

Performance Comparison of Pseudo-Random and Orthogonal Spreading Sequences

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Abstract— Orthogonal Spreading and pseudo-noise or Pseudo-Random sequences are widely used in the Wideband Code Division Multiple Access (WCDMA) systems using Rake receiver. This paper presents the theoretical aspect for direct sequences DS-CDMA systems and reviews the main characteristics of the maximal length, Gold, barker and Kasami sequences, also the variable and fixed-length orthogonal codes are discussed. Finally, comparisons between these methods is given by using auto-correlation, cross-correlation functions, The mean square aperiodic auto-correlation (MSAAC), the mean square aperiodic cross-correlation (MSACC) measures and the Merit facto (MF).

Keywords— pseudo-noise codes, orthogonal codes, WCDMA, DS-CDMA, auto-correlation, cross-correlation, merit factor.

I. INTRODUCTION

UMTS based on WCDMA access technique meets the 3G requirements, allowing a variety of services with variable bandwidths to be freely mixed and handed simultaneously with good level of Quality of services (QoS). Each WCDMA device can access to several services simultaneously. The method of spread spectrum communication gained a great deal of prominence, and involves spreading the desired signal over a bandwidth much larger than the minimum bandwidth required sending the signal.

Spread spectrum is a radio communications system in which the baseband signal bandwidth is intentionally spread over a larger bandwidth by injection of a higher-frequency signal. As a direct result, the energy used in the transmitting the signal is spread over a wider bandwidth, and appears as noise. The ratio between the spread baseband and the original signal is called processing gain.

The spread spectrum technique includes contains several methods such as: Direct sequence Spread spectrum (DS for short), Frequency Hopping (FH), Time Hopping (TH), and linear FM (Chirp) [1]. In the 3G mobile communication, DS, FH and FH / DS are the most used, for more details about Spread Spectrum Technique refer to [2] [3] [4]. In this paper, we will focus on DS method. In figure 1 the principle of spreading and despreading is illustrated.

Direct Sequence CDMA (DS-CDMA) uses Direct Sequence Spread Spectrum (DSSS) technology to spread the spectrum [5] [6]; spreading is carried out using a pseudorandom noise (PN) sequence, See Figure 2. At transmitter, each user data bit is coded with a PN sequence code called chips. The chip rate is much higher than the bit

rate, thus making the bandwidth occupied by the signal much greater than the necessary bandwidth the information needs. The ratio of PN sequence bit rate to data bit rate is known as the Spreading Factor (SF). The receiver must know the correct code sequence for recover the original data from the signal sent inside the used frequency range. This technology allows a narrowband signal to be spread several times creating a wideband signal. The resulting waveform is wideband, noise-like, and balanced in phase. Spreading helps the signal resist interference as well as allows primary data to be recovered if data bits are damaged due to transmission [5] [6].

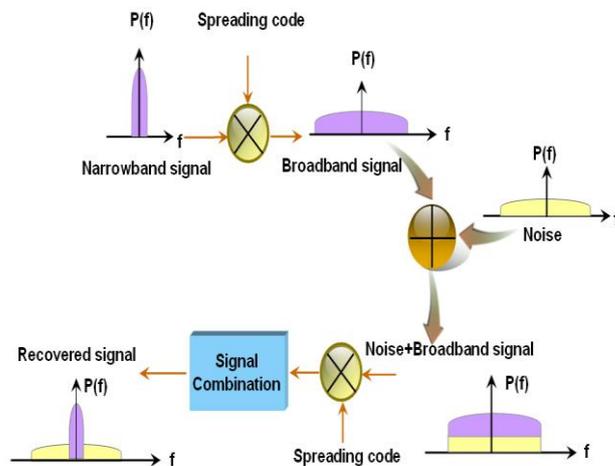


Fig 1. Spectrum Analysis of Spreading & Despreading

As well, the notion of PN codes can be used for encryption. Although the transmission of information (voice, image or other data) through insecure channels, if this is not set properly attached, there may be unwanted disclosure and unauthorized modification of data. So, some mechanisms are needed to protect information in insecure channel [2] [3].

One way to guarantee this protection is to convert intelligible data into unintelligible form before transmission and such a conversion process with a key is known encryption [7]. At the receiver side, the encrypted message is converted back to its original form via the reverse process of encryption called decryption. For these two important areas of application, PN sequences have received much attention recently. In this Paper, an analytical approach is adopted to show the generation, code length, autocorrelation and cross correlation properties of some important PN and orthogonal sequences [8][9] [10].

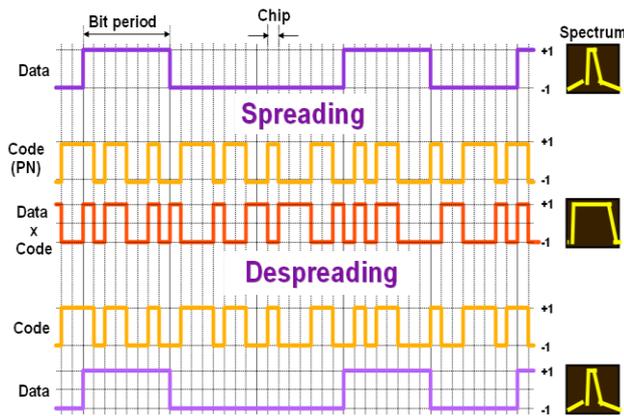


Fig 2. Spreading and Despreading using PN Codes

The spread spectrum communication technology has the following characteristics:

- 1) A high anti-interference capacity.
- 2) Multi-access communication
- 3) Good security
- 4) Anti-multi-path interference

The spreading codes candidates for use in DS-SS must have a series of properties, from which, the most important are:

- The Length high for spreading the spectrum and ensure the information safety.
- The high number of sequences access to authorize maximum of users.
- The Balancing Factor which indicates the difference between the number of '1' and the number of '0' in a sequence data should be the lowest possible for synchronization.
- The correlation of two different codes (cross-correlation) should be the lowest as possible to eliminate interference due to simultaneous transmission of multiple users.
- The correlation of a code with its time-delayed version (auto-correlation) must equal 0 for any time delay other than zero to eliminate multi-path interference.
- A length of recursion, indicating the complexity of generate the sequence, should be high compared to the length of the sequence to ensure the safety of information.

The rest of the paper is organized as follows. In Section 2 and 3, we deal with different kind of Pseudo Noise (PN) and orthogonal sequence. Section 4 describes the techniques to measure the correlation properties, while Section 5 presents the merit factor. Simulation Results and discussions are presented in section 6. Finally, section 7 draws some conclusions.

II. PSEUDONOISE(PN) SEQUENCE

A pseudo-noise (PN) sequence is a sequence of 1's and 0's and it is periodic. It is not random (deterministic) but it

looks randomly for the user who doesn't know the code. The larger the period of the PN spreading code, will be more random binary wave and it is harder to detect it. PN sequences have some characteristics that are similar to random binary sequences (having equal # of 0's and 1's), very low correlation between any two shifted version of the sequence and low cross-correlation between any two sequences. The PN sequences satisfy the following three properties [11]:

Balance Property: Any PN sequence of length $N=2^n - 1$, the number of 1s is always one more than the number of 0s. So there must be 2^{n-1} ones and $2^{n-1} - 1$ zeros in a full period of the sequence.

Run Property: The "run" represents a subsequence of identical symbols (1's or 0's) within one period of the sequence. The length of this subsequence is the length of the run. Among the runs of 1's and 0's in each period of a PN sequence, one-half the run of each kind are of length one, one-fourth are length two, one-eighth are of length three, etc. For a PN sequence generated by a linear feedback shift register of length n , the total number of runs is $(N+1)/2$ where $N=2^n - 1$.

Correlation Property: The autocorrelation function of a m-sequence is binary valued, periodic and has a period $T=NT_c$ where T_c is chip duration. This property is called the correlation property. The autocorrelation function $r(i)$ is defined as:

$$r(i) = \begin{cases} 1 & \text{for } i = 0 \\ -\frac{1}{N} & \text{for } 1 \leq |i| \leq N-1 \end{cases} \quad (1)$$

Due to these three properties, PN sequences are measured efficient for data encryption. However, having a correlation of adjacent bits much less, is the direct result of the third property, which making the PN sequences more effective to be used in DS-SS systems. Consequently, useful PN sequences should have very good auto-correlation and cross-correlation properties in addition to maintaining some randomness properties. The Welch bound places a lower limit on the maximum level of the correlation function (cross-correlation levels and auto-correlation of sidelobes). For a set of K sequences with each sequence of length N ($N \geq K$), the Welch bound is given by:

$$\phi_{\max} \geq \sqrt{\frac{N-K}{NK-K}} \quad (2)$$

Note that this bound is no longer achievable when $N > K$ ($K+1 = 2$ for real cases).

