

Thermo-Structure of Dual-Bell Nozzle

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Abstract— A thermo-structural analysis was performed on a nozzle throat insert composed of a 3D carbon carbon ablative coating on 4D carbon carbon substrate. The analysis considered an environment produced by a solid rocket propellant at a chamber pressure of 1000 psi for duration of sixty seconds. The analysis showed the effect on the thermo-structural predictions and of the sensitivity of the stresses on the backside constraint, a dual-bell nozzle is considered to be feasible devices to improve performance of booster engines for near future reusable launch vehicles. Hot firing tests were conducted at a high altitude test stand.

The objective of this paper is to establish the thermal stability and structural integrity of the nozzle. The combustion gas flow was sonic, supersonic, & hypersonic respectively, in the inlet, throat, and exit sections of the nozzle. The exit plane Mach number was 5.021. Thermal analysis by APDL Code and structural analysis by ANSYS. The recession life of thermal layer was 60 sec for 100 load steps. Stress analysis of the nozzle due to aerodynamic pressure and temperature resulted in low strains and presents no concern

Key words- 3D carbon carbon ablative coating on 4D carbon carbon, Mach number, APDL Code and structural analysis by ANSYS,

I. INTRODUCTION

A Nozzle is a device designed to control the direction or characteristics of a fluid flow (especially to increase velocity) as it exits (or enters) an enclosed chamber or pipe. A nozzle is often a pipe or tube of varying cross sectional area and it can be used to direct or modify the flow of a fluid (liquid or gas). Nozzles are frequently used to control the rate of flow, speed, direction, mass, shape, and the pressure of the stream that emerges from them.



FIG 1. NOZZLE

Frequently, the goal is to increase the kinetic energy of the flowing medium at the expense of its pressure and energy. Nozzles can be described as convergent (narrowing down from a wide diameter to a smaller diameter in the direction of the flow) or divergent (expanding from a smaller diameter to a larger one). A de Laval nozzle has a convergent section followed by a divergent section and is often called

a convergent-divergent nozzle ("con-di nozzle"). Convergent nozzles accelerate subsonic fluids. If the nozzle pressure ratio is high enough, then the flow will reach sonic velocity at the narrowest point (i.e. the nozzle throat). In this situation, the nozzle is said to be choked.

Increasing the nozzle pressure ratio further will not increase the throat Mach number above one. Downstream (i.e. external to the nozzle) the flow is free to expand to supersonic velocities however Mach 1 can be a very high speed for a hot gas because the speed of sound varies as the square root of absolute temperature. This fact is used extensively in rocketry where hypersonic flows are required and where propellant mixtures are deliberately chosen to further increase the sonic speed. Divergent nozzles slow fluids if the flow is subsonic, but they accelerate sonic or supersonic fluids.

Convergent-divergent nozzles can therefore accelerate fluids that have choked in the convergent section to supersonic speeds. This C-D process is more efficient than allowing a convergent nozzle to expand supersonically externally. The shape of the divergent section also ensures that the direction of the escaping gases is directly backwards, as any sideways component would not contribute to thrust.

A. PRINCIPLE OF OPERATION

- A nozzle operates by using its narrowest part, or 'throat', to increase pressure within the engine by constricting airflow, then expanding the exhaust stream to, or near to, atmospheric pressure, and finally forming it into a high speed jet to propel the vehicle.
- The energy to accelerate the stream comes from the temperature and pressure of the gas- the gas adiabatically, when done against a nozzle, this largely reversibly and hence efficiently cools, expands, and accelerates the gas, with the heat and pressure of exhaust gas being proportional to its speed.
- Air-breathing engines create forward thrust on the airframe by imparting a net rearward momentum onto the air via producing a jet of exhaust gas, which, when fully expanded, has a speed that exceeds the aircraft's airspeed.
- Engines that are required to generate thrust quickly from idle use propelling nozzles with variable area. While at idle, the nozzle is set to its open configuration for minimum thrust and high engine rpm, but when thrust is needed, (e.g., while initiating a go-around) constricting the nozzle will quickly generate thrust.

A stepped nozzle is a De Laval rocket nozzle which has properties. The characteristic of this kind of nozzle is that part of the way along the inside of the nozzle there is a

straightening of the curve of the nozzle contour, followed by a sharp step outwards.

At low altitude this causes the jet to separate at the step, and ambient pressure maintains the jet at this place, avoiding jet instabilities and avoiding massive over-expansion. As the altitude rises the jet becomes progressively under-expanded and grows until it fills the nozzle, at which point the gas provides more pressure against the rest of the nozzle and thrust and specific impulse increases.

