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Probability Distributions, Flow Model and Wavelet Transform for Growing Areas in Irrigation Districts of Hidalgo State, Mexico

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Abstract— ***The purpose of this research is to find for corn, vegetables, fodder, and grains, and low irrigation with wastewater in the Mezquital Valley, the probability distributions that give structure to these agricultural data. Karl Pearson's method, based on the first four central moments is applied. Distributions appear to be of Beta type, so it might be based on a simple model of the combination of risk and likelihood of success; but for grains is an asymmetric Student distribution. Consequently, for products, a proportional flow model at the prices rates is proposed. Wavelet transform is used and a singularity is located in 1996, signaling a trend change. Data were grouped into two classes, and the trends are obtained by the method of least squares. It is noted that the singularity is due to the entry into force of the standard for wastewater use, which is our second goal. Changing of trends shows a stimulus for fodder at the expense of a disincentive for planting grains and vegetables, but with the economic implication of falling incomes for farmers.***

Index Terms—**Statistical analysis, Beta distribution, crop areas, product flow.**

I. INTRODUCTION

We want to solve an inverse problem for the probability distribution of agricultural data. It is conceivable that immersed in the data, a probability distribution is hidden, which gives the same data structure, so the goal is to identify its analytical form. In addition, we apply wavelet transform to find singularities that can make turns in trends. We propose a stratified flow model products in one direction, and money in the opposite direction, with a given by the two form factors coefficient. More than a century ago, the use of wastewater in agriculture has also emerged as a partial substitute for first-use water. The explanation may be based on the chemical composition, because these waters have a rich content of nitrogen and phosphorus so that they are highly valued by farmers because of its wealth of nutrients and energy that are usable by crops. But on the other hand, these waters carry waste containing pathogenic microorganisms such as bacteria, viruses and parasites, and constitute a significant risk to public health and should therefore be used in a planned and regulated way, meeting the reference guides for official use and current standards of wastewater. But an adequate characterization of soils is then also required to avoid contamination risks because they are the main assets of farmers and are a valuable resource of the nation. Also for preservation purposes, quality groundwater and aquifers should be frequently monitored. In this paper, data from the Irrigation District 003, in Hidalgo State (Mexico), are discussed, especially since 1996, with the entry into force of the NOM-001-ECOL-96 standard, [1]-[5]. We pursue three objectives: first, solve the inverse problem for the probability distribution of crops areas, second, application of wavelet transform to find singularities to explain trend changes that are observed, and the third, to propose a law for goods and capital flow determined by the shape parameters. The first aims to understand how crop areas are distributed, and find a simple model to explain them. The second seeks to locate some uniqueness, noting the influence of NOM-001-ECOL-96, now called NOM-001-SEMARNAT-96 in cropping patterns and investigate their economic consequences. The third, model goods and capital exchange as a of diffusion law.

II. METHODS

The method corresponds to Karl Pearson (1895), [6]. Probability densities ρ are found as solutions to the differential equation (1), wherein the four parameters (a_i, b_j) are in correspondence with the first four central moments of the data set under consideration, [1]. Then classification parameters must be found. However, the first moment, the media, is set to zero, so are the other three moments who determine the classification. Also the skewness and kurtosis are calculated and provide information on the shape of the distribution [7]-[9],

$$\frac{d}{dx}\ln\rho(x) = \frac{a_0 + x}{b_0 + 2b_1 x + b_2 x^2} \qquad (1)$$

Sequence and grouping are created, because results showed analogies: 1. corn, vegetables, beans, 2. wheat, forage and 3. grains, we calculate the first four central moments, as well as the coefficients of skewness and kurtosis, and got the two shape parameters, denoted by m and n (see Table 1 and 2).

III. RESULTS AND DISCUSSION

In Table 1, for **corn**, it is observed that the parameter associated with the fourth central moment, the kurtosis (*Ku*), is less than 3, so that the density is of platykurtic type, which means that, compared to normal, data shifts from the peak and/or queues to the central region are presented, or that the variance is getting quite frequent departures, but relatively modest, and/or infrequent but with intense deviations. Looking at the third central moment, we see that asymmetry is positive, so it is expected that the right tails are thicker than the left, so that deviations that feed the variance comes predominantly from that in above mode [10]. Parameters *D* and λ are qualifiers, so that when their sign are negative, probability density is classified as Type *I*. But also, as the two shape exponents are positive, the density is unimodal with its peak coinciding with its mode, but it is shifted to the left with respect to logistic serving as a reference, [8]. For **vegetables**, the first four central moments produce a skewness and kurtosis which manifest in the form of density, the latter being platykurtic, although only slightly less than that of the Gaussian, and with thicker right tails. In the case of central moments and coefficients, it can be seen that the qualifying parameters *D* and λ located to the density as Type *I*. Positive shape parameters again involve a unimodal density; mode offset to the left with respect to the center line, logistic characteristic mode. For **beans**, the features that are inferred from the skewness and kurtosis are of the same type as corn. On the basis of qualifier parameters, *D* and λ, it is inferred that this is a Type *I* density, while the two parameters of shape correspond to an unimodal density, however, compared to corn, the mode is moving toward the right because there is a decrease in the second of these parameters with respect to the first, and it can be seen most clearly that the tail on the right side is more prominent than the left (see Figure 1).