A. Maximal Length Sequences

The Maximal length sequences or m-sequences are pseudo-noise sequences generated by linear feedback shift registers (LFSR) [12] [13]. The actual period generated, however, depends on the logic used in the feedback. The feedback function, also called as characteristic polynomial, determines the length and type of the sequence generated.

Say, $g(x) = x^n + \alpha_{n-1}x^{n-1} + \dots + \alpha_0$. In A finite field, also called a Galois field (GF(p)) of order p, where p is a prime number, if g(x) cannot be factored into a product of lower degree polynomials, it called irreducible. It becomes primitive if the smallest integer l for which g(x) divides $f(x) = x^l - 1$ is $l = q^n - 1$ [12] [13].

The general structure of a m-sequence generator is shown in Figure 3. It contains n shift registers and is initiated with a starting seed, which is generally transmitted to the intended users through a protected channel. The outputs of the shift registers are multiplied with the coefficients of a primitive polynomial with respect to modulo-2 operation. The output's resultant obtained by the modulo-2 operation is then fed back to the first shift register and is called an m-sequence. The length of the generated m-sequence (which depends on the length of the LFSR used for the generation) is $N = 2^n - 1$. For binary fields (GF (2^n)), feedback connections with their set of primitive polynomials up to n = 12 are shown in Table 1. For Primitive polynomials of much higher degree see [14].

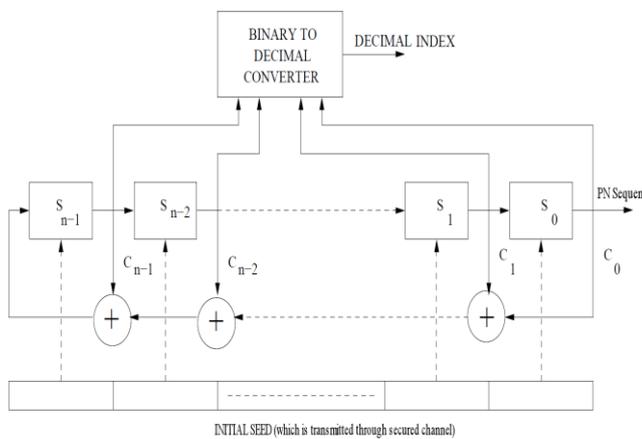


Fig 3. Structure of an m-sequence generator

B. Preferred pair of m-sequences

The cross-correlation between two m-sequences generated from different LFSR_s provides, in general, very poor results. However for Certain pairs of m-sequences have very good cross-correlation properties are known as preferred pairs. Two m-sequences **d** and **d'** of length $N = 2^n - 1$ are called the *preferred pair* if:

1. $n \neq 0 \pmod 4$ (n is odd or $n = 2 \pmod 4$).
2. $\mathbf{d} = \mathbf{d}'[q]$ where q is odd and either $q = 2^k + 1$ or $q = 2^{2k} - 2^k + 1$ for integer k.
3. The greatest common divisor of n and k, $\gcd(n, k) = 1$ for n odd and $\gcd(n, k) = 2$ for $n = 2 \pmod 4$.

The preferred pairs of m-sequences have three-valued cross-correlation function defined as $\{-1, -t(n), t(n) - 2\}$, where:

Degree (n)	$N=2^n - 1$	Feedback Taps for m-sequences	g(x)	Number of primitive polynomials
1	1	[1,0]	$x + 1$	1
2	3	[2,1]	$x^2 + x + 1$	1
3	7	[3,1]	$x^3 + x + 1$	2
4	15	[4,1]	$x^4 + x + 1$	2
5	31	[5,3] [5,4,3,2] [5,4,2,1]	$x^5 + x^2 + 1$	6
6	63	[6,1] [6,5,2,1] [6,5,3,2]	$x^6 + x + 1$	6
7	127	[7,1][7,3][7,3,2,1][7,4,3,2] [7,6,4,2][7,6,3,1][7,6,5,2] [7,6,5,4,2,1] [7,5,4,3,2,1]	$x^7 + x + 1$	18
8	255	[8,4,3,2][8,6,5,3][8,6,5,2] [8,5,3,1][8,6,5,1][8,7,6,1] [8,7,6,5,2,1] [8,6,4,3,2,1]	$x^8 + x^6 + x^5 + x + 1$	16
9	511	[9,4][9,6,4,3][9,8,5,4][9,8,4,1] [9,5,3,2][9,8,6,5][9,8,7,2] [9,6,5,4,2,1][9,7,6,4,3,1] [9,8,7,6,5,3]	$x^9 + x^4 + 1$	48
10	1023	[10,3][10,8,3,2] [10,4,3,1] [10,8,5,1][10,8,5,4][10,9,4,1] [10,8,4,3][10,5,3,2][10,5,2,1] [10,9,4,2][10,6,5,3,2,1] [10,9,8,6,3,2][10,9,7,6,4,1] [10,7,6,4,2,1][10,9,8,7,6,5,4,3] [10,8,7,6,5,4,3,1]	$x^{10} + x^3 + 1$	60

TABLE 1. FEEDBACK CONNECTIONS WITH SET OF PRIMITIVE POLYNOMIALS FOR GENERATING M-SEQUENCES

$$t(n) = \begin{cases} 2^{\frac{n+1}{2}} + 1 & \text{for } n \text{ odd} \\ 2^{\frac{n+2}{2}} + 1 & \text{for } n \text{ even} \end{cases} \quad (3)$$

These sequences can be utilized to produce several well known families of binary sequences with good cross-correlation properties. These families called Quadratic Form Sequences include the Gold and Kasami sequences [15]. Table 2 shows the preferred pairs of m-sequences.

TABLE 2. PREFERRED PAIRS OF M-SEQUENCES

degree (n)	N=2n-1	preferred pairs of m-sequences
5	31	[5,3] [5,4,3,2]
6	63	[6,1] [6,5,2,1]
7	127	[7,3] [7,3,2,1] [7,3,2,1] [7,5,4,3,2,1]
8*	255	[8,7,6,5,2,1] [8,7,6,1]
9	511	[9,4] [9,6,4,3] [9,6,4,3] [9,8,4,1]
10	1023	[10,9,8,7,6,5,4,3] [10,9,7,6,4,1] [10,8,7,6,5,4,3,1] [10,9,7,6,4,1] [10,8,5,1] [10,7,6,4,2,1]
11	2047	[11,2] [11,8,5,2] [11,8,5,2] [11,10,3,2]

C. Gold Sequences

Gold sequences can be constructed by the modulo-2 operation of two different preferred pair of m-sequences of length $N=2^n$. The block diagram of a Gold sequence generator is presented on the Figure 4. The two m-sequences are able to generate a family of many non maximal product codes; however a preferred maximal sequences can only produce Gold codes [16]. For defining set of Gold sequences it is necessary to find preferred pair of m-sequences. Since both m-sequences have equal length N, the generated gold sequence is also the same length N. From a pair of preferred sequences, the Gold sequences are generated by the modulo-2 sum of the first sequence with shifted versions of the second. There are N possible circular shifts for a period of $N = 2^n - 1$. Thus, one can get N sequences with two preferred m-sequences, and these are called Gold sequences [11] [17]. Consider two preferred pair of m-sequence represented by a binary vector f of length N, and a second sequence h obtained by sampling every q^{th} symbol of f. The set of Gold sequences generated with the two preferred pair of m-sequences f and h is defined as:

$$G(f, h) = \{f, h, f \oplus h, f \oplus T.h, f \oplus T^2.h, \dots, f \oplus T^{N-1}.h\} \quad (4)$$

Where T denotes the cyclic shift operator and \oplus is the XOR operation.