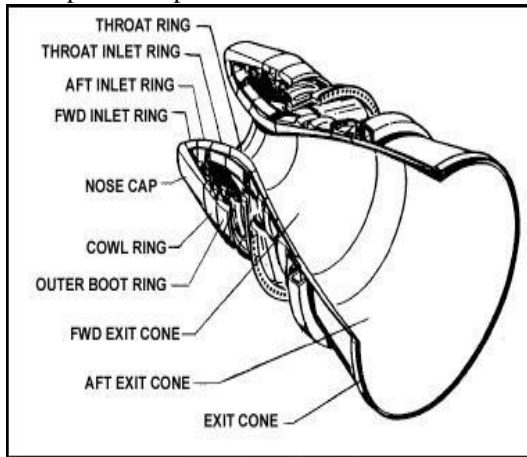


FIG 2 COMPONENTS OF DUAL-BELL NOZZLE

II. ADVANTAGES OF THE DUAL-BELL NOZZLE

Dual-Bell Nozzle This nozzle concept was first studied at the Jet Propulsion Laboratory in 1949. In the late 1960s, Rocket-dyne patented this nozzle concept, which has received attention in recent years in the U.S. and Europe. Figure below illustrates the design of this nozzle concept with its typical inner base nozzle, the wall inflection, and the outer nozzle extension.

This nozzle concept offers an altitude adaptation achieved only by nozzle wall inflection. In low altitudes, controlled and symmetrical flow separation occurs at this wall inflection which results in a lower effective area ratio.

For higher altitudes, the nozzle flow is attached to the wall until the exit plane, and the full geometrical area ratio is used. Because of the higher area ratio, an improved vacuum performance is achieved. However, additional performance losses are induced in dual-bell nozzles.

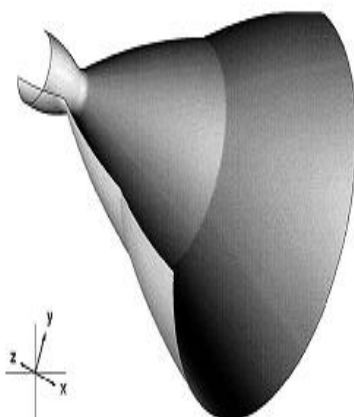


FIG 3 DUAL BELL NOZZLE (DOUBLE STEP NOZZLE)

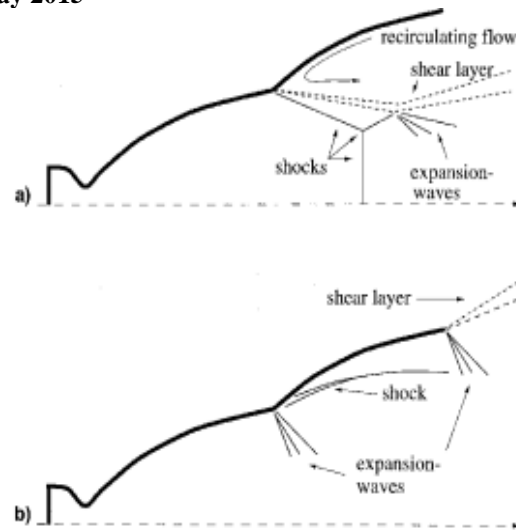


FIG 4. FLOW FIELD PHENOMENON IN DUAL BELL NOZZLE

- a) At sea-level mode with flow separation at the inflection point,
- b) Altitude mode with full flow nozzle.

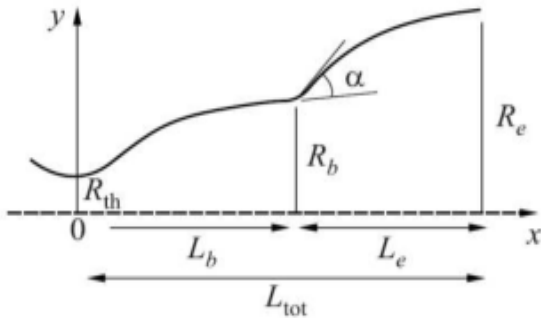
As compared with two baseline nozzles having the same area ratio as the dual-bell nozzle at its wall inflection and in its exit plane. Fig 4 above illustrates the performance of a dual-bell nozzle as a function of flight altitude in comparison with both baseline bell-type nozzles. (Design parameters of the dual-bell nozzle are taken from a launcher analysis published in Ref. 18: Propellants hydrogen / oxygen, wall inflection at area ratio EB = 30, and total area ratio EE = 100.) The pressure within the separated flow region of the dual-bell nozzle extension at sea-level operation is slightly below the ambient pressure, inducing a thrust loss referred to as “aspiration drag”. In addition, flow transition occurs before the optimum crossover point, which leads to further thrust loss as compared to an ideal switch over.

The non-optimum contour of the full flowing dual-bell nozzle results in further losses at high altitudes. To gain insight into the performance and flow behavior of dual-bell nozzles at different ambient pressures, extensive numerical simulations with parametrical variations of contour design parameters were performed. An optimized bell nozzle with equal total length and area ratio was used as the reference nozzle for comparison. As a result the vacuum performance of the dual-bell nozzles has degradation because of the imperfect contour, and this additional loss has the same order of magnitude as the divergence loss of the optimized bell nozzle.

