\ \dots & \dots & -1 & 1 & \ddots & & & \\ & & & \ddots & \ddots & \ddots & & \\ & & & & \ddots & 1 & 0 & \dots \\ & & 0 & & & -1 & 1 & 0 \\ & & & & & 0 & -1 & 1 \\ & & & & & & & \end{pmatrix} \quad (44)$$

Consequently the differential motion equation in the inter-storey system considering a ground seismic acceleration are:

$$\begin{cases} \mathbf{MT^{-1}}\ddot{\overline{\Delta}}_X + \mathbf{CT^{-1}}\dot{\overline{\Delta}}_X + \mathbf{K}_L\mathbf{T^{-1}}\overline{\Delta}_X + \mathbf{K}_{Hy}\mathbf{T^{-1}}\overline{\Delta}_Z = -\mathbf{M}\bar{r}\ddot{X}_g \\ \dot{\overline{Z}} = \mathbf{C}^{eq}\dot{\overline{\Delta}}_X + \mathbf{K}^{eq}\overline{Z} \\ \ddot{X}_f(t) + 2\xi_f\omega_f\dot{X}_f(t) + \omega_f^{\,2}X_f(t) = -W(t)\varphi(t) \end{cases} \quad (45)$$

Where $\bar{r} = [1,1,1,\dots,1]^T$ is the drag vector and $\mathbf{M}$, and $\mathbf{C}$ are respectively principal structure mxm *mass* and *viscosity* matrices while $\mathbf{K}_L$ and $\mathbf{K}_{Hy}$ represent, in that order, the linear stiffness *and* hysteretic *stiffness mxm* building matrices. Mechanical filter parameters are ground damping ratio ξ_f and frequency ω_f and are supposed to be time varying during all seismic duration.

System equations motion (45) can be rephrased as:

$$\begin{cases} \ddot{\overline{\Delta}}_X + \mathbf{TM^{-1}CT^{-1}}\dot{\overline{\Delta}}_X + \mathbf{TM^{-1}K}_L\mathbf{T^{-1}}\overline{\Delta}_X + \mathbf{TM^{-1}K}_{Hy}\mathbf{T^{-1}}\overline{Z} = -\mathbf{T}\bar{r}\ddot{X}_g \\ \dot{\overline{Z}} = \mathbf{C}^{eq}\dot{\overline{\Delta}}_X + \mathbf{K}^{eq}\overline{Z} \\ \ddot{X}_f(t) + 2\xi_f\omega_f\dot{X}_f(t) + \omega_f^{\,2}X_f(t) = -W(t)\varphi(t) \end{cases} \quad (46)$$

By introducing the following matrices:

$$\begin{cases} \mathbf{C}^* = \mathbf{TM^{-1}CT^{-1}} \; ; \\ \mathbf{K_L}^* = \mathbf{TM^{-1}K}_L\mathbf{T^{-1}} \; ; \\ \mathbf{K_{Hy}}^* = \mathbf{TM^{-1}K}_{Hy}\mathbf{T^{-1}} \; ; \end{cases} \quad (47)$$

eq. (47) can be re-written in its canonical form:

$$\begin{cases} \ddot{\overline{\Delta}}_X + \mathbf{C}^*\dot{\overline{\Delta}}_X + \mathbf{K}_L{}^*\overline{\Delta}_X + \mathbf{K}_{Hy}{}^*\overline{Z} = -\bar{u}\ddot{X}_g \\ \dot{\overline{Z}} = \mathbf{C}^{eq}\dot{\overline{\Delta}}_X + \mathbf{K}^{eq}\overline{Z} \\ \ddot{X}_f(t) + 2\xi_f\omega_f\dot{X}_f(t) + \omega_f^{\,2}X_f(t) = -W(t)\varphi(t) \end{cases} \quad (48)$$

where the matrices $\mathbf{C}^*, \mathbf{K_L}^*, \mathbf{K_{Hy}}^*, \mathbf{C^{eq}}, \mathbf{K^{eq}}$, are written comprehensively in appendix B and $\bar{u} = \mathbf{T}\bar{r} = [0,0,\dots1]^T$.

For this system the energy balance equation will still have the form of equation (11), but the hysteretic dissipated energy term, in this case, can be expressed as:

$$E_H = \int_0^t \dot{\overline{\Delta}}_x{}^T \mathbf{K}^*_{\mathbf{Hy}} \overline{Z} dt \quad (49)$$

Introducing the two m+1 inter storey displacement and velocity vectors $\overline{U} = \begin{pmatrix} \overline{\Delta}_x \\ X_f \end{pmatrix}$ and $\dot{\overline{U}} = \begin{pmatrix} \dot{\overline{\Delta}}_x \\ \dot{X}_f \end{pmatrix}$ is possible defining the space state 3m+2 vector $\overline{Q} = \begin{pmatrix} \overline{U} \\ \overline{Z} \\ \dot{\overline{U}} \end{pmatrix}$.

The dynamic equations system has the linearized first order form:

$$\dot{\overline{Q}}(t) = \mathbf{A_{eq}}(\mathbf{R}_{QQ},\; t)\overline{Q}(t) + \overline{F}(t) \quad (50)$$

Where the 3m+2 linearized square system matrix has the following form:

$$\left[A_{\mathbf{eq}}(R_{qq},\; t)\right]^{(3m+2\text{ x }3m+2)} = \begin{pmatrix} [0]^{mxm} & \overline{\varnothing}^T & [0]^{mxm} & [I]^{mxm} & \overline{0}^T \\ \overline{0} & 0 & \overline{0} & \overline{0} & 1 \\ [0]^{mxm} & \overline{0}^T & -\left[K^{eq}(R_{qq},\; t)\right]^{mxm} & -\left[C^{eq}(R_{qq},\; t)\right]^{mxm} & \overline{0}^T \\ -[H_1]^{mxm} & \overline{+\omega_f^2} & -[H_2]^{mxm} & -[H_3]^{mxm} & \overline{+2\xi_f\omega_f} \\ \overline{0} & -\omega_f^2 & \overline{0} & \overline{0} & -2\xi_f\omega_f \end{pmatrix} \quad (51)$$

being $\mathbf{H}_1 = \mathbf{M}^{-1}\mathbf{K}_L$, $\mathbf{H}_2 = \mathbf{M}^{-1}\mathbf{K}_{Hy}$,and finally $\mathbf{H}_3 = \mathbf{M}^{-1}\mathbf{C}$.