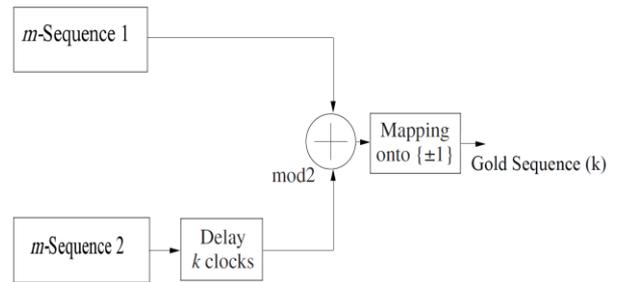


Fig 4. The block diagram of a Gold sequence generator.

The auto-correlation properties of Gold sequences are not as excellent as that of m-sequences. Apart from the two original sequences, the other are not m-sequences; so the auto-correlation is not two valued. The Gold sequences set have three valued auto-correlation spectrum, however these sequences provides additional security compared to m-sequences. While its cross-correlations have the same properties as the sequences m. Note that these properties satisfy $N = 2^n - 1$ different sequences so that the problem of the number of sequences available is reduced greatly.

Gold sequences auto-correlation function $r_{xx}(\tau)$ and the cross-correlation function $r_{xy}(\tau)$ are given as:

$$r_{xx}(\tau) = \begin{cases} 1 & \text{for } \tau = 0 \\ \left\{ -\frac{t(n)}{N}, -\frac{1}{N}, \frac{t(n)-2}{N} \right\} & \text{for } \tau \neq 0 \end{cases} \quad (5)$$

$$r_{xy} = \left\{ -\frac{t(n)}{N}, -\frac{1}{N}, \frac{t(n)-2}{N} \right\} \quad (6)$$

Where:

$$t(n) = \begin{cases} 1 + 2^{\frac{n+1}{2}} & \text{for } n \text{ odd} \\ 1 + 2^{\frac{n+2}{2}} & \text{for } n \text{ even} \end{cases} \quad (7)$$

for Gold sequences, The peak correlation value is $t(n)=N$, and from the equations of $r_{xx}(\tau)$ and $r_{xy}(\tau)$, Gold sequences with even value of LFSR length have more correlation values compared to that of odd value of n, since the value of t(n) for even value of n is less. while its cross-correlations have the same properties as the sequences m. Note that these properties satisfy $N = 2^n - 1$ different sequences so that the problem of the number of sequences available is reduced greatly.

D. Gold-Like Sequences

Gold-like sequences are other types of sequences that allow the existence of large sets with good correlation properties and have parameters similar to those of Gold sequences [18].

Let f denote an m -sequence of length $N = 2^n - 1$ generated by a primitive polynomial of degree n and let q be an integer such that $\gcd(q, N) = 3$ and let $h^{(k)}$, $k = 0; 1; 2$, be the sequences obtained by decimating $T^k f$ by q . The sequences $h^{(k)}$ are periodic with period $N' = N/3$. This novel class of sequences called Gold-like sequences can be defined as [19]:

$$G_q(f, h) = \left\{ \begin{matrix} f, h \oplus h^{(0)}, f \oplus Th^{(0)}, f \oplus T^1 h^{(0)}, \dots, f \oplus T^{N'-1} h^{(0)}, f \oplus h^{(1)}, f \oplus Th^{(1)}, f \oplus T^1 h^{(1)} \\ \dots, f \oplus T^{N'-1} h^{(1)}, f \oplus h^{(2)}, f \oplus Th^{(2)}, f \oplus T^1 h^{(2)}, \dots, f \oplus T^{N'-1} h^{(2)} \end{matrix} \right\} \quad (8)$$

The Gold-like sequence set contains 2^n sequences and each sequence has a period of $N = 2^n - 1$. The peak correlation value for the set $G_q(f, h)$ is $\phi_{\max} = t(n)$ similar to that of the Gold sequences.

E. Barker Sequences

Optimal binary sequences are binary sequences whose autocorrelation peak sidelobes is the minimum possible for a given code length. Barker sequences are short length codes that offer good correlation properties. A Barker code is a binary $\{-1, +1\}$ sequence, $\{c_i\}$, of some finite length N such that the discrete auto-correlation function $r(\tau)$, can be defined as [20]:

$$r(\tau) = \sum_{i=0}^{N-\tau} c_i c_{i+\tau} \quad (9)$$

Satisfies:

$$|r(\tau)| \leq 1 \quad \text{for } (\tau \neq 0) \quad (10)$$

Barker sequences have several advantages over other PN sequences. These sequences satisfy the run property and the balance property of the PN sequences. Also they have uniformly low autocorrelation sidelobes (≤ 1), but the size of these families is small. Table III lists all known Barker codes. However, the longest known Barker codes are of length 13.

TABLE 3. EXISTING BARKER CODE

Code Length (bits)	Sequence
2	[1 -1], [1 1]
3	[1 1 -1]
4	[1 -1 1 1], [1 -1 -1 -1]
5	[1 1 1 -1 1]
7	[1 1 1 -1 -1 1 -1]
11	[1 1 1 -1 -1 -1 1 -1 -1 1 -1]
13	[1 1 1 1 1 -1 -1 1 1 -1 1 -1 1]

For Barker codes, only sequences with length $N = 2, 3, 4, 5, 7, 11$ and 13 exist. Turyn and Storer [21] have shown that there are no Barker sequences of $N > 13$, for an N odd or for N between $4 < N < 1898884$, for an N pair. While for N even greater than 1898884 , it is not clear that there are sequences Barker. In [22], it has been shown that codes of even length must have a length which is a perfect square (except $N = 2$).

Figure 5 shows the structure of a Barker sequence generator.

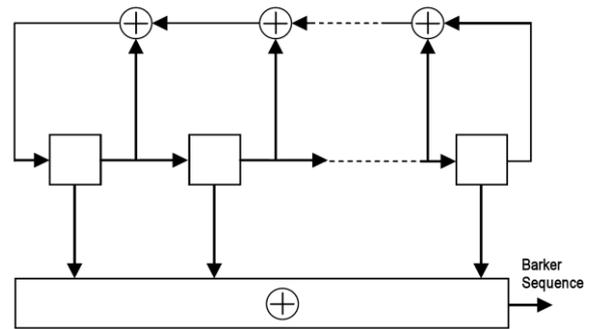


Fig 5. The structure of a Barker sequence generator.

F. Barker-Like Sequences

Barker sequences have good correlation properties with the peak correlation value being bounded by 1. The numbers of existing Barker sequences, however, are very less. We can generate more number of sequences by making certain relaxation on the peak value of the correlation function along with a maximum allowed shift between the sequences. These newly generated sequences are called Barker-like sequences [23].

Barker sequences have good correlation properties with the peak correlation value being bordered by 1. The numbers of existing Barker sequences, however, are very less. By making some relaxation on the peak value of the correlation function, along with a maximum allowed shift between the sequences, more number of sequences can be generated. These newly generated sequences are called Barker-like sequences [24] which are defined as follows:

$$r(\tau) \leq m \quad 0 < \tau \leq \tau' < N \quad (11)$$

Where $r(\tau)$ denote the auto-correlation function for a sequence of length N as defined in (8). The variables τ' is the maximum allowed shift between the sequence and m is the magnitude of the upper bound on the peak correlation function, both of them are design parameters for the Barker like sequences. If $m = 1$ and $\tau' = N$, the generated sequences becomes Barker sequences.

G. Kasami Sequences

Kasami codes are derived from m -sequences. The series of Kasami sequences are important because of its very low cross-correlation [25]. The most important property of the Kasami codes is that the peak values of the cross correlation functions are even smaller compared with the Gold codes.

The degree n of the primitive polynomial used to generate Kasami sequences and these sequences are defined for even values of n ($n = 2^k$). In this case, the length $N = 2^n - 1$ can be factorized in the following manner: $N = (2^k - 1)(2^k + 1)$, where $k = n/2$ [26]. Figure 6 illustrates the Kasami code sequences generator scheme.