Table 1. First column: Statistical parameters; First rows: crops.

	Corn	Vegetables	Beans
Sk	0.20127	0.31508	-0.55530
Ku	2.6475	2.8426	2.4478
D	-3.8337×105	-27111	-2.4091×106
λ	-0.031963	-0.099254	-0.10902
m	2.9901	3.6754	0.97985
n	4.6947	7.6920	0.034660

	Wheat	Forages
Sk	2.1171	0.64246
Ku	7.8139	2.2997
D	-1.3914×107	-1.6202×109
λ	-0.98618	-0.11508
m	-0.54875	0.19686
n	1.6068	-0.38480

For **wheat**, kurtosis is somewhat elevated, reflecting the ownership of a leptokurtic density, or that the variance is nourishing, though infrequent extreme deviations, and/or small deviations but very frequent. The positive skew indicates that the right tails are thicker, i.e., deviations admitted to bulk variance do predominantly with deviations below mode. Moreover, the density is leptokurtic may mean that there is overlap data, and therefore there is no reliability in the structure thereof. Within relationships between coefficients and central moments, and judging from the signs of the parameters *D* and λ, it can be seen that this is again a density of Type *I*. However, the sign of the first parameter points out that it has a decreasing density, similar to a distribution tail.

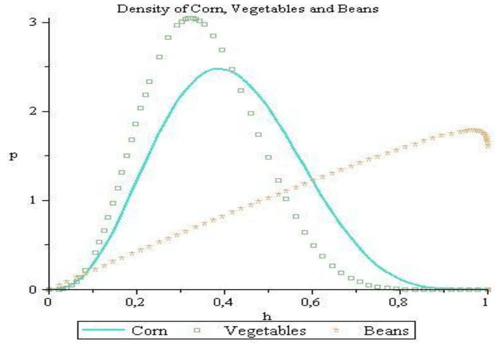

Fig. 1. a) Probability density for crops: corn, vegetables and beans.

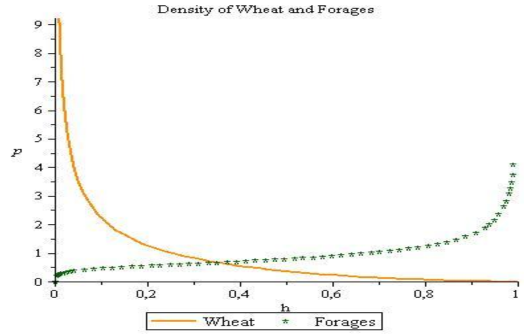

Fig. 1. b) Probability density for crops: wheat and forage.

It can be compared with a tail with fractal characteristics and fractal dimension equal to $-m$, and therefore may correspond to scattered and irregular data. For **forages**, the density has a platykurtic shape, asymmetric, with thicker tails at right, so the variance is fed with extreme and rare deviations and/or small but frequent deviations, but also predominate deviations below average. Relations between coefficients and central moments show that the density is of Type *I*. By the sign of the first exponent, it follows that it is an increasing density. For **grains** (see Table 2), the parameters of skewness (*Sk*) and kurtosis (*Ku*) reflect that this is a leptokurtic distribution, with more prominent tails at right. However, the sign of the parameter *D* and the vicinity of λ to zero but less than 1, indicate that it is a Type *IV* density, analogous to a Student distribution, but asymmetrical, see Figure 2. Parameters α and ν determine a modulating step for asymmetry by $e^{-\nu \arctan h/\alpha}$, [9].