The auto-covariance matrix is:

$$\mathbf{R}_{QQ}(t) = \begin{pmatrix} \mathbf{R}_{\overline{U}\overline{U}} & \mathbf{R}_{\overline{U}\overline{Z}} & \mathbf{R}_{\overline{U}\dot{\overline{U}}} \\ \mathbf{R}_{\overline{Z}\overline{U}} & \mathbf{R}_{\overline{Z}\overline{Z}} & \mathbf{R}_{\overline{Z}\dot{\overline{U}}} \\ \mathbf{R}_{\dot{\overline{U}}\overline{U}} & \mathbf{R}_{\dot{\overline{U}}\overline{Z}} & \mathbf{R}_{\dot{\overline{U}}\dot{\overline{U}}} \end{pmatrix} \quad (52)$$

Both statistics of response and mean values energy evaluation needs knowledge of the evolutive covariance matrix $R_{QQ}(t)$ in space state $\overline{Q}(t)$. It is evaluated by solution of the Lyapunov Covariance Matrix Differential Equation, whose general expression, as previously pointed out is:

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$$\dot{\mathbf{R}}_{QQ}(t) = \mathbf{A}_{\mathbf{eq}}(\mathbf{R}_{QQ},t)\mathbf{R}_{QQ}(t)+\mathbf{R}_{QQ}(t)\mathbf{A}_{\mathbf{eq}}(\mathbf{R}_{QQ},t)^T+\mathbf{B}(t) \quad .$$
(53)

In this case, when the forcing vector is a white noise uncorrelated nonstationary processes, the square matrix 3m+2 $\mathbf{B}(t)$ has all the elements equal to zero except the last one that has the following form:

$$2\pi S_0 \varphi^2(t) \tag{54}$$

VI. NUMERICAL APPROACH FOR LYAPUNOV EQUATION

The main question in solving no stationary non linear Lyapunov equation is related to its covariance matrix structure. To overcome this problem many approach has been proposed, but no one isn't able to take into account matrix symmetry.

In this work, without changing formally Lyapunov covariance equation, at each time step it is transformed in a vectorial one by a simple algorithm, then solved by a standard tools, and finally reconstructed in a matrix form. Moreover, in order to consider structural non linearity, system matrix is renewed at each integration step with the values of linearized coefficients k^{eq} and c^{eq}.The transformation between matrix and vectorial form consist to assembly all the row (or alternative columns) from diagonal element. With reference to a m-DoF non linear system added to a m-DoF filter, only $2\left(4m^2+m\right)$ elements are independent in the space state covariance matrix, instead of $16m^2$ total elements. To make this operation, the vector of independent covariance elements is assembled as sequence of single row vectors (or alternatively columns). These are defined as the sequence of the elements taken in every line (or column) of $\mathbf{R}$, that goes from the principal diagonal element to the end (fig. 4). Naturally, the same order must be applies for the inverse symmetric matrix transformation into a vector. Such procedure allows to treat the no stationary *Lyapunov* equation in a vectorial differential system form.

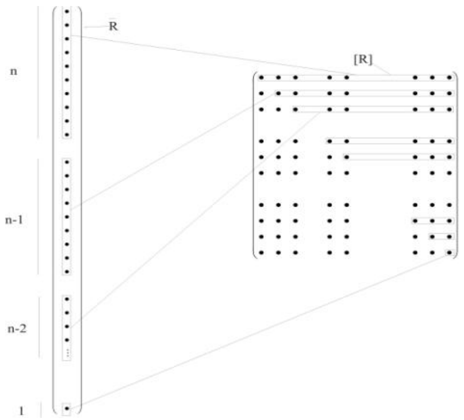

Fig. 4: vectorialization of a symmetric square matrix R

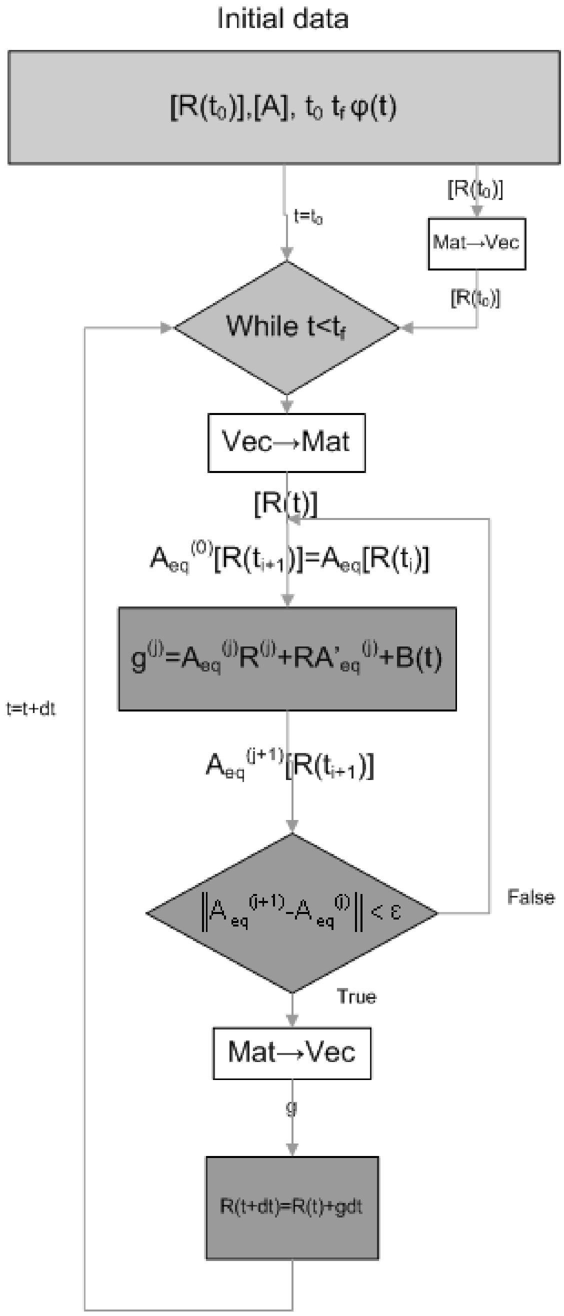

Fig. 5: integration scheme for matrix Lyapunov differential equation

To build up this approach, a specific recursive Matlab code has been developed: its flow chart is showed in fig.5. First of all, the covariance matrix is used for evaluation of the left terms of eq.(53), then is vectorialised, is performed the integration by a standard time step algorithm used by Matlab, then is evaluated the covariance at new time step in matrix form; finally the system matrix $\mathbf{A}_{eq}$ is refreshed with the values of linearized coefficient until a specific tolerance is achieved.

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VII. NUMERICAL TESTING

A mathematical application of proposed computational approach for no stationary covariance analysis of non linear structure is developed. In fact, several simulations are carried out in order to estimate the sensitivity of response and to determine the time evolution of dissipated energy. Primarily, the influence of used solvers in determining the solution is evaluated by estimating variation of integration time step Δt during seismic duration (fig.6). It is manifest that there are critical zones in correspondence of some discontinuities in modulation function: ode45 solver presents the worst trend that is obtained for main structural period T=0.25 (sec).

Then, an analysis concerning the estimation of covariance response and dissipated energy of a SDoF system is achieved, in addition with the values of linearization coefficient k^{eq}, c^{eq}. Both different time period and diverse amplitudes of modulation function t_d are considered. The results are plotted taking into account also distinct values of the rigid ratio α (see fig. from 7 to 13). As shown in mentioned figures, it is remarkable to notice the trend of displacement covariance changes considerably as period increases: in fact when the main structural period is T=0.15 (sec) the several curves are almost superposed and the elastic displacement is the maximum one nearly t=15 (sec) when it tends to zero more rapidly than the other curves. The more the main structural period raises, the more the curves are detached each other; at T=2.0 (sec) the trend becomes more irregular till T=3.0 (sec) where an unexpected result is attained: the covariance displacement presents a peak in correspondence of α =0.2 curve between t= 6 (sec) and t=12 (sec). This situation doesn't agree completely with theoretical assumptions according to which the elastic displacement has to be the upper limit.