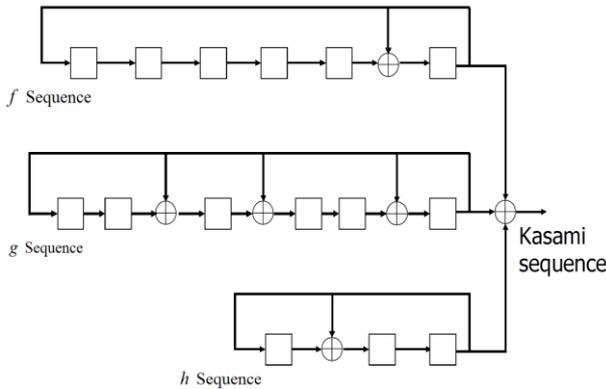


Fig 6. Kasami code sequences generator scheme

There are two kinds of Kasami sequences:

1. Small set of Kasami sequences,
2. Large set of Kasami sequences.

a) *Small set of Kasami sequences*

Small set of Kasami sequences are optimal in the sense of matching Welch’s lower bound for correlation functions. A small Kasami sequence comprise $2^{n/2}$ binary sequences with period $N=2^n-1$ which is even. To generate a large series, a M-sequence f with period $N=2^n-1$ is decimated by $s(n)=2^{n/2}+1$ in order to form a binary sequence g with period $N/\text{gcd}(N, s(n))= 2^{n/2} - 1$. A Kasami sequence with period $N=2^n-1$ is provided from the modulo-2 addition of f and a cyclic shift of g from 0 to $2^{n/2}+1$. The small set of Kasami sequences is defined as:

$$k_s(f) = \left\{ f, f \oplus g, f \oplus Tg, \dots, f \oplus T^{2^{n/2}-2} g \right\} \quad (12)$$

The correlation function of small set of Kasami sequences is too three valued comparable to the Gold codes. The correlation functions for the sequences take on from the values $\{-1, -s(n), s(n)-2\}$, the auto-correlation function of small set of Kasami sequences is defined as:

$$r_{xx}(\tau) = \begin{cases} 1 & \text{for } \tau = 0 \\ \left\{ -\frac{s(n)}{N}, -\frac{1}{N}, \frac{s(n)-2}{N} \right\} & \text{for } \tau \neq 0 \end{cases} \quad (13)$$

b) *Large Set of Kasami Sequences*

As we have seen, the Small set of Kasami sequences are optimal sequences because they have better correlation properties relative to Gold sequences. In counterpart, his set contains less number of sequences. The number of possible sequences for the small set of Kasami sequence relatively to the shift register of length n is only $2^n - 2$ sequences, while a Gold code set contains $2^n + 2$. Therefore, by manufacturing some relaxation on the correlation values of the sequences, the number of sequences can be increased. This new set of sequences is called large set of Kasami sequences [9] [19].

The large set of Kasami sequences contains supplementary number of sequences compared to Gold codes but these sequences have additional correlation values compared to Gold codes. The large set of Kasami sequences contains all the sequences in the small set along with the Gold codes.

For $\text{mod}(n,4) = 2$, Let h be the sequence formed by decimating the sequence f by $t(n)$. The large set of Kasami sequences is then defined as follows:

$$k_l(f) = G(f, h) \cup \left\{ \bigcup_{i=0}^{2^{n/2}-2} \{T^i g \oplus G(f, h)\} \right\} \quad (14)$$

The correlation functions for this sequences takes on the values $\{-t(n); -s(n); -1; s(n)-2; t(n)-2\}$. For even value of n , The maximum value of the correlation function is $t(n)$ which is similar as that of Gold codes. And for odd value of n , the large set of Kasami sequences have more correlation values compared to the Gold codes. Table IV shows a comparison of the length of PN sequences, the number of possible PN sequences of a class as well as the maximum correlation value for different PN sequences according to the degree of primitive polynomial used to generate the sequences.

TABLE 4. A COMPARISON OF DIFFERENT PN SEQUENCES.

Sequenc e	n	Leugt h	Number of Sequences	ϕ_{\max}
Maximal	even/odd	$\frac{2n-1}{1}$	1	1
Gold	odd	$\frac{2n-1}{1}$	$2n-1$	$1+2^{(n+1)/2}$
	even	$\frac{2n-1}{1}$	$2n-1$	$1+2^{(n+2)/2}$
Gold-Like	$\text{gcd}(n, k) = 3$	$\frac{2n-1}{1}$	$2n$	$1+2^{(n+2)/2}$
Barker-Like	even/odd	$\frac{2n-1}{1}$	$f(n, m)$	m
Small Kasami	even	$\frac{2n-1}{1}$	$2^{n/2}$	$1+2^{(n+2)/2}$
Large Kasami	even and $\text{mod}(n, 4) = 2$	$\frac{2n-1}{1}$	$[2^{n/2} (2^n + 1) - 1]$	$1+2^{(n+2)/2}$

III. ORTHOGONAL CODES

Orthogonal codes are a set of sequences whose all pairwise cross-correlations are zero when the inner product between two sequences is zero, those are said to be orthogonal. If $c_i(k\tau)$ and $c_j(k\tau)$ are, respectively, the i^{th} and j^{th} orthogonal members of an orthogonal set of length N , and τ is the symbol duration, then the orthogonal property

$$\sum_{k=0}^{N-1} c_i(k\tau) c_j(k\tau) = 0 \quad i \neq j \quad (15)$$

affirms that:

There are two types of orthogonal codes which obey this relation:

1. Fixed length Orthogonal Codes.
2. Variable length Orthogonal Codes.

A. Fixed Length Orthogonal Codes

a) Walsh Hadamard Codes

Walsh-Hadamard (WH) codes [27] are binary orthogonal and can easily be generated from Hadamard matrices. The orthogonal sequences generated from Hadamard matrices are called Walsh-Hadamard matrices [10], [28]. And they satisfy the unitary property given by:

$$H_N H_N^T = N I_N \tag{16}$$

Where H_N^T denotes the transposed Hadamard matrix. of order N, and I_N is the N x N unity matrix. Walsh sequences are the rows of a Hadamard matrix HN, which is a square matrix of order N. in each set of the Hadamard matrices, the first row of the matrix consist all 1's and rest of the rows contains N/2 0's (mapped to +1) and N/2 1's (mapped to -1). Also row N/2 starts with N/2 -1's and ends with N/2+1's Walsh sequences can be constructed for block length N = 2ⁿ (n is an integer). In general, Hadamard matrices are generated based on the Kronecker product recursion as follows:

$$H_{2N} = H_N \otimes H_N \tag{17}$$

Where \otimes represents the Kronecker product operator and

$$H_1 = [1]_{1 \times 1}, H_2 = \begin{bmatrix} H_1 & H_1 \\ H_1 & -H_1 \end{bmatrix}_{2 \times 2}, \dots, H_N = \begin{bmatrix} H_{N/2} & H_{N/2} \\ H_{N/2} & -H_{N/2} \end{bmatrix}_{N \times N} \tag{18}$$

This method of construction is called as Sylvester construction [29]. Orthogonality is the most significant property of Hadamard-Walsh codes. Due to this orthogonality property, and when system is perfectly synchronized, the cross-correlation between any two WH codes of the same matrix (set) is zero. However, the auto-correlation function of WH codes does not have good characteristics, it can have more than one peak and these sequences do not satisfy the run property.

b) Modified Walsh Hadamard Codes

Modified Walsh Hadamard (MWH) sequences are derived from the original sequences and then by multiplying Walsh-Hadamard HN by a diagonal matrix with a diagonal matrix DN of same order [28], [30], then the new set of sequences is based on a matrix WN, given by:

$$W_N = H_N D_N \tag{19}$$

The matrix WN is also orthogonal, since:

$$W_N W_N^T = H_N D_N (H_N D_N)^T = H_N D_N D_N^T H_N^T \tag{20}$$

And because of the orthogonality of matrix DN, we have

$$D_N D_N^T = k I_N \tag{21}$$

Where k is a real constant. Substituting (20) into (21) yields:

$$W_N W_N^T = k H_N H_N^T = k N I_N \tag{22}$$

Moreover, if k = 1, then the sequences defined by the matrix WN are not only orthogonal, but possess the same normalization such as the Walsh-Hadamard sequences. However, other correlation properties of the sequences defined by WN can be substantially different to those of the original Walsh-Hadamard Sequences. Note that these sequences have better correlation properties compared to original Walsh-Hadamard sequences.