Table 2. Statistical parameters, first row, for Grains

	Sk	Ku	D	λ
Grains	0.17345	3.1993	42201	0.14791

	m	α	N
Grains	42.112	-0.40999	-1.4457×10^{6}

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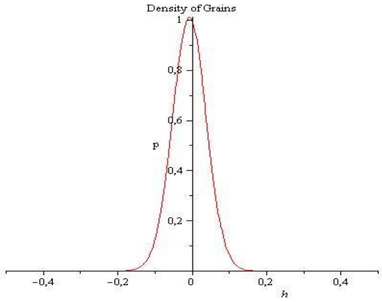

Fig. 2. Horizontal axis: probability of success; vertical axis: probability

Influence of the standard

The analysis is performed on data from the growing areas in the Mezquital Valley, irrigated with wastewater, and corresponds to the Irrigation District 003, Tula, Hidalgo, [1], [11]. When wavelet transform is applied to data series for planted surfaces with grains, vegetables and forage, we can identify singularities; for example, we find the date 1996, which coincides with the beginning of the implementation of NOM-001-SEMARNAT-96 standard. Figure 3 shows the singularity for planted surfaces of grains since 1986, (Coifman wavelet 6, level 1); also are found similar singularities for vegetables and corn, [12], [13].

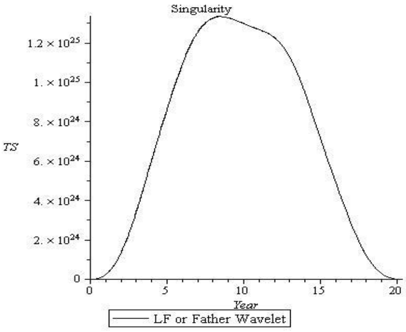

Fig. 3. Horizontal axis: year (since 1986); vertical axis: transform planted surface (grains).

The trend changes, from that date, are determined by the least squares method. Data were grouped into two classes, separated by the singularity detected by wavelet transforms, or, before and after the standard entry into force. For grains, the random variable is denoted by G, the tendency, before of the standard, by TB_G, and after by TA_G. The results are shown on the left of eq. (2). Graphical representations of data and trend are shown in Figure 4, (Grains). A noticeable shift downward, from before to after of the standard is observed and quantified by the change in slope. For the case of vegetables, the random variable is represented by V, also the tendency before the standard is TB_V, and thereafter TA_V. The two trends are illustrated in the equations shown in eq. (2). A clear decrease in the slope of the trend is observed, or change in trend downward; this is plotted in Figure 5, in where the trend is gentler before the application of the standard (TB), while the lower slope corresponds to the later (TA).

$$\begin{aligned} TB_G &= 16233 + 675.65 \cdot G, \quad TB_V = 1857 - 19.489 \cdot V \\ TA_G &= 21352 - 102.39 \cdot G, \quad TA_V = 2169.4 - 62.148 \cdot V \end{aligned} \quad (2)$$

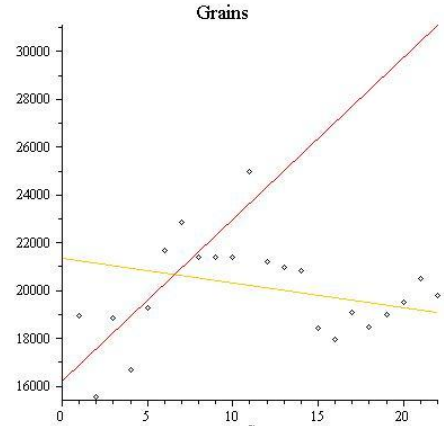

Fig. 4. Trends for grains: Before 1996 and after, (TB) and (TA).

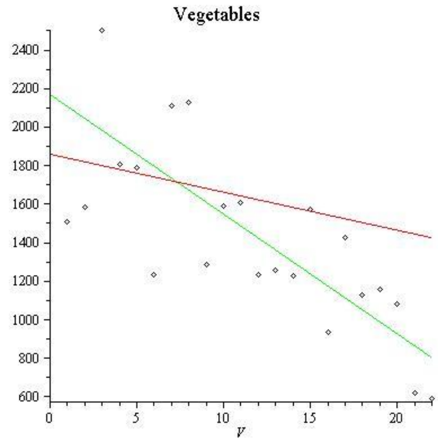

Fig. 5. Trends for vegetables: Before 1996 and after, (TB) and (TA).

Finally, in the case of forages, the random variable is denoted by F, and the two trends are shown in the following equation (3), (TB) and (TA). Now an increase in slope, quantifiable, showing an upward shift is observed, and is illustrated in Figure 6, entitled Forages.

$$\begin{aligned} TB_F &= 22039 - 4.1886 \cdot F \\ TA_F &= 4058.5 + 1293.6 \cdot F \end{aligned} \quad (3)$$

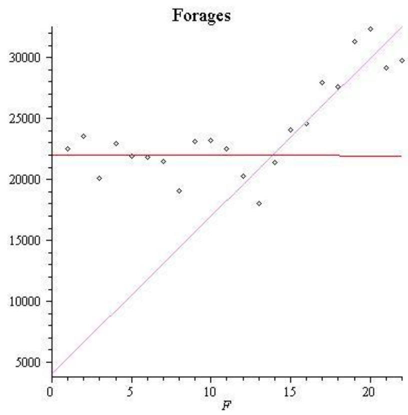

Fig. 6. Trends for forages: Before 1996 and after, (TB) and (TA).