SALIENT FEATURES OF THE NOZZLE

The nozzle hardware is designed as two components i.e. integral throat and entrance (ITE) and the exit cone. The nozzle is designed so as to get the required clearance space for seating the ITE inside the counter bore. The exit cone is threaded to the ITE by means of buttress threads. The throat diameter is kept constant as 50 mm. The cone inflection angle is selected as 11.49 degrees to obtain an area ratio of 16 that accelerates the exhaust to a Mach of 5.021 The thermal

protection system is designed as a single component namely carbon-phenolic liner that acts as backup to the ITE and the exit cone (till the threaded portion) and limits the temperature raise of the metallic portion.



**FIG 5. DESIGN SPECIFICATION
DESIGNING & MODELLING OF THE NOZZLE**

At first the profile of the nozzle is created using the designing software “CATIA V5 R20”. It is the simplest of all the software’s to produce complex profiles easily. The profile of the nozzle created is now saved as Initial Graphics Exchange Specification (IGES) file with an extension “igs” now the IGES file, which is the suitable format to import into the “ANSYS”, analysis software to analyze the nozzle.

THROAT RADIUS	R_{th}	50mm
BASE LENGTH	L_b/R_{th}	4.1
EXTENSION LENGTH	L_e/R_{th}	4.9
AREA RATIO	E_b	16
INFLECTION ANGLE	Alpha	11.49deg

TABLE 1 DESIGN SPECIFICATIONS

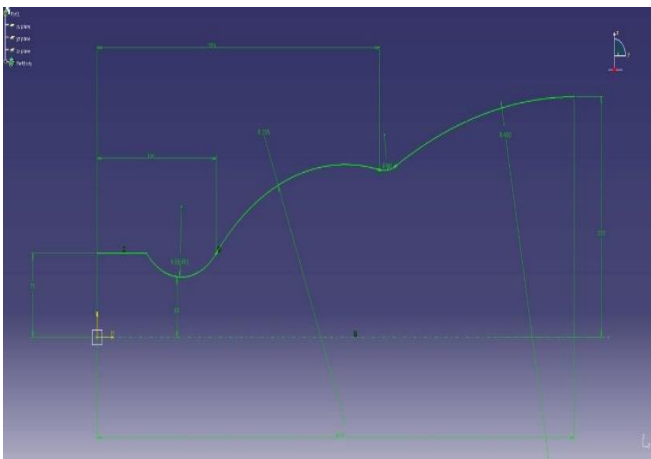


FIG 6 SKETCH OF DUAL BELL NOZZLE

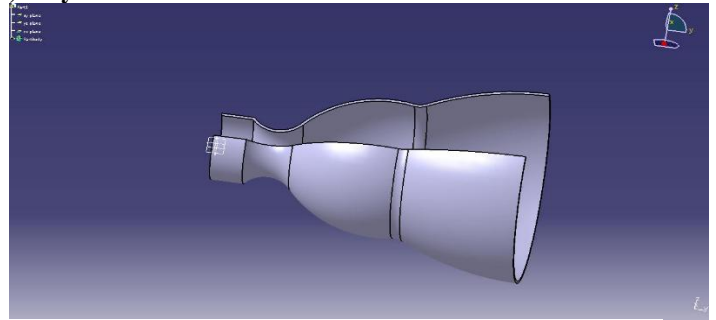


FIG 7 DESIGN OF DUAL BELL NOZZLE (270 DEGREES)

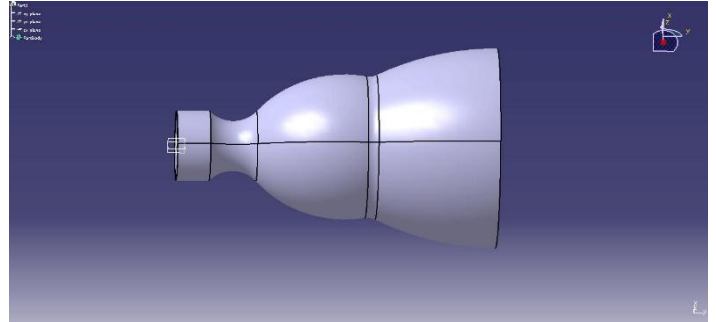


FIG 8 DESIGN OF DUAL BELL NOZZLE (360 DEGREES)

III. NOZZLE MATERIAL

A. MATERIALS USED FOR NOZZLE & ITS PROPERTIES

The present interest in high speed and high altitude aircraft, missiles and rockets has created a need for structural elements that are characterized by having the requisite dimensional stability and structural strength to withstand the severe stresses and strains encountered during operation within the high temperature environment

Typically, rocket nozzles, combustion chambers and integral rocket motors are made of composite materials with two-dimensional reinforcing fibers. A number of methods are known for laying the fibers and include a tape wrap and flat wrap in which woven cloth forms are laid perpendicular to the nozzle/motor axis. Another method is the Dixie cup wrap in which the cloth forms are canted to the axis. A chopped fabric method, in which the material is randomly put into a mold, compressed and molded into a matrix material, is also employed. Conventional filament wound or braided structures have also been used.

1. 3D AND 4D CARBON-CARBON

3D carbon carbon is a unique form of graphite manufactured by decomposition of a hydrocarbon gas at very high temperature in a chemical vapor deposition furnace. The result is an ultra-pure product which is near theoretical density and extremely anisotropic.