From equation (19) it is not clear how to chose the matrix DN to achieve the desired properties of the sequences defined by the WN. In addition, there are only a few known methods to construct the orthogonal matrices, such as those used for the Hadamard matrices themselves. However, another simple class of orthogonal matrices are diagonal matrices with their elements d_{m,n} satisfying the condition:

$$|d_{m,n}| = \begin{cases} 0 & \text{for } m \neq n \\ k & \text{for } m = n \quad m, n = 1, 2, \dots, N \end{cases} \tag{23}$$

Where $|k| = 1$

To preserve the normalization of the sequences, the elements of DN, being in general complex numbers, must be of the form:

$$d_{m,n} = \begin{cases} 0 & \text{for } m \neq n \\ \exp(j\Phi_m) & \text{for } m = n \quad m, n = 1, 2, \dots, N \end{cases} \tag{24}$$

Where Φ_m denote the phase coefficients (for m = 1, 2, ..., N), are real numbers whose values are taken from the interval [0, 2π], and $j^2 = -1$. The values of Φ_m can be optimized to achieve the preferred correlation as a minimal value of peaks in cross-correlation functions or minimum out-of-phase auto-correlation.

B. variable Length Orthogonal Codes

DS-CDMA systems are designed to support a variety of data services from low to very high bit rates. since the spread signal bandwidth is the same for all users, These multi-rate systems require multiple spreading factors (SF). Therefore, the fixed length orthogonal codes (WH codes and MWH codes), presented earlier, are not able to perform this task. The Variable length orthogonal codes are those with different lengths satisfying the orthogonal property [16], they are able to fulfill the demands of the multiple spreading factors (SF) [31].

The Variable length orthogonal codes, known as Orthogonal Variable Spreading Factor (OVSF), can be derived by rearranging Walsh functions. The codes can be generated recursively using the tree structure as shown in Figure 7. New levels in the code tree are generated by concatenating a root codeword with a replica of itself.

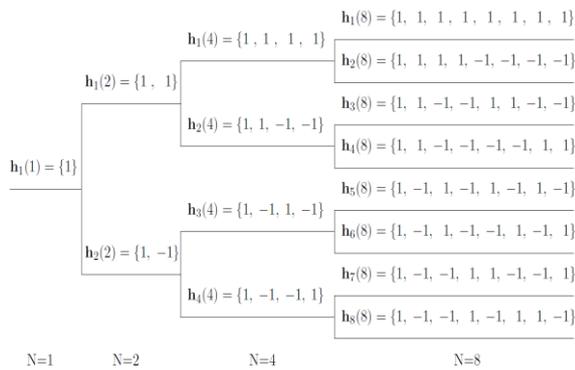


Fig 7. OVFSF code tree for WH codes

Let C_k be a matrix of size $N \times N$ and denote the set of N codes, $\{c_k(i)\}$ $i = 1, 2, \dots, 2^k$. Each of length N bits and the subscript k in $c_k(i)$ represents the layer of recursion. A set of $N=2^k$ codes can be generated at the k^{th} layer using the recursive relation, with the initial condition $c_0 = \{1\}$, as follows:

$$C_k = \begin{bmatrix} c_k(1) \\ c_k(2) \\ c_k(3) \\ \vdots \\ c_k(2^k - 1) \\ c_k(2^k) \end{bmatrix} = \begin{bmatrix} c_{k-1}(1)c_{k-1}(1) \\ c_{k-1}(1)c_{k-1}(1) \\ c_{k-1}(2)c_{k-1}(2) \\ c_{k-1}(2)c_{k-1}(2) \\ \vdots \\ c_{k-1}(2^{k-1})c_{k-1}(2^{k-1}) \\ c_{k-1}(2^{k-1})c_{k-1}(2^{k-1}) \end{bmatrix} \quad (25)$$

Where any $\bar{c}(\cdot)$ denotes the complement of $c(\cdot)$. The significant result from this tree is that all codes from different branches are orthogonal except when one of the codes is a mother code [31], and that the codes in the same layer constitute the Walsh functions, hence, are orthogonal.

As noted above, MWH codes of given length N are generated by multiplying the Hadamard matrix with a diagonal matrix of size $N \times N$. So, we have 2^N different diagonal matrices with diagonal elements equal to +1's and -1's. Each diagonal matrix can restore the fixed length orthogonal property of the MWH code set, but not the variable length orthogonal property. To obtain variable length orthogonal property for MWH code set, the diagonal elements of the diagonal matrix should be the repetitive sequence of length equal to the minimum spreading factor required in the DS-CDMA system. The code tree for generate a variable length orthogonal codes with $\{1, 1, 1, -1\}$ as repetitive sequence is presented in figure 8.

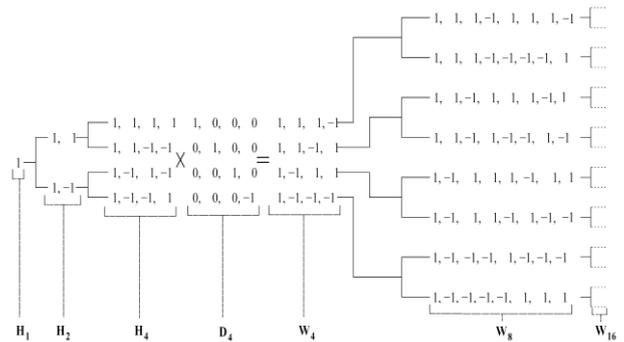


Fig 8. OVFSF code tree for MWH codes (with $\{1, 1, 1, -1\}$ as repetitive sequence)

C. Orthogonal Gold Codes

One can see that many cross-correlation values of Gold codes are "-1". Through padding one zero to the original Gold codes [9], this suggests that it may be possible to make cross-correlation values to "0" at no shift between the two sequences. Indeed, by this simple zero padding, $2^n + 1$ orthogonal codes can be obtained. These codes are called orthogonal Gold codes. The length of these orthogonal Gold codes is 2^n , what makes these sequences more suitable for different applications. The correlation values of these new orthogonal Gold codes are nearly equal to that of the original Gold codes [32][33].

IV. MEAN SQUARE CORRELATION

The Mean Square Aperiodic Auto-Correlation (MSAAC) and Mean Square Aperiodic Cross-Correlation (MSACC) measures are widely accepted performance measures for correlation properties of sequences applied in DS-CDMA. These correlation measures have been introduced by Oppermann and Vucetic [34].

Let c_i presents a sequence of length N , $c_i(n)$ denotes non-delayed version of $c_k(i)$, and $c_j(n+\tau)$ denotes the delayed version of $c_k(j)$ by ' τ ' units. The discrete aperiodic correlation function is given by:

$$r_{i,j}(\tau) = \frac{1}{N} \sum_{\tau=1-N}^{N-1} c_i(n)c_j(n+\tau) \quad (26)$$

The mean square aperiodic auto-correlation (MSAAC) value R_{AC} for a given code set containing M sequences is defined as:

$$R_{AC} = \frac{1}{M} \sum_{i=1}^M \sum_{\tau=1-N, \tau \neq 0}^{N-1} |r_{i,i}(\tau)|^2 \quad (27)$$

$$R_{CC} = \frac{1}{M(M-1)} \sum_{i=1}^M \sum_{j=1, j \neq i}^M \sum_{\tau=1-N}^{N-1} |r_{i,j}(\tau)|^2 \quad (28)$$

Auto-correlation relates to the level of correspondence between a sequence and its phase-shifted replica version, whereas cross-correlation is the measure of agreement between two different codes. These two measures have been used as the basis for comparing the sequence sets previously

presented. The sequences which have good auto-correlation properties will have poor cross-correlation properties, and vice-versa, and they have wide and flat frequency spectrum. The sequences which have less MSAAC values removes the correlation between the bits with in a sample, and those which have less MSACC values removes the sample to sample correlation, and make the information signal less intelligible.

V. THE MERIT FACTOR

In studying the properties of aperiodic auto-correlation of a family of codes, the price for being able to select good cross-correlation properties signifies degradation in the auto-correlation properties of the set of sequences. Note also that a degradation of the auto-correlation properties has a direct relation on the frequency spectrum of the sequences in the set, which is means if the R_{AC} values are less, the sequence spectrum will not be wide-band and flat. so in addition to correlation measures (R_{AC}), and to determine quantitatively how significant this degradation for a given set of sequences, there is another criterion called Merit Factor (MF) which provides a measure of all the sidelobes of the R_{AC} compared to the main peak [35], Sequences with low MF has narrow flat spectrum and they are neither suitable for CDMA [35].