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Theoretical model

To develop the theoretical model, we should look the density of the harvested areas, and their modal profile, as exhibited in Figure 1. The sequence of accumulated production is imagined by us as an inverse cascade process. Moreover, we must eliminate the asymmetry manifesting the different densities which can be achieved by introducing a quadratic dual variable to absorb changes sign, and insert a vertical axis of symmetry, this is achieved with a Laplace transform, so must be passed to the dimensionless variable h to its dual s. As it was shown, the Pearson method produces two numbers that are the exponents of form m and n, and in accordance with the classification as Type *I*, we have: $m+1>0$ and $n+1>0$, or: $m+1>0$ and $m+n+2>0$.

Therefore, a theoretical model, able to reproduce these results, it is a process of generalized Cantor, [14]. It involves the formation of a sequence based on the two shape parameters and a sequence of powers of the resolution h:

$$p_j = \frac{(m+n+2)_j}{(m+1)_j} h^{-(j+1)} \quad , \quad (m+1)_j = \frac{\Gamma(m+1+j)}{\Gamma(m+1)} \quad ,$$

where Γ is the gamma function; and its subsequent discretization $h_l = \frac{l}{k^2}$, that produces the $M(m+1, m+n+2, k^2)$ Kummer function. And with change: $k^2 \mapsto -s^2$, the generating function of moments, associated with Beta density, $M(m+1, m+n+2, -s^2)$ is obtained, [14]. Considering again the dates shown in Table 1, Figure 7 is elaborated for the Generating Function of Moment of: Corn, Vegetables and Beans.

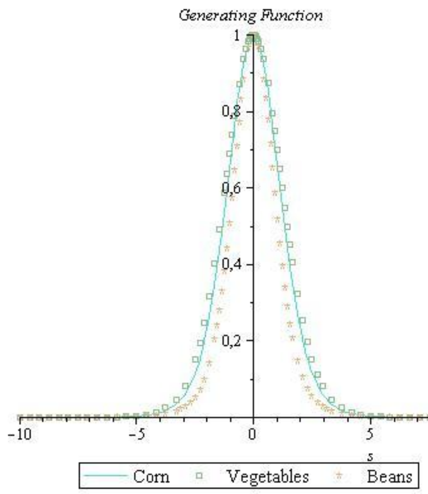

Fig. 7. Moment generating function of crops: corn, vegetables and beans.

Conversely, now we imagine a market as the scalar field of prices, which could be displayed as a layered medium in the vertical direction. A flow of products is established from one price level to the next higher level. We say, therefore, that similar to the Darcy flow, this is proportional to the prices gradient, but with the proportionality expressed by a coefficient given by $D(m,n;s) = \left(M\left(m+1, m+n+2, -s^2\right)\right)^{-1}$, the inverse of the Kummer function. Thus flow law is set as follows: $q_D = D\frac{\partial}{\partial y}\psi$, where y is the vertical coordinate; and ψ the scalar field of prices. If ψ is expressed in dimensionless form, D and y in their usual units, then q_D has velocity units. Finally, if we grant a non-local character to the flow, and β index denote the spatial occupation, then the flow is described by (4):

$$q_{\beta,D} = D_\beta \frac{\partial^\beta}{\partial y^\beta}\psi \qquad (4)$$

Similarly, we imagine a mass of money, or capital stratified in the vertical direction, so that the advance from top to bottom, means a direct cascade process, where the initial mass is progressively becoming more fractionating small in size, or resolution, but with increasing the number of parts or features, [15].

IV. CONCLUSIONS

We can set a model for product flow in one direction, and money in the opposite direction, with a coefficient that depends on the two form factors, same as those calculated from the data using the first four central moments. The transform of Beta density is the inverse of the $D(m,n;s)$ coefficient, and the density is determined from the two exponents m and n. We find very important that for group 1: corn, vegetables and beans, due to the characteristics of their densities of distribution, happens to be in correspondence with the traditional and ancient crop grouping in the cornfields: corn, vegetables and beans [16]. In the case of the Irrigation District 003, when the standard entry into force, it produced the reduction of planting vegetables irrigated with wastewater, which showed a moderate decrease in planted area, from 7.9% in 1997 to 4.9% in 2007. While for forage crops, it was observed a trend of sustained growth, referring to the planted area, going from 45.9% in 1997 to 57.5% in 2007.

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