Carbon/Carbon composites have demonstrated a capability to act as an ablative and structural material in solid rocket nozzles replacing the existing standard materials such as carbon or silica cloth phenolic used for ablative liners and other metals used for the structural shells. The Carbon/Carbon composites allow new nozzle design concepts to be achieved with only one material that serves jointly as the structure and

the ablative liner. With this new nozzle concept and a single design material, the nozzle weight, volume envelope are reduced that leads to increased missile thrust performance.

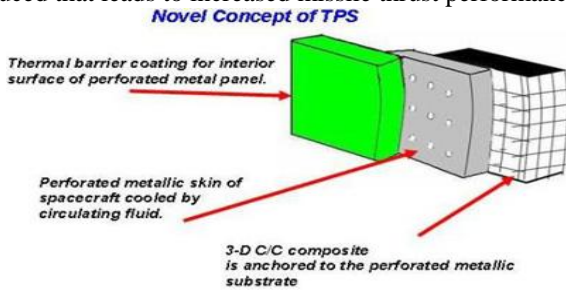


FIG 9. TEMPERATURE PROTECTIVE SYSTEM

IV. THERMAL ANALYSIS OF THE NOZZLE (ANSYS)

A. METHODOLOGY

The coupled field analysis is chosen to solve for the thermal and structural loads one after the other. The model is meshed and then the thermal problem is defined. The thermal conductivity, specific heat and density values are given as material properties and are written in the thermal physics file. The thermal environment is cleared and then the structural problem is defined. The Young's modulus, expansion coefficients are given as material properties and are written in the structural physics file. The material properties at different temperatures used for the analysis are given in the appendix. The heat transfer coefficients are given as inputs along with the bulk temperature of the fluid at various axial positions of the nozzle. Solving the thermal problem gives the temperature distribution after the stipulated time. The pressure load and displacement constraints are applied. The temperature distribution from thermal results is also read. This model is solved to obtain the stresses and displacements.

A two dimensional, axisymmetric finite element model was employed for the sake of simplicity. This model is also used to simulate the nonlinear and orthotropic material properties. Coupled field analysis is chosen to solve for thermal and structural loads one after the other. Thermal model is meshed with plane 55 element. This element has four nodes with a single degree of freedom, temperature, at each node. This element switches to plane 77 when the structural analysis is run. Plane 183 is used for 2-D modeling of solid structures. This element is defined by four nodes having two degrees of freedom at each node: translations in the nodal x and y directions.

The loading due to the thermal gradients is a function of the local heat transfer coefficients (htc). The htc at various locations are calculated using bartz equation at an average pressure of 35 ksc (lower bound time). The boundary conditions for the thermal model are the htc's at various axial locations along with the local gas temperature. Initial temperature of 327K is applied.]

The transient thermal analysis is run to get the temperature profile of the nozzle at the end of 60 seconds. The values of heat transfer coefficients and the gas temperatures at various

locations that are used for the thermal analysis are provided in the appendix. The finite element model along with the thermal boundary conditions is depicted. The temperature plot of the ITE and the exit cones after 60 seconds of motor operation. Temperature distribution and heat flux Test facility M5.021 offers only short test duration (up to 60 s). Consequently, it is difficult to reach thermal steady state conditions. To circumvent this limitation, the tests were conducted in series with short interval the temperature distribution recorded at the end of a series in altitude mode. The temperatures were recorded in the wall at 1mm and 3 mm from the hot flow side. The temperature decreases progressively along the base nozzle wall. Downstream the contour inflection, the flow is highly expanded, leading to a faster decrease of the wall temperature.

In addition to the present study on a 2D nozzle model, an axisymmetric nozzle model has been experimentally investigated. The model is represented with its geometrical parameters. The tests were conducted under similar conditions at M5.021 facility. The region of the nozzle throat featured 20mm wall thickness, the remaining part, from the middle of the base nozzle to the end of the extension, only 3mm. The thin wall permitted temperature measurements using thermography from the outer side of the nozzle. Thermocouples were also placed in the wall along the nozzle contour. Figure illustrates the temperature distribution along the outer wall obtained with this method in sea level and altitude mode.

The tests conducted in this study were very short (only 15 s), so that no thermal steady state could be reached. Besides, the measurement uncertainties using thermography are higher. The consequence is that the experimental results obtained for this campaign feature a significant inaccuracy margin. However, the thermal flux distribution calculated out of the measurements confirms the effect seen for the planar nozzle in classical axisymmetric dual bell nozzle.