The Merit Factor for a sequence, $c_i(n)$, of length N having the auto-correlation function $r_i(\zeta)$ is defined as follows:

$$F_x = \frac{r_{i,j}^2(0)}{\sum_{\tau \neq 0} |r_{i,j}(\tau)|^2} = \frac{N^2}{2 \cdot \sum_{\tau=1}^{N-1} |r_{i,j}(\tau)|^2} \quad (29)$$

The Merit Factor may be regarded as the ratio of the square of the in-phase autocorrelation, to the sum of the squares of the out-of-phase aperiodic auto-correlation values. Also, the MF may be extended to the whole sequence set as each sequence in the set has the same absolute value of auto-correlation value. Therefore we may use the inverse of the value calculated for the R_{AC} as the MF [35].

VI. RESULTS AND DISCUSSIONS

Pseudo-noise (PN) and orthogonal sequences are generated as described previously, and the MSAAC and MSACC measures as well as Merit Factor are implemented in MATLAB. Note that for all the PN sequences, the codes length has been taken as 63 bits (except the Barker sequence of length. $N = 13$), while for orthogonal codes, it has been taken as 64 bits.

TABLE 5. A PERIODIC CORRELATION MEASURES AND MERIT FACTOR FOR PSEUDO-NOISE AND ORTHOGONAL SEQUENCES

Sequence	MSAAC	MSACC	FM
Maximal	0.4429	-	2.2577
Gold	0.9750	0.9849	1.0256
Gold-like	0.9227	0.9859	1.0838

Barker (13bit)	0.0710	-	14.0833
Barker-like	0.6547	1.0546	1.5274
Small Kasami	0.7604	0.9098	1.3151
Large Kasami	0.9148	0.9979	1.0932
Walsh Hadamard	10.3906	0.8531	0.0962
NWH	5.3281	0.9154	0.1877
OVSF	5.3281	0.9154	0.1877
Orthogonal Gold	0.9739	0.9848	1.0268

Figure 9 (a) shows the aperiodic auto-correlation of m-sequences, and (b) shows the cross-correlation functions of preferred pair each of length 63 bits. Figure 10 shows the aperiodic auto-correlation function of Barker sequence of length 13 bits.

Figure 11(a)-(b), Figure 12 (a)-(b), Figure 13 (a)-(b), Figure 14 (a)-(b), Figure 15 (a)-(b) and Figure 16 (a)-(b) show the same functions for Gold, Barker-like, large Kasami, Walsh-Hadamard, MWH and orthogonal Gold sequences, respectively. While Table V shows the Aperiodic Correlation Measures and Merit Factor relating to the same sequences (Note that the correlation values of OVSF codes are for the repetitive sequence {1; 1; 1;-1}). The Histograms comparing those measures is presented on the figure 17.

From the results, between all PN sequences m-sequences have low MSAAC values given that these sequences have single peak auto correlation function. Because there is only one possible m-sequence of given LFSR length, even if the preferred pairs of m-sequences have three-valued cross-correlation function, m-sequences are not recommended for multi-user environments because to the number N that comprise the set is very small. Performance of Gold code is good as compared to m-sequence, so that the Gold codes have less MSAAC and MSACC values and for a given length of m-sequence one can generate more number of Gold codes. The Barker sequences have many advantages over other PN sequences. These sequences have uniformly low auto-correlation (MAAC = 0.0710) and have a merit factor equal to 14.0833. But they are very limited in number and are not. Similar to the Gold sequences, the Gold-like sequences have four valued auto-correlation and three valued cross-correlation functions. However, the MF of Gold-like sequences is more than that of Gold sequences (even if the number of codes that can be generated is similar); consequently these sequences are more preferable than Gold [36].

The correlation values of Barker-like sequences depend on the value of m . For length $N = 63$ and for $m = 15$, the corresponding values of MSAAC and MSACC are 0.6547 and 1.0546, respectively. These sequences have less auto-correlation values and therefore high MF value (MF= 1.5274) and also they have less autocorrelation values [36].

The autocorrelation and cross-correlation functions of kasami sequences provide excellent properties, as good or

better, than Gold Codes. Small Kasami sequences have less MSAAC value making the MF of these sequences high. But the numbers of sequences that can be generated are less. Thus the security provided by these sequences is less compared to Barker sequences. In contrast, the large Kasami sequences have many unique features .their MF is similar to that of Barker-like sequences. The possible numbers of large Kasami sequences are more compared to all other PN sequences. All these features make large Kasami sequences effective for multi-user environments.

Once there is no time shift between the two orthogonal codes sequences, they have zero cross-correlation. However the correlation values are high when there is a shift between the sequences. The correlation values of orthogonal codes are high compared to that of PN sequences. And and it is seen that the auto-correlation values of Walsh-Hadamard codes are very high (10.3906) and the MF value is less (0.0962), consequently the spectrum of these sequences is not so wide and flat. But, compared to WH codes, the MWH codes have less correlation values (5.3281) making the cross-correlation values of these codes high (0.9154) [36].

OVSF codes have the correlation functions, the MSAAC and MSACC values almost same as that of MWH codes. The correlation value of OVSF codes depends on the repetitive sequence, with the repetitive sequence {1; 1; 1;-1}; the obtained OVSF codes have less correlation values. The orthogonal Gold codes have the correlation values similar to that of original Gold codes [36].

VII. CONCLUSION

In the paper we presented different PN as well as fixed- and variable-length orthogonal sequences that can be used in DS-CDMA systems.

The comparison of PN and orthogonal codes are also derived. It can be concluded from the simulation results that the sequences which have less correlation values have make the data signal less intelligible by encrypting the data signal with those sequences. Also, the signal is better encrypted if the code set contains more number of sequences. And it is observed that large Kasami sequence has both good correlation values and high MF, which make these sequences to have wide flat spectrum that is better suited to be used in the WCDMA uplink transmission. The WH, MWH and orthogonal Gold codes have zero cross-correlation when there is no shift in the sequences. However, they have very high correlation values if there is any shift in the sequences. In the downlink of WCDMA, variable data rate is supported by using orthogonal variable spreading factor (OVSF) codes.

REFERENCES

[1] Valery P. Ipatov, "Spread Spectrum and CDMA Principles and Applications," Jhon Wiley & Sons Ltd, 2005.

[2] K. Feher, Wireless Digital Communications, "Modulation and Spread Spectrum Applications", Prentice Hall, New Jersey, USA, 1995.

[3] J. G. Proakis, Digital Communications, McGraw-Hill Series, New York, USA, third edition, 1995.

[4] B. Sklar, Digital Communications: Fundamentals and Applications, Prentice Hall, Englewood Cliffs, 1988.

[5] Dixon, R. C. , "Spread Spectrum Systems", John Wiley & Sons Inc, 1976.

[6] Simon, Marvin K., Omura, Jim K., Scholtz, Robert A., Levitt, Barry K. "Spread Spectrum Communications". Computer Science Press, 1986.

[7] W. Diffie and M. E. Hellman, "New directions in cryptography," IEEE Trans. Inform. Theory, vol. 22, pp. 644-654, Nov. 1976.

[8] R. L. Pickholtz, D. L. Schilling and L. B. Milstein, "Theory of spread spectrum communications- A tutorial," IEEE Trans. Commun., vol. COM-30, no. 5, May 1982.

[9] X. Wang, Y. Wu and B. Caron, "Transmitter identification using embedded pseudo random sequences", IEEE Trans., Broadcasting, vol. 50, no. 3, pp. 244-252, Sep. 2004.

[10] V. Milosevic, V. Delic and V. Senk, "Hadamard transform application in speech scrambling," Proc. IEEE, vol. 1, pp. 361-364, July 1997.

[11] B. Sklar, "Digital Communications: Fundamentals and Applications", 2nd Ed., Prentice Hall, 2001.

[12] L. T. Wang and E. J. McCluskey, "Linear feedback shift register design using cyclic codes," IEEE Trans. Comput., vol. 37, pp. 1302-1306, Oct. 1988.

[13] A. Fuster and L. J. Garcia, "An efficient algorithm to generate binary sequences for cryptographic purposes", Theoretical Computer Science, vol. 259, pp. 679-688, May 2001.