Consequently, structural response of a multilevel buildings subjected to earthquake loads is considered and three cases are analysed: in the first is assumed that the building is made up by ten floors, each storey has a constant mass equal to $2.50\text{x}10^5$ (kg); lateral inter-storey stiffness and damping are assumed also constant for each floors, whose values are $k=4.50\text{x}10^8$ (N/m) and $c=1.06\text{x}10^6$ (N•sec/m) (fig.14). The second case regards a building with ten floors also, but considering different values of lateral stiffnesses for each storey as shown in tab.1(fig.15).

The last instance holds a six levels structure with both different masses and lateral stiffnesses (see tab.2) (fig 16).

For all tests carried out, a constant value of damping ratio ξ = 0.05 % is kept up. All simulations are characterized by ground acceleration having a PGA equal to 0.3 (g).

Floor	m [kg•10^4]	k [N/m•10^7]
1	2,50	9,26
2	2,50	8,57
3	2,50	7,88
4	2,50	7,20
5	2,50	6,51
6	2,50	5,83
7	2,50	5,14
8	2,50	4,45
9	2,50	3,77
10	2,50	3,08

Tab. 1

Floor	m [kg•10^4]	k [N/m•10^7]
4,80	8,34	4,80
4,80	8,34	4,80
4,32	5,34	4,32
4,32	5,34	4,32
3,84	3,24	3,84
3,84	3,24	3,84

Tab. 2

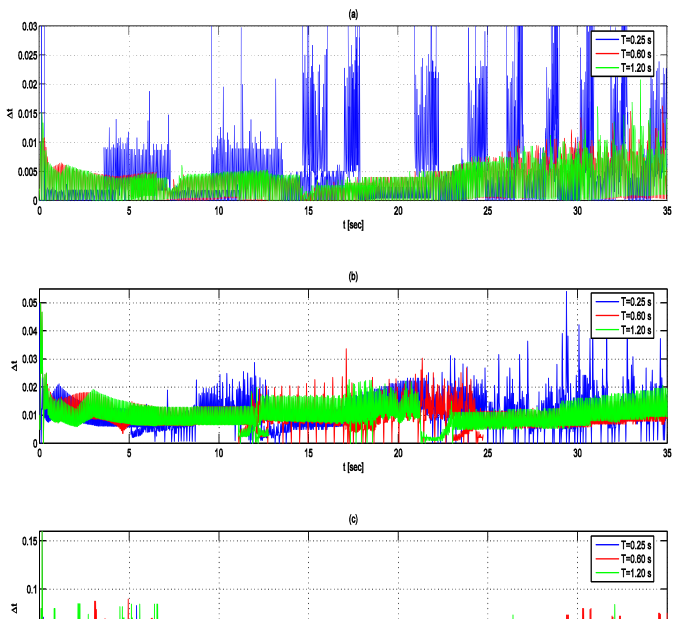

Fig.6: Integration step by using different Matlab solvers by three different values of period: a) ode45 b)ode23 c)ode113. The blue lines are for T=0.25 s the red lines are for T=0,60 s and the green lines are for T=1,20 s

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Time variation of ground frequency is expressed by:

$$\omega_f(t) = \pi F_c(t)$$

$$F_c(t) = F_0 + F_1(e^{-b_1 t} - e^{-b_2 t})$$

Where F_0=3 $(sec)^{-1}$, F_1=19.01 $(sec)^{-1}$, b_1=0.0625 $(sec)^{-1}$ and b_2=0.15 $(sec)^{-1}$ [8]

As integration algorithm has been used an explicit Runge-Kutta [7] formula, and an Adams one.

Finally, modulation function is assumed to be a Jennings one (A.4).

Solver	Period T	Mean value	Total integration time
	[s]	[s]	[s]
Ode45	0,25	0,00632	9,093
(55)	0,60	0,00338	6,954
	1,20	0,00345	7,500
Ode23	0,25	0,01036	7,079
(56)	0,60	0,00917	5,437
	1,20	0,00975	5,562
Ode113	0,25	0,01026	6,703
	0,60	0,01299	5,250
	1,20	0,01339	5,000

Tab. 3: Mean values of Δt

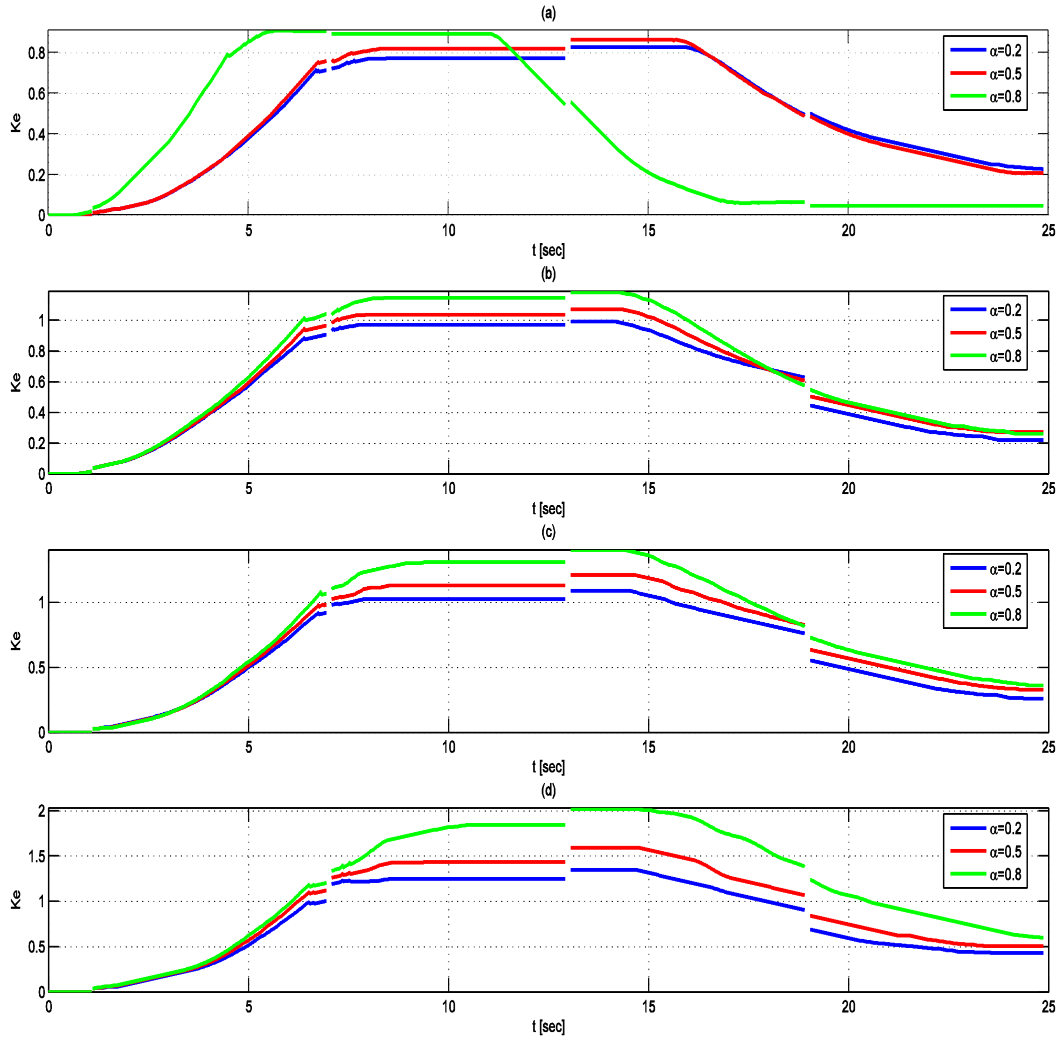

Fig.7: Time variation of linearized coefficient k^{eq} for a SDoF system by four different values of period a) T=0,25 (s) b) T=0,50 (s) c) T=0.80 (s) d) T=1,5 (s) and by three different values of the rigid ratio α . The blue lines are for α =0,2 the red lines are for α =0,5 and the green lines are for α =0,8