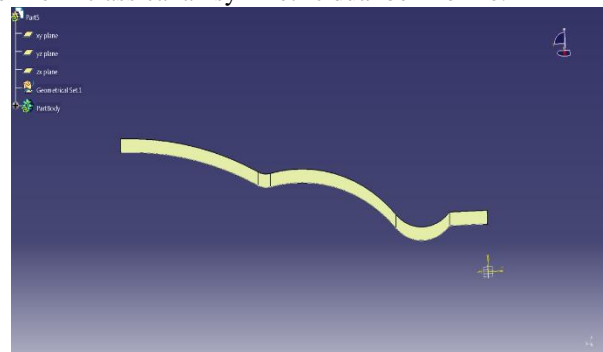


FIG 10 AXI-SYMMETRIC DESIGN OF THE NOZZLE

The conditions reached on the test position M5.021 (P0, max= 0.772bar and T0, max= 623K, sub scale nozzle) are still far from real flight conditions. The main advantage however is the possibility of testing a small nozzle model without cooling system. A film cooling, for example, would produce a secondary flow which significantly influences the flow separation condition (as seen during the test campaign CALO at DLR Lampoldshausen5), and hence the dual bell operating

mode. A cooling system would also disturb the investigation of thermal loads at the nozzle wall by increasing the number of influence parameters.

The test bench was designed for stationary operation, but it was possible to reproduce transient NPR conditions by manually varying the hydrogen mass flow during the test run. The total pressure variations were then linked to a temperature variation in the opposite direction.

For stiffness reason, the wall thickness was 10 mm in the subsonic part and in the throat region. In the region directly upstream the contour inflection and in the extension, the wall thickness was reduced to 3 mm. This permitted to keep a low thermal response time of the wall (0.7 s for a response at 80%) to flow temperature variation. Furthermore, the Biot number is kept low.

$$Bi = h / L * k$$

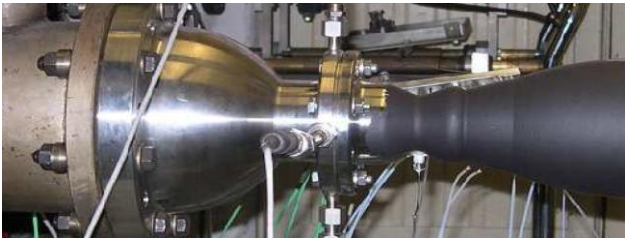


FIG 11. TESTING OF NOZZLE FOR THERMAL ANALYSIS

With h the heat transfer coefficient of the material, L the characteristic length (here the wall thickness) and k the thermal diffusivity of the material. The value of Bi is lower than 0.1, which means that the wall can be considered as thin, that is with an almost constant temperature through the wall.

The base nozzle was designed as a truncated ideal nozzle (TIC) and the extension on an isobar (constant pressure extension). The nozzle was constituted of two walls corresponding to the nozzle contour and two exchangeable side plates. The base nozzle was designed as a non-truncated ideal nozzle, to limit the 3D effects due to the side walls. Like for the axisymmetric model, the extension was designed on an isobar. The nozzle depth was 45 mm and it was kept constant from the convergent to the nozzle end. A window of 45 mm diameter was integrated in the side plates of the planar model, in the region of the contour inflection.

Now the model imported from the CATIA as an IGES file is given element types and is meshed in the preprocessor and then the problem is defined. The meshed figure is shown below:

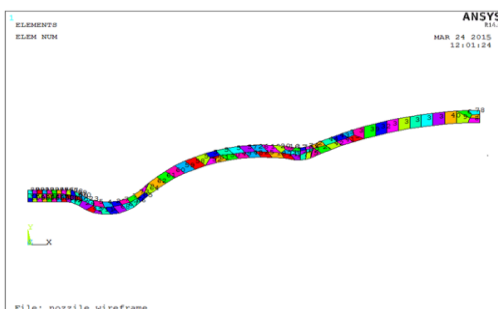


FIG 12 MESHED NOZZLE

B. ANSYS PARAMETRIC DESIGN LANGUAGE CODE FOR THERMAL ANALYSIS

```
sfe,99,3,conv,,60
sfe,99,3,conv,2,327
sfe,101,3,conv,,60
sfe,101,3,conv,2,431.5
sfe,104,3,conv,,60
sfe,104,3,conv,2,472
sfe,107,3,conv,,60
sfe,107,3,conv,2,484
sfe,109,3,conv,,60
sfe,109,3,conv,2,527
sfe,73,3,conv,,60
sfe,73,3,conv,2,541
sfe,63,3,conv,,60
sfe,63,3,conv,2,554
sfe,42,1,conv,,60
sfe,42,1,conv,2,567
sfe,57,2,conv,,60
sfe,57,2,conv,2,580
sfe,52,3,conv,,60
sfe,52,3,conv,2,584
sfe,45,2,conv,,60
sfe,45,2,conv,2,593
sfe,49,2,conv,,60
sfe,49,2,conv,2,608
sfe,12,3,conv,,60
sfe,12,3,conv,2,623
sfe,68,3,conv,,60
sfe,68,3,conv,2,565
sfe,69,3,conv,,60
sfe,69,3,conv,2,565
sfe,38,4,conv,,60
sfe,38,4,conv,2,513
sfe,36,4,conv,,60
sfe,36,4,conv,2,513
sfe,29,1,conv,,60
sfe,29,1,conv,2,467
sfe,27,2,conv,,60
sfe,27,2,conv,2,451
sfe,31,1,conv,,60
sfe,31,1,conv,2,418
sfe,34,3,conv,,60
sfe,34,3,conv,2,386
sfe,74,2,conv,,60
sfe,74,2,conv,2,339
```

The wall thickness for the side plates and the contour was of 20 mm. The geometrical parameters of the planar dual bell are summarized above and illustrated figure nozzle model was made of heat resistant.