[14] E. J. Watson, "Primitive Polynomials (mod 2)", Mathematics of Computation, vol. 16, pp. 368-369, 1962.

[15] T. Helleseth, P. Vijay Kumar, "Sequences with Low Correlation," preprint, 1996.

[16] E. H. Dinan and B. Jabbari, "Spreading codes for direct sequence CDMA and wideband CDMA cellular networks," IEEE Commun., Magazine, vol. 36, no. 4, pp. 48-54, Sep. 1998.

[17] S. Glisic, "Adaptive WCDMA, Theory and Practice", Wiley: Chichester 2003.

[18] M.Simon, J.Omura, 6SUHDG_ 6SHFWUXP_ &RPPXQLFDWLRQV, Computer Science Press, Maryland, USA, 1985.

[19] D. V. Sarwate and M. B. Pursley, "Correlation properties of pseudo random and related sequences," Proc. IEEE, vol. 68, no. 5, pp. 593- 619, May 1980.

[20] S. W. Golomb and R. A. Scholtz, "Generalized Barker sequences," IEEE Trans. Inform. Theory, vol. IT-11, no. 4, pp. 533-537, Oct. 1965.

[21] R. Turyn and J. E. Storer, "On binary sequences," Proc. Am. Math. Soc, vol. 12, pp. 394-399, June 1961.

[22] D. G. Luenberger, "On Barker codes of even length," Proc. IEEE, vol. 51, pp. 230-231, Jan. 1963.

- [23] V. Milosevic, V. Delic and V. Senk, "Hadamard transform application in speech scrambling," Proc. IEEE, vol. 1, pp. 361-364, July 1997.
- [24] C. K. Chan and W. H. Lam, "Generalised Barker-like PN sequences for quasisynchronous spread spectrum multiple access communication systems," IEEE Proc. Commun., vol. 142, no. 2, pp. 91-98, April 1995.
- [25] M., Gudmundson, "Correlation Model For Shadow Fading in Mobile Radio Systems", Electronic Letters, Vol. 27, No. 23, pp. 2145-2146, November 1991.
- [26] Rappaport S., Theodore, "Principles of Communication Systems Simulation with Wireless Applications", Prentice Hall, 2003.
- [27] J. L. Walsh, "A closed set of normal orthogonal functions", American Journal of Mathematics, 45(1):5-24, enero 1923.
- [28] Tai-Kuo Woo, "Orthogonal variable spreading codes for wideband CDMA," IEEE Trans. Vehicular Techn., vol. 51, no. 4, pp. 700-709, July 2002.
- [29] Todd K. Moon, Wynn C. Stirling, "Mathematical Methods and Algorithms for Signal Processing", Pearson Education. Année?????
- [30] B J Wysocki, T A Wysocki, "Modified Walsh-Hadamard sequences for DS CDMA wireless systems", International Journal of Adaptive Control and Signal Processing, 2002, 16:589-602
- [31] E.H. Dinan and B. Jabbari, "Spreading Codes for Direct Sequence CDMA and Wideband CDMA Cellular Networks", IEEE Communications Magazine, vol. 36, no. 9, pp. 48-54, September 1998.
- [32] S. Tachikawa. "Recent spreading codes for spread spectrum communications systems", Electronics and Communications in Japan, 75(6):41-49, marzo 1992.
- [33] H. Donelan and T. O'Farrell, "A new method for generating sets of orthogonal sequences for a synchronous CDMA system", IEEE Electronics Letters, 35(8):1537-1538, Sep. 1999.
- [34] I. Oppermann and B. S. Vucetic, "Complex spreading sequences with a wide range of correlation properties," IEEE Trans. Commun., vol. COM-45, pp. 365-375, March 1997.
- [35] M. J. Golay, "Sieves for low auto-correlation binary sequences", IEEE Trans, on Information Theory, 23(1):43-51, enero 1977.
- [36] A. ZIANI, A. MEDOURI, "ANALYSIS OF DIFFERENT PSEUDO-RANDOM AND ORTHOGONAL SPREADING SEQUENCES IN DS-CDMA", Proc. IEEE Xplore Digital Libraries, Cat. Number: CFP12050-PRT, pp. 558 – 564, May 2012.

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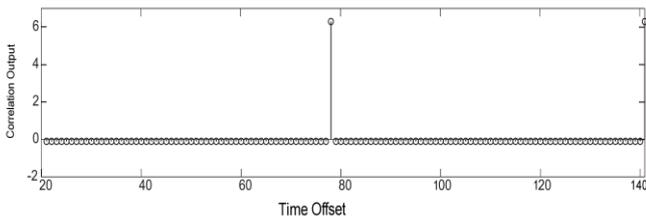
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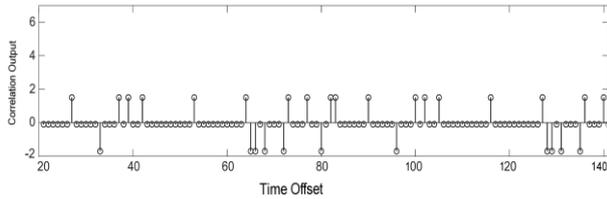
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APPENDIX

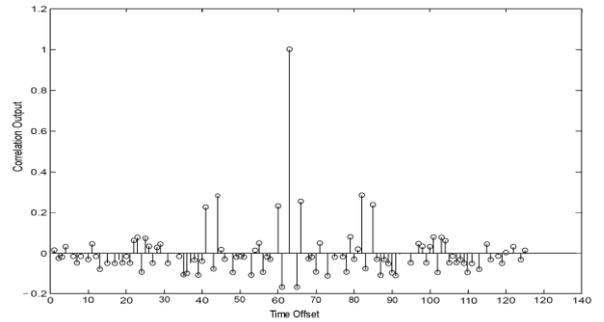


(a)

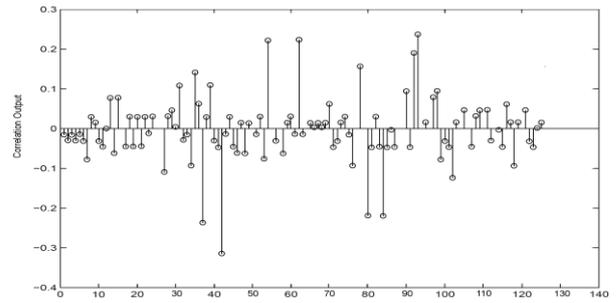


(b)

Fig 9. (a) Aperiodic auto-correlation function of m-sequences of length 63 bits. (b) Aperiodic cross-correlation function of preferred pair of length 63 bits.



(a)



(b)

Fig 12 (a) Aperiodic auto-correlation function of Barker-Like sequence. (b) Aperiodic cross-correlation function of Barker-Like of length 63 bits.

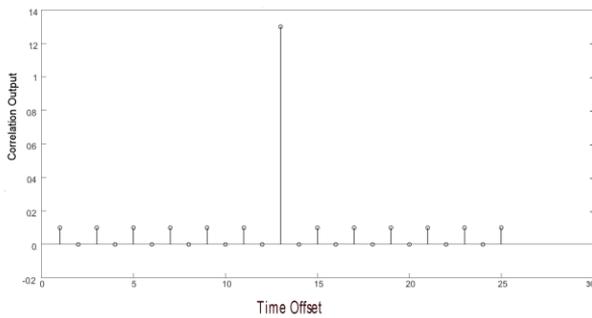
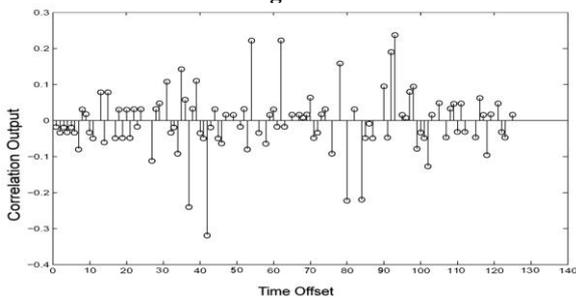
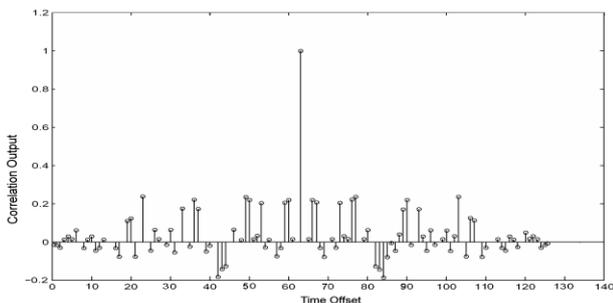


Fig 10. Aperiodic auto-correlation function of Barker sequence of length 13 bits.