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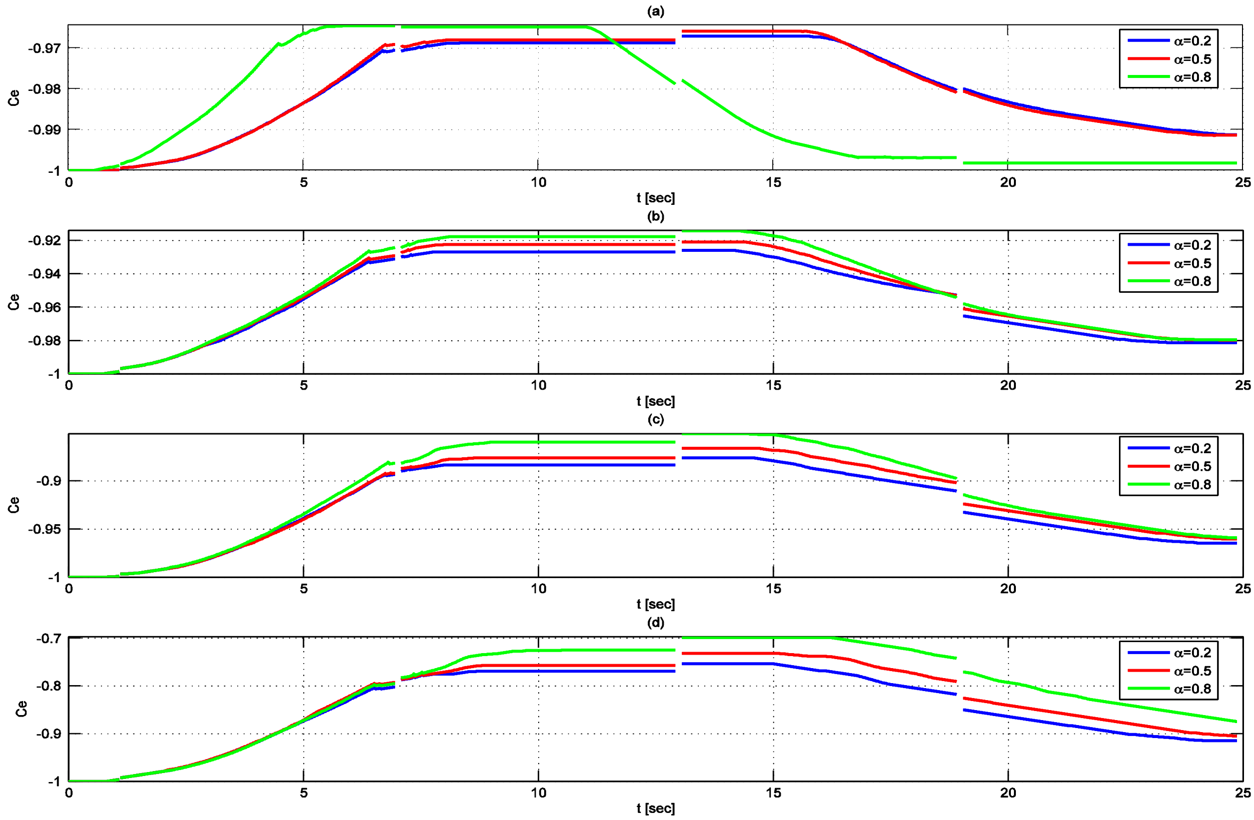

Fig.8: Time variation of linearized coefficient c^{eq} for a SDoF system by four different values of period a) T=0,25 (s) b) T=0,50 (s) c) T=0.80 (s) d) T=1,5 (s) and by three different values of the rigid ratio α . The blue lines are for α =0,2 the red lines are for α =0,5 and the green lines are for α =0,8

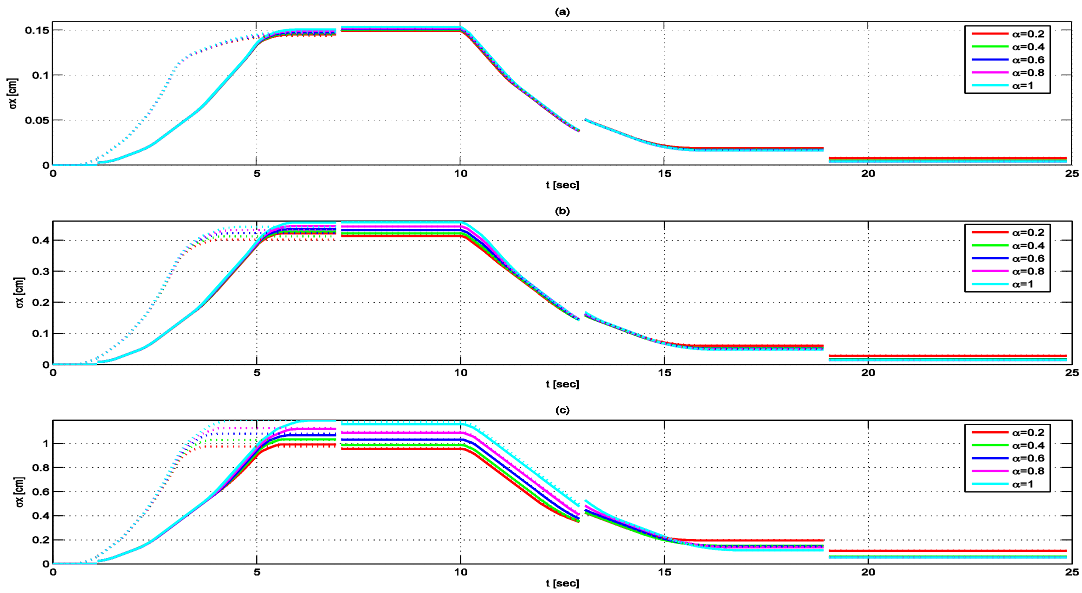

Fig.9: Single degree of freedom building displacement covariance response evaluated by three different values of period and for various values of α : a)T=0.15 s b) T=0.25 s c) T=0.50 s. The red lines are for α =0.2, green lines are for α =0.4, blue lines are for α =0.6, magenta lines are for α =0.8 and cyan lines are for the elastic case α =1.0. Continuous lines are for t_d = 5 s while dotted lines are for t_d = 7 s