Temperature measurements were also made along the wall of nozzle. Thermocouples were placed at various depths in the wall. The first objective was to determine the temperature distribution along the contour, particularly in the vicinity of the inflection. In future evaluation, the various depths at which the thermo-couples were placed will yield information on thermal loads. The following APDL code was written in

order to apply thermal loads along the nozzle wall as illustrated below:

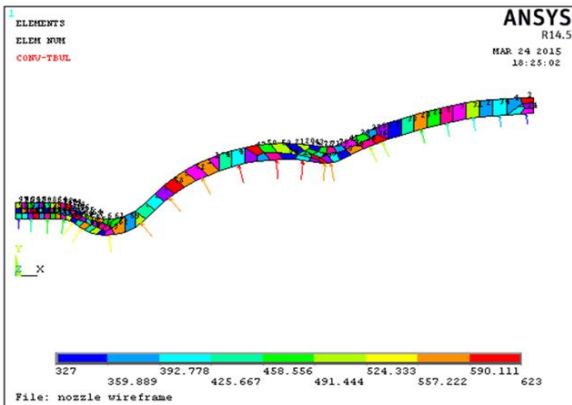


FIG 13. THERMAL BOUNDARY CONDITIONS

V. STRUCTURAL ANALYSIS OF THE NOZZLE (ANSYS)

A. METHODOLOGY

A two dimensional, axisymmetric finite element model was employed for the sake of simplicity. This model is also used to simulate the nonlinear and orthotropic material properties. Coupled field analysis is chosen to solve for thermal and structural loads one after the other. Thermally loaded model is now selected for performing structural analysis. This element switches to plane 42 when the structural analysis is run. Plane 183 is used for 2-D modeling of solid structures. This element is defined by four nodes having two degrees of freedom at each node: translations in the nodal x and y directions.

After switching from thermal to structural analysis we now directly apply the pressure loads over the thermal loads applied on the material previously as the project is all about thermo-structural analysis (i.e.) the loads are applied at all same elements were thermal loads were applied the following is the coding written in ANSYS (APDL).

B. ANSYS PARAMETRIC DESIGN LANGUAGE FOR STRUCTURAL ANALYSIS

- sfe,99,3,pres,,5.52e+3
- sfe,101,3,pres,,7.16e+3
- sfe,104,3,pres,,9.69e+3
- sfe,107,3,pres,,1.59e+4
- sfe,109,3,pres,,2.17e+4
- sfe,73,3,pres,,2.94e+4
- sfe,63,3,pres,,3.84e+4
- sfe,42,1,pres,,6.09e+4
- sfe,57,2,pres,,7.44e+4
- sfe,52,3,pres,,7.72e+4
- sfe,45,2,pres,,7.67e+4
- sfe,49,2,pres,,4.96e+4
- sfe,12,3,pres,,3.61e+4
- sfe,68,3,pres,,3.39e+4
- sfe,69,3,pres,,3.39e+4
- sfe,38,4,pres,,3.15e+4
- sfe,36,4,pres,,3.15e+4
- sfe,29,1,pres,,9.69e+3
- sfe,27,2,pres,,7.67e+3

- sfe,31,1,pres,,6.02e+3
- sfe,34,3,pres,,4.52e+3
- sfe,74,2,pres,,3.14e+3

The following picture depicts the loads applied along the nozzle contour.

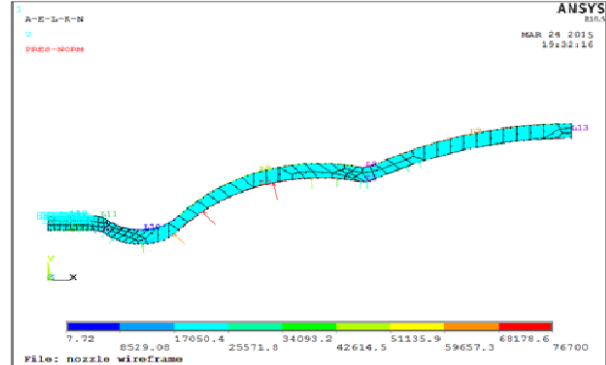


FIG 14 STRUCTURAL BOUNDARY CONDITIONS

VI. RESULTS FOR THERMO-STRUCTURAL ANALYSIS OF THE NOZZLE

A. RESULTS FOR THERMAL ANALYSIS OF THE NOZZLE

The temperature distribution along the contour is found from the nodal solution in the figure below & it is found that the material temperature decreases progressively along the base nozzle wall. Downstream the contour inflection, the flow is highly expanded, leading to a faster decrease of the wall temperature.