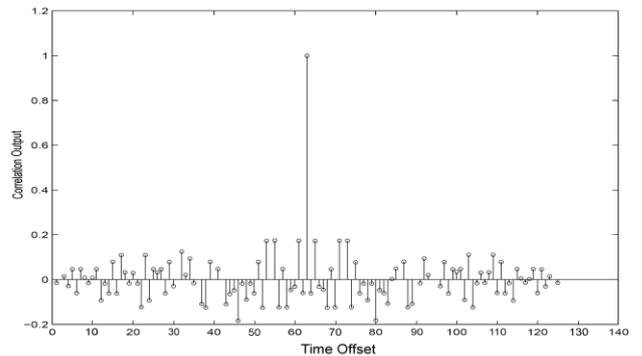


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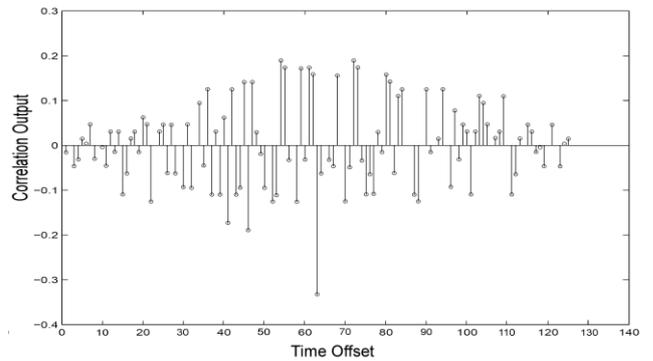


(b)

Fig 11 (a) Aperiodic auto-correlation function of Gold sequence. (b) Aperiodic cross-correlation function of Gold sequence of length 63 bits.

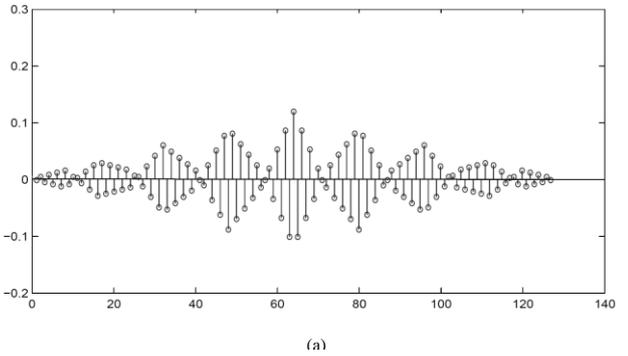


(a)

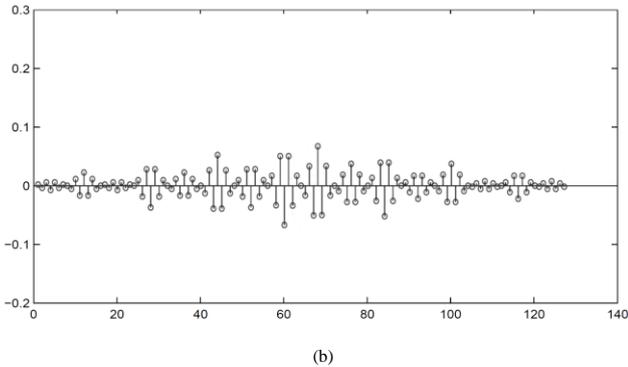


(b)

Fig 13 (a) Aperiodic auto-correlation function of large Kasami sequence. (b) Aperiodic cross-correlation function of large Kasami of length 63 bits.

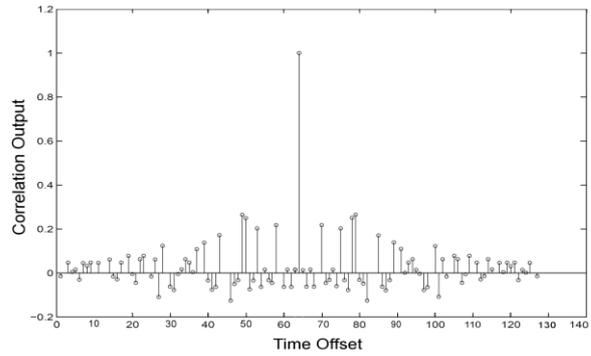


(a)

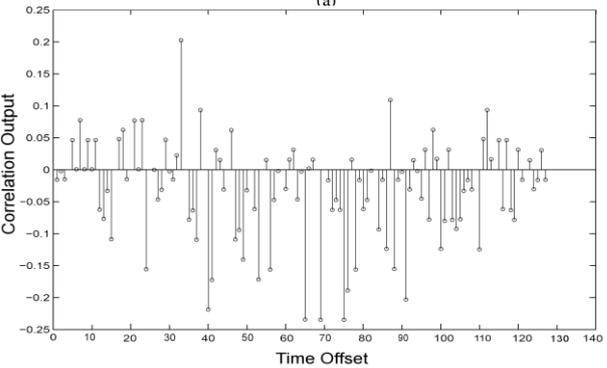


(b)

Fig 14.(a) Aperiodic auto-correlation function of WH sequence. (b) Aperiodic cross-correlation function of WH of length 64 bits.

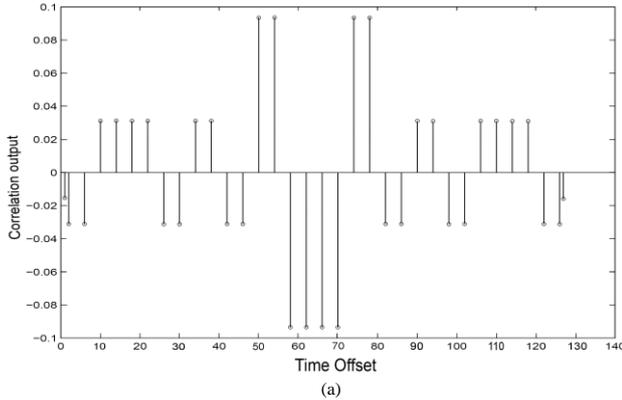


(a)

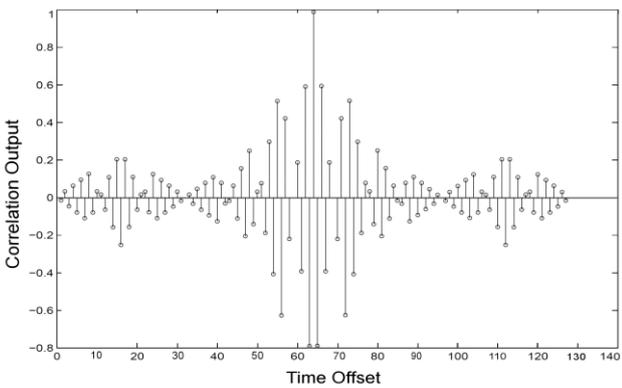


(b)

Fig 16 (a) Aperiodic auto-correlation function of Orthogonal Gold sequence. (b) Aperiodic cross-correlation function of Orthogonal Gold sequence of length 64 bits



(a)



(b)

Fig 15. (a) Aperiodic auto-correlation function of MWH sequence. (b) Aperiodic cross-correlation function of MWH sequence of length 64 bits

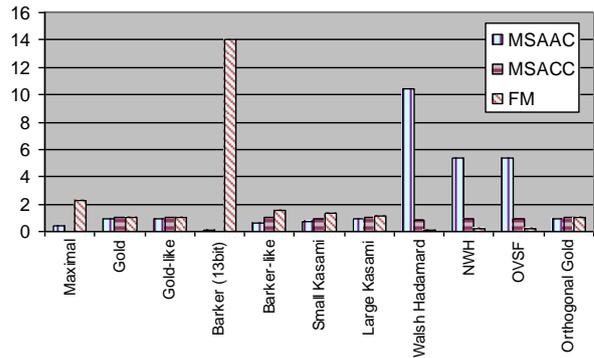


Fig 17. Histograms comparing APERIODIC CORRELATION MEASURES and Merit Factor for Pseudo-noise and orthogonal sequences