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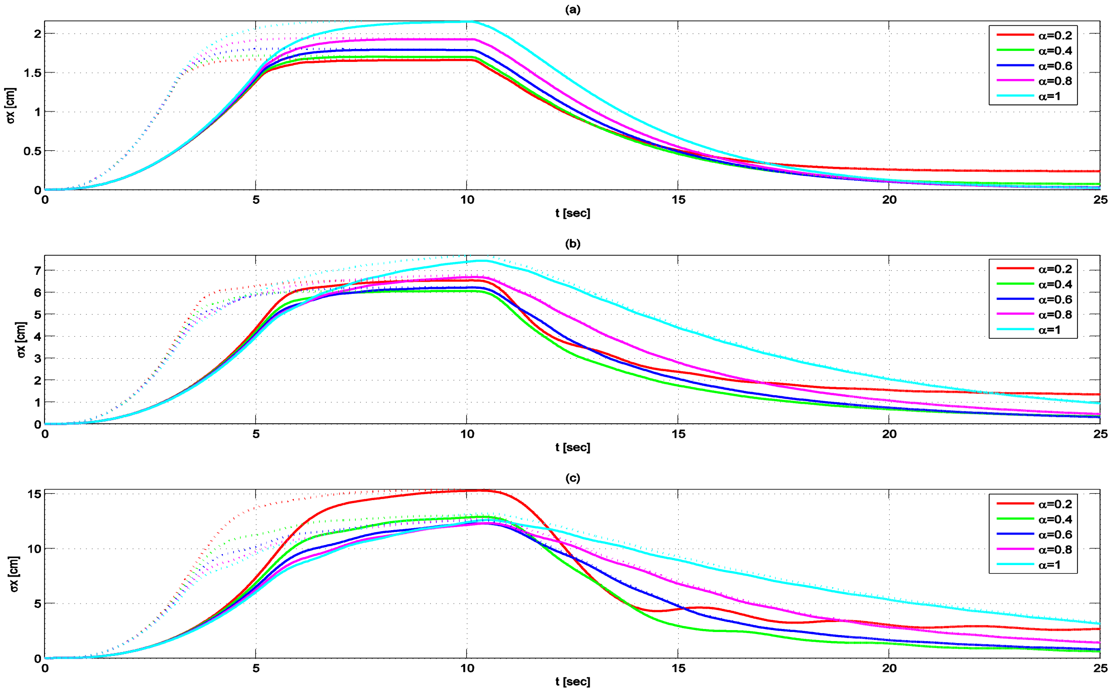

Fig.10: Single degree of freedom building displacement covariance response evaluated by three different values of period and for various values of α : a)T=0.80 s b) T=2.00 s c) T=3.00 s. The red lines are for α =0.2, green lines are for α =0.4, blue lines are for α =0.6, magenta lines are for α =0.8 and cyan lines are for the elastic case α =1.0. Continuous lines are for t_d = 5 s while dotted lines are for t_d = 7 s

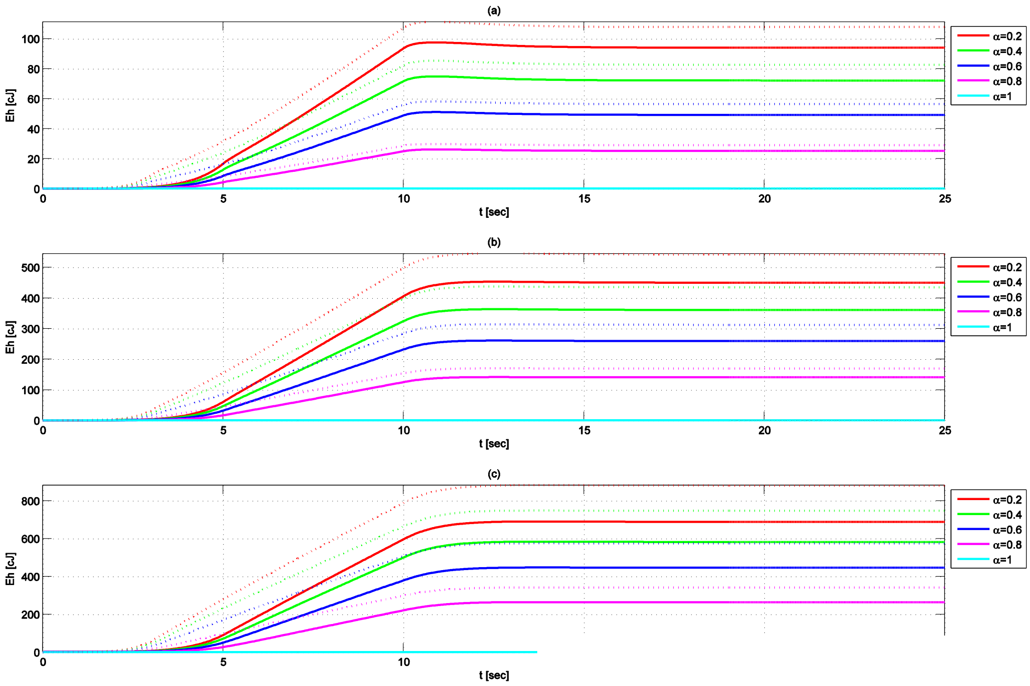

Fig.11: Single degree of freedom building dissipated hysteretic energy evaluated by three different values of period and for various values of α : a) T=0.15 s b) T=0.25 s. c) T=0.50 s. The red lines are for α =0.2, green lines are for α =0.4, blue lines are for α =0.6, magenta lines are for α =0.8 and cyan lines are for the elastic case α =1.0. Continuous lines are for t_d = 5 s while dotted lines are for t_d = 7 s

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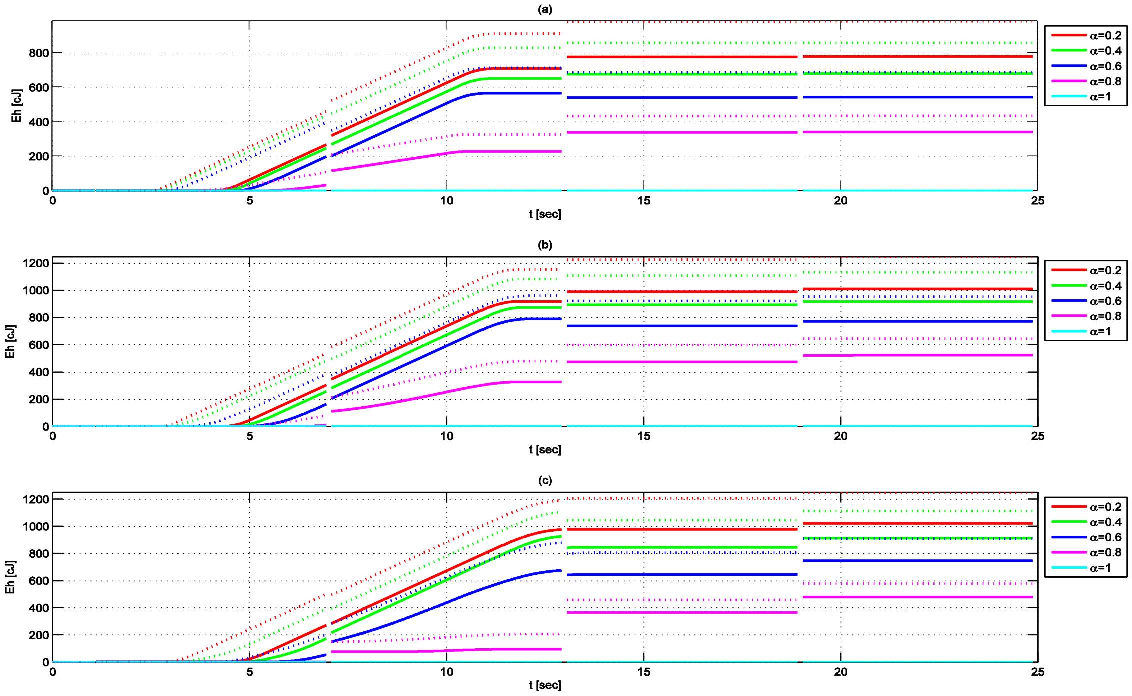