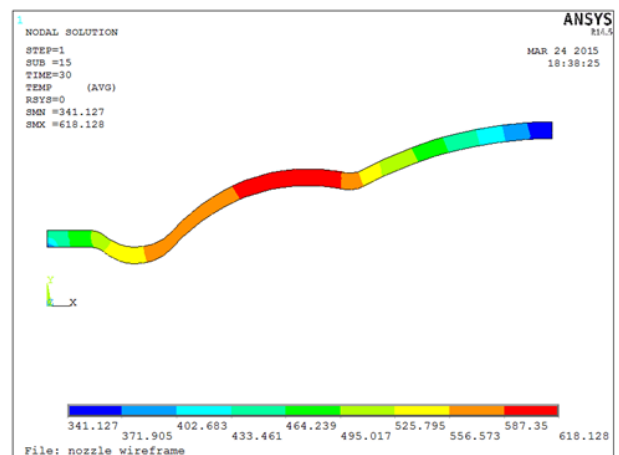


FIG 15 TEMPERATURE DISTRIBUTION

A. RESULTS FOR STRUCTURAL ANALYSIS OF THE NOZZLE

Dynamic pressure is the pressure that is exerted by a fluid. Specifically, it is the pressure measured when the fluid is still, or at rest. The above figure reveals the fact that the gas gets expanded in the nozzle exit. The Dynamic pressure in the inlet is observed to be 5.52 e+03 Pa and as move towards the throat there is an increase and the value at the throat is found out to be 7.72 e+04 Pa. After the throat, there is a sudden increase in the Dynamic pressure at the axis which indicates the occurrence of the shock. After the shock there is a slight

decrease in the pressure but it again rises at the second shock. Then it reduces to a value of 3.14×10^4 Pa at the exit section due to the expansion of the fluid towards the exit of the nozzle.

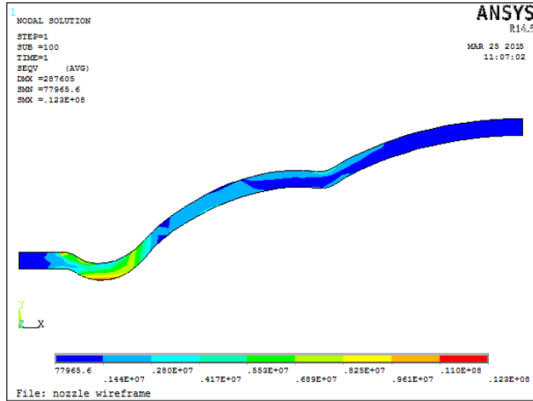


FIG 16. VON MISES STRESS

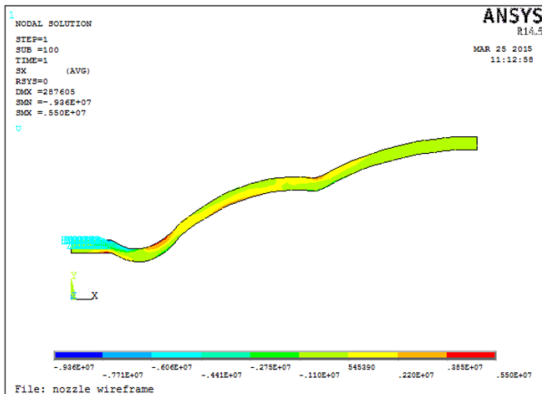


FIG 17. X-COMPONENT OF STRESS

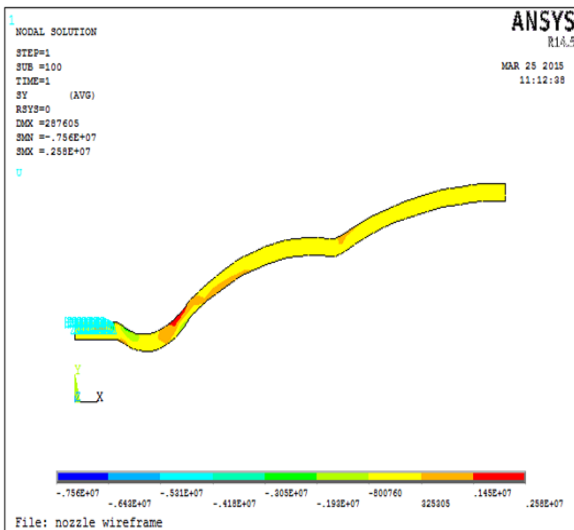


FIG 18. Y-COMPONENT OF STRESS

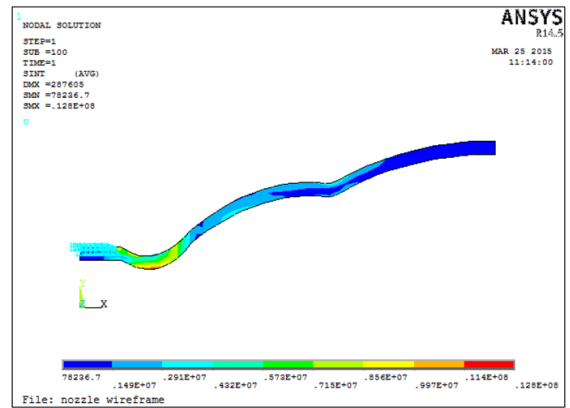


FIG 19 STRESS INTENSITY

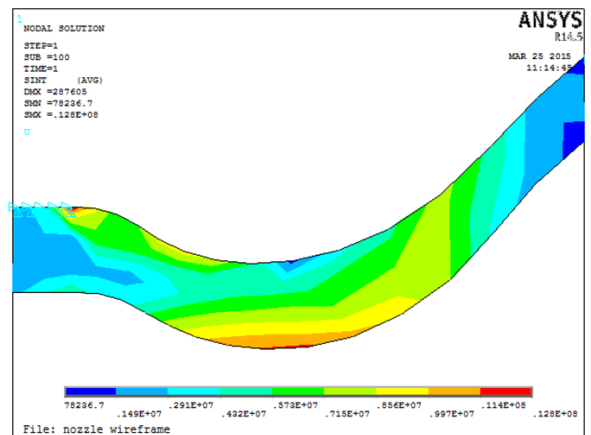


FIG 20 STRESS INTENSITY AT THROAT

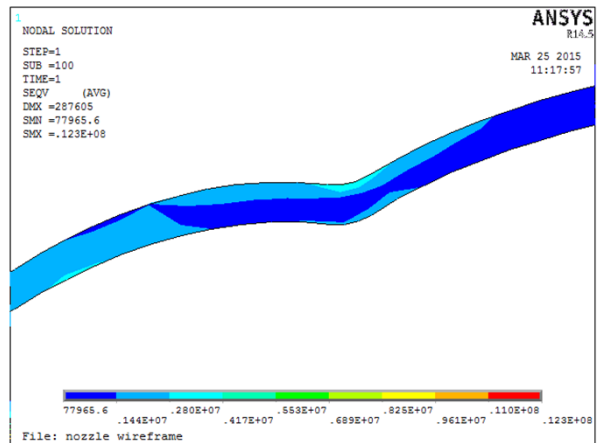


FIG 21 STRESS INTENSITY AT INFLECTION POINT

VII. CONCLUSION

A nozzle is a device designed to control the direction or characteristics of a fluid flow (especially to increase velocity) as it exits (or enters) an enclosed chamber or pipe.

Here we considered Dual-Bell nozzle due to its enormous advantages. The dual bell is a nozzle concept for altitude adaption. The flow separates at the contour inflection in certain altitude, mode in a mainly controlled and symmetrical way, reducing the side load generation and increasing the

thrust. The transition to altitude mode is reached when the flow suddenly attaches to the extension for an improved altitude thrust.

Both the thermal and structural analyses are done by applying the loads one after the other. The model is first designed using the design software CATIA. Due to some exceptions in the modeling data, the model was modified a little bit and was saved as an IGES file, the file was saved in such way so as to import the model created in CATIA in to ANSYS for the purpose of analysis.

Now the model imported from the CATIA as an IGES file is given element types and is meshed in the preprocessor and then the problem is defined. After the modeling is completed the complete problem is solved in processor and we can get the plotted and list of results at each node in the general post processor.

First the thermal analyses is done by giving the material properties thermal conductivity, specific heat, density and are written in the thermal physics file. The boundary conditions that are applied on the model are the values given in the appendix. The boundary conditions for the thermal model are the heat transfer coefficients at various axial locations along with the local gas temperature.

Now the structural analysis is done by giving the material properties Young's modulus, expansion coefficients are written in the structural physics file. The boundary conditions that are applied on the model are the values given in the appendix. The results of various stresses can be in the post processor.

VIII. FUTURE SCOPE

The dual bell nozzle has 90% overall better performance than the conventional bell-shaped nozzle. The efficiency at low altitudes is much higher because the atmospheric pressure restricts the expansion of the exhaust gas. A vehicle using a dual bell nozzle also saves 25-30% more fuel at low altitudes. At high altitudes, the dual bell nozzle is able to expand the engine exhaust to a larger effective nozzle area ratio. Hence the dual bell design is suitable for Single Stage to Orbit (SSTO) flight with reduced orbit cost. It has higher thrust efficiency and thus a higher average specific impulse. Hence engines using dual bell nozzle can give higher performance with lower complexities.

CALCULATIONS

EXIT MACH NUMBER

$$\frac{A_e}{A^*} = \left(\frac{\gamma + 1}{2}\right)^{-\frac{(\gamma + 1)}{2(\gamma - 1)}} \frac{\left(1 + \frac{\gamma - 1}{2} M_e^2\right)^{\frac{(\gamma + 1)}{2(\gamma - 1)}}}{M_e}$$

$$\frac{200^2}{50^2} = (1.2)^{-0.3} \frac{(1 + 0.2 M_e^2)^{0.3}}{M_e}$$

$$43.904 = \frac{(1 + 0.2 M_e^2)^{0.3}}{M_e}$$

Thus by solving the above equation, we get,
 $M_e = 5.021$

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