Fig.12: Single degree of freedom building dissipated hysteretic energy evaluated by three different values of period and for various values of α : a) T=0.80 s b) T=2.00 s. c) T=3.00 s. The red lines are for α =0.2, green lines are for α =0.4, blue lines are for α =0.6, magenta lines are for α =0.8 and cyan lines are for the elastic case α =1.0. Continuous lines are for t_d = 5 s while dotted lines are for t_d = 7 s

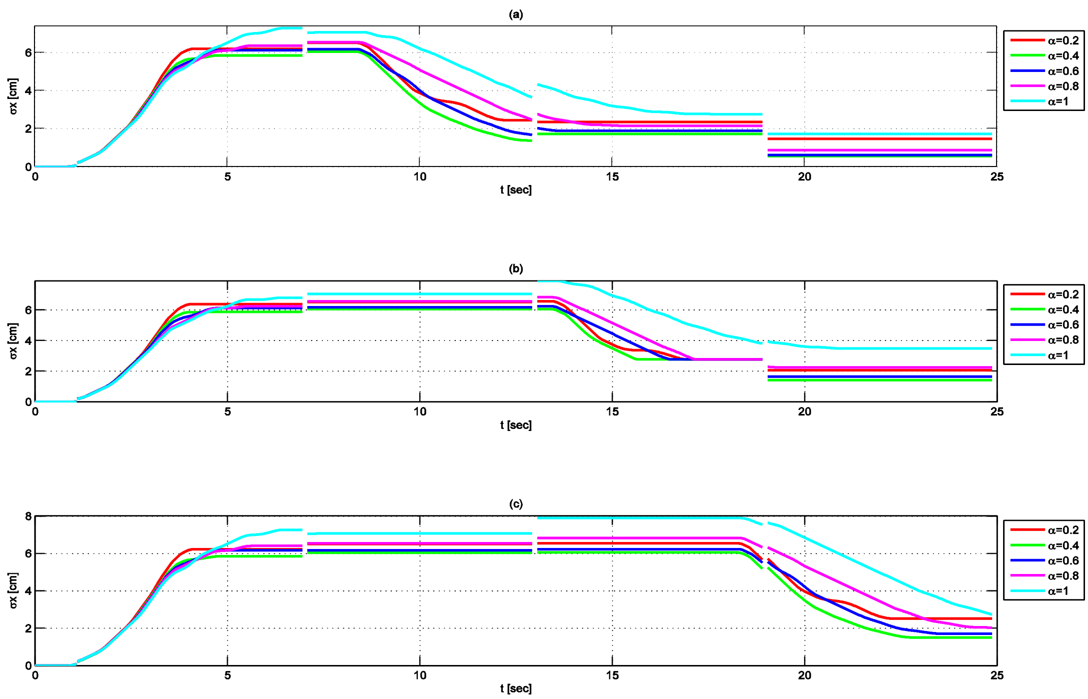

Fig. 13: Single degree of freedom building displacement covariance sensitivity response evaluated by three different amplitude of modulation function: a)t_d = 5 s b)t_d = 10 s c)t_d = 15 s and for period T=2 s. The red lines are for a=0.2, green lines are for a=0.4, blue lines are for a=0.6, magenta lines are for a=0.8 and cyan lines are for the elastic case a=1.0.

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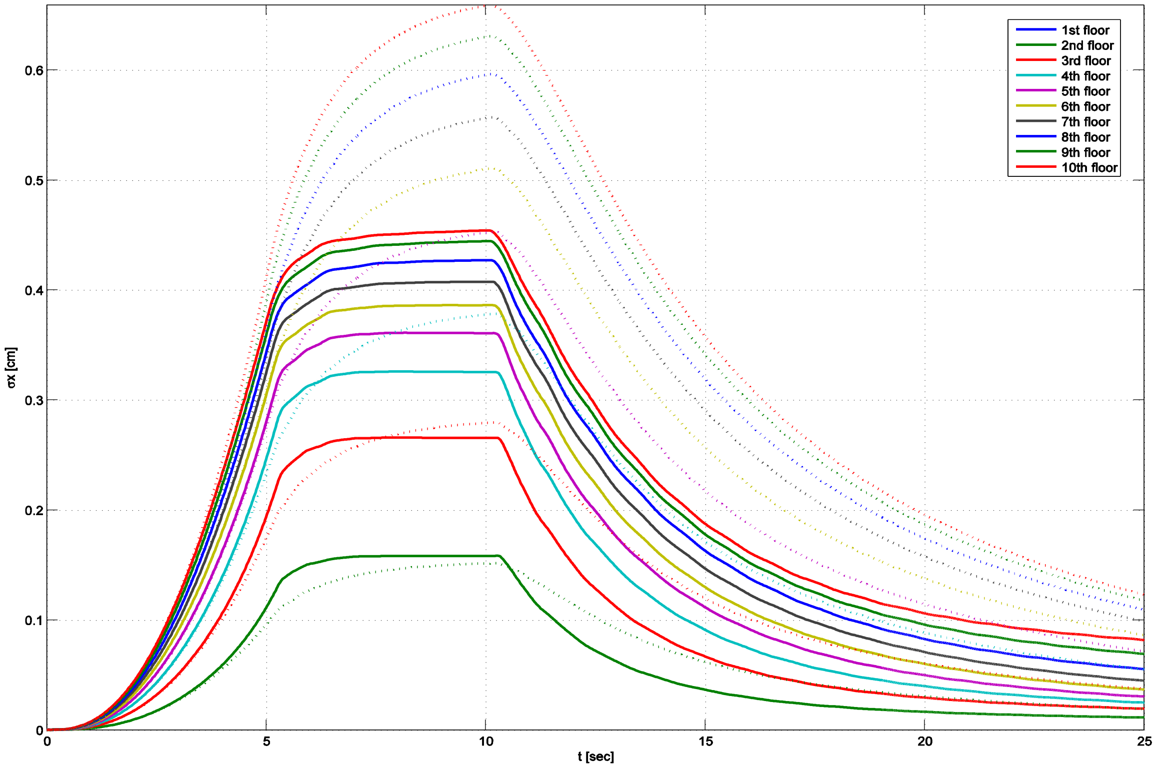

Fig.14bis In this image, a more detailed representation of covariance response from 2nd to 10th floor of above building is provided.

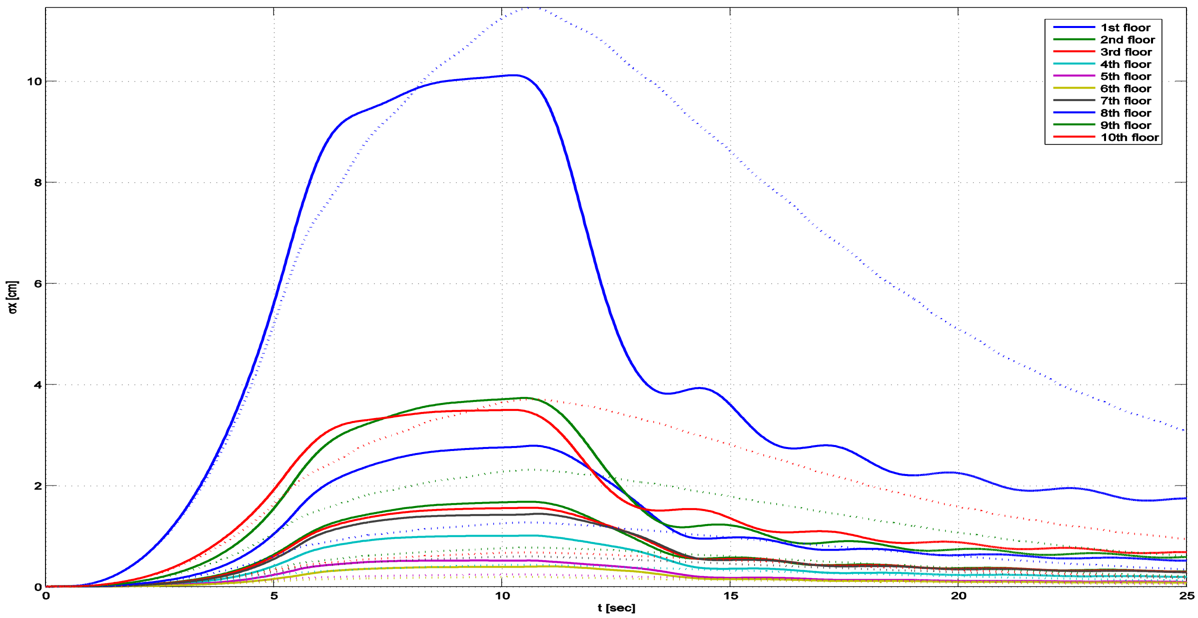

Fig. 15: Ten degree of freedom building displacement covariance response evaluated considering a strong hysteretic behaviour characterized by α =0.2 (continuous line) and a softly one represented by α =0.8 (dotted line). Building data are available in tab.1, damping ratio is ξ =0.05%

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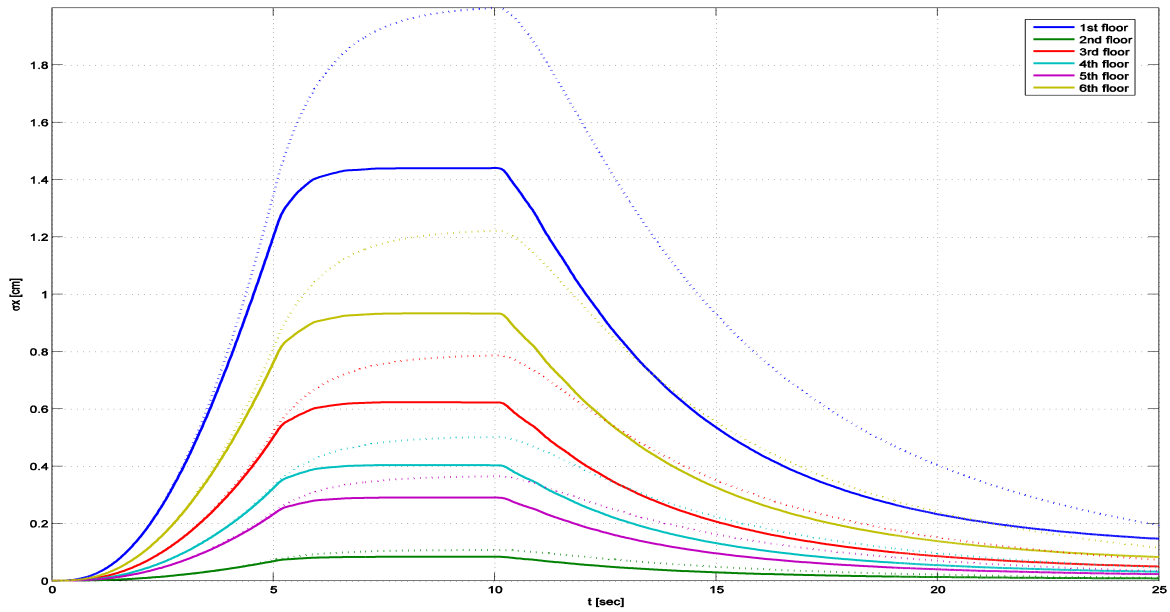

Fig. 16: Six degree of freedom building displacement covariance response evaluated considering a strong hysteretic behaviour characterized by α =0.2 (continuous line) and a softly one represented by α =0.8 (dotted line). Building data are available in tab.2, damping ratio is ξ =0.05%

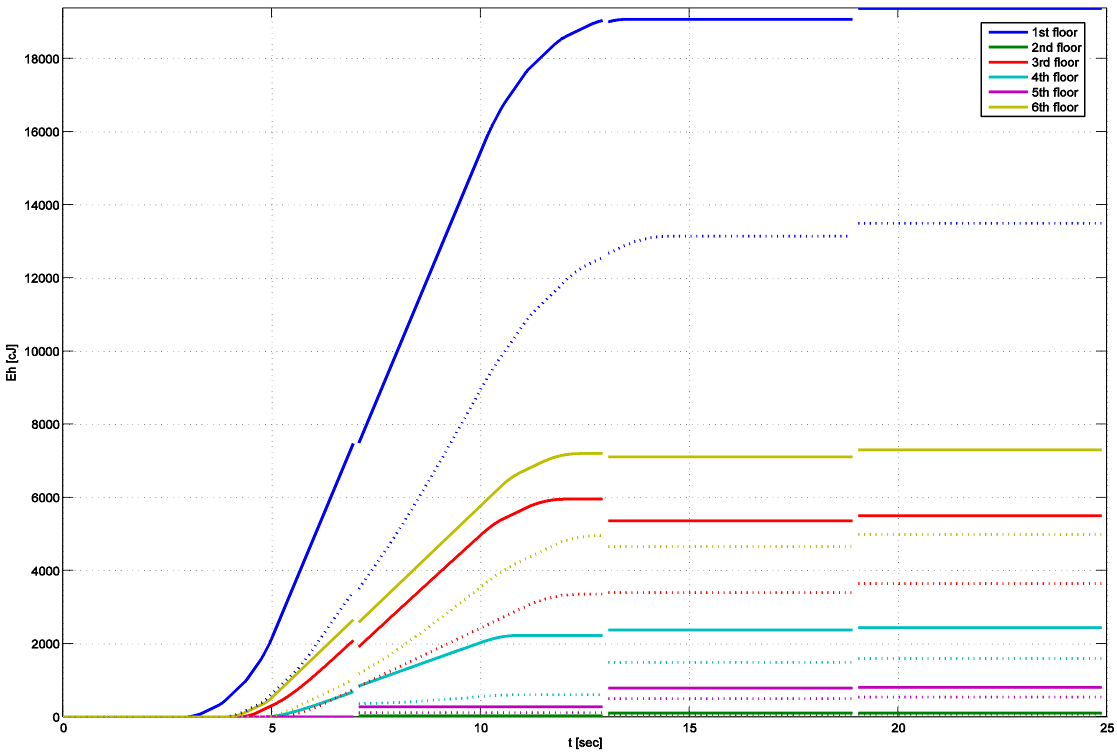

Fig. 17: Six degree of freedom building hysteretic dissipated energy evaluated considering a strong hysteretic behaviour characterized by α =0.2 (continuous line) and a softly one represented by α =0.8 (dotted line). Building data are available in tab.2, damping ratio is ξ =0.05%

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VIII. CONCLUSION

The proposed numerical approach has been adopted to evaluate the structural response of a non linear structural system subjected to an earthquake input modelled as a Kanai and Tajimi filtered non stationary process. A formal justification has been provided, in order to give a reasonable expression of the matrix $\mathbf{B}(t)$ in the Lyapunov matrix differential equation, supposing a white noise excitation. Moreover, this procedure is based on statistical linearization, and it is able to minimize computational effort taking into account matrix properties of symmetry. The method is adopted to evaluate maximum displacement covariance of inter storey drift, assuming both a uniform distribution of floor masses and inter storey stiffnesses and dampings, and a variable distribution of same quantities, considering different values of the rigid ratio α and diverse amplitudes of modulation function. The numerical results have shown the consistency and the effectiveness of the procedure.

I. APPENDICES

A. *Appendix A: Amplitude modulation functions*

The modulation functions mentioned in previous sections are:

Shinozuka and Sato [17]:

$$\varphi(t) = \alpha_1\left(e^{-\beta_1 t} - e^{-\beta_2 t}\right) \qquad \beta_1, \beta_2 > 0$$

Hsu and Bernard [10]:

$$\varphi(t) = \alpha_2 t e^{-\beta_3 t} \qquad \beta_3 > 0$$

Iwan and Hou [11]:

$$\varphi(t) = \alpha_3 t^m e^{-\beta_4 t} \qquad m, \beta_4 > 0$$

Jennings et al. [12]:

$$\varphi(t) = \begin{cases} \left(\dfrac{t}{t_1}\right)^2 & t < t_1 \\ 1 & t_1 \le t \le t_2 \\ e^{-\beta_5 (t - t_2)} & t > t_2 \end{cases}$$

1.1 Appendix B: Expressions of matrices

$$\mathbf{C}^* = \begin{pmatrix} 2\xi_1\omega_1 & -2\xi_2\omega_2\mu_1 & 0 & \cdots & \cdots & 0 \\ -2\xi_1\omega_1 & 2\xi_2\omega_2(\mu_1+1) & -2\xi_3\omega_3\mu_2 & 0 & \cdots & 0 \\ 0 & -2\xi_2\omega_2 & 2\xi_3\omega_3(\mu_2+1) & -2\xi_4\omega_4\mu_3 & 0 & \vdots \\ \vdots & 0 & & \ddots & & \\ & \vdots & & & 2\xi_{n-1}\omega_{n-1}(\mu_{n-2}+1) & -2\xi_n\omega_n\mu_{n-1} \\ 0 & 0 & & & -2\xi_{n-1}\omega_{n-1} & 2\xi_n\omega_n(\mu_{n-1}+1) \end{pmatrix}$$

(B.1)

$$\mathbf{K}_\mathbf{L}^* = \begin{pmatrix} \alpha_1\omega_1^2 & -\alpha_2\omega_2^2\mu_1 & 0 & \cdots & \cdots & 0 \\ -\alpha_1\omega_1^2 & \alpha_2\omega_2^2(\mu_1+1) & -\alpha_3\omega_3^2\mu_2 & 0 & \cdots & 0 \\ 0 & -\alpha_2\omega_2^2 & \alpha_3\omega_3^2(\mu_2+1) & -\alpha_4\omega_4^2\mu_3 & 0 & \vdots \\ \vdots & 0 & & \ddots & & \\ & \vdots & & & \alpha_{n-1}\omega^2_{n-1}(\mu_{n-2}+1) & -\alpha_n\omega^2_n\mu_{n-1} \\ 0 & 0 & & & -\alpha_{n-1}\omega^2_{n-1} & \alpha_n\omega^2_n(\mu_{n-1}+1) \end{pmatrix}$$

(B.2)

$$\mathbf{K}_\mathbf{Hy}^* = \begin{pmatrix} (1-\alpha_1)\omega_1^2 & -(1-\alpha_2)\omega_2^2\mu_1 & 0 & \cdots & \cdots & 0 \\ -(1-\alpha_1)\omega_1^2 & (1-\alpha_2)\omega_2^2(\mu_1+1) & -(1-\alpha_3)\omega_3^2\mu_2 & 0 & \cdots & 0 \\ 0 & -(1-\alpha_2)\omega_2^2 & (1-\alpha_3)\omega_3^2(\mu_2+1) & -(1-\alpha_4)\omega_4^2\mu_3 & 0 & \vdots \\ \vdots & 0 & & \ddots & & \\ & \vdots & & & (1-\alpha_{n-1})\omega^2_{n-1}(\mu_{n-2}+1) & -(1-\alpha_n)\omega^2_n\mu_{n-1} \\ 0 & 0 & & & -(1-\alpha_{n-1})\omega^2_{n-1} & (1-\alpha_n)\omega^2_n(\mu_{n-1}+1) \end{pmatrix}$$

(B.3)

$$\mathbf{C}^{\mathbf{eq}} = \begin{pmatrix} -c_1^{eq} & 0 & \cdots & & & 0 \\ 0 & -c_2^{eq} & 0 & & \cdots & 0 \\ & 0 & -c_3^{eq} & 0 & & \\ \vdots \text{(A.1)} & \vdots & 0 & \ddots & & \vdots \\ & & \vdots & & -c_{n-1}^{eq} & \\ 0 & 0 & 0 & & 0 & -c_n^{eq} \end{pmatrix}$$

(B.4) (A.2)

$$\mathbf{K}^{\mathbf{eq}} = \begin{pmatrix} -k_1^{eq} & 0 & \cdots & & & 0 \\ 0 & -k_2^{eq} & 0 & & \cdots & 0 \\ & 0 & -k_3^{eq} & 0 & & \\ \vdots \text{(A.3)} & \vdots & 0 & \ddots & & \vdots \\ & & \vdots & & -k_{n-1}^{eq} & \\ 0 & 0 & 0 & & 0 & -k_n^{eq} \end{pmatrix}$$

(B.5)

where:

$$\begin{cases} \omega_i^2 = \dfrac{k_i}{m_i} \text{(A.4)} \\ \xi_i = \dfrac{c_i}{2\omega_i m_i} \qquad i = 1, 2 \ldots m \\ \mu_i = \dfrac{m_{i+1}}{m_i} \end{cases} \qquad \text{(B.6)}$$

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