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Energy Conversation Law in the free Atmosphere for Compressible Flows

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***Abstract*— This paper is presented energy conversation law in the free atmosphere for compressible flows given by the Cauchy problem for the three dimensional Navier-Stokes equations with Holder continuous coefficients in the Cartesian coordinate system. Using the parametrix method of theory nonlinear partial differential equations were obtained the velocity components of the transient compressible flow which provided a description of the constitutive relationships between three physical quantities: the velocity vector, the external and internal forces, the pressure distribution.**

***Index Terms*— Navier-Stokes equations, compressible flow, parametrix method, turbulent motion, potential field, pressure distribution**

I. INTRODUCTION

Turbulence is flow characterized by recirculation and eddies. Mathematically, turbulent flow is represented via a Reynolds decomposition, in which the flow is broken down into the sum of an average component. Turbulent flows can be described well through the use of the Navier-Stokes equations. The conservation laws are fundamental laws of mechanics which are particularly for turbulent motion. We have focused on the global existence, uniqueness and smoothness of the Navier-Stokes problem for compressible flows. Examples of weak solution for incompressible flows were given by L. Caffarelli [1], V.Sheffer [2]. A critical analysis for many analytic and numerical solutions of Navier-Stokes equations was given by C.L.Fefferman [3]. We will extent this unique idea of existence of solution given in [3] by using the energy conservation law for the external and internal forces, the gradient of pressure. This paper has focused on the parametrix method with Holder continuous coefficients of the kinematic and dynamic viscosity. This result has shown the next step to mathematical understanding of the elusive compressible phenomena of turbulence .There we have got some additional fundamental information about behavior of potential, kinetic and static energies for the description their general mechanics of the turbulent compressible motion. In the present work we derive a number of results concerned with the mathematical theory of the Navier-Stokes equations for compressible fluids. We shall deal with the following problems: on the one hand, a description of the known results on the existence in the non-linear and time-dependent cases; on the other hand, the uniqueness and the regularity of solutions for the three dimensional Navier-Stokes equations with Holder continuous coefficients.

II. MATHEMATICAL FORMULATION OF THE PROBLEM

Turbulence model is considered in the three-dimensional infinite space R^3. Denote by $x=(x_1,x_2,x_3)$ a point in R^3, let us consider the velocity vector in a point $M(x,t)$ of the three spatial coordinates at a given time t by the formula

$$\vec{u}(x,t)=\begin{cases}\vec{u}_\nu(x,t) & if\ div\vec{u}(x,t)=0\\ \vec{u}_\alpha(x,t) & if\ div\vec{u}(x,t)\neq 0\end{cases}$$

Suppose that infinite spaces $\Omega=R^3$, $\Omega_T=R^3\times(0<t<\infty)$,

$$\vec{u}(x,t)=u_1(x,t)\vec{i}+u_2(x,t)\vec{j}+u_2(x,t)\vec{k}$$

is the velocity vector, $p(x,t)$ is the fluid pressure field.

Let us consider a function $\nu(x,t)$ defined on a bounded closed set S of R^3. The function $\nu(x,t)$ is said to be Holder continuous of exponent $\alpha(0<\alpha<1)$ in S if there exists a const A such that

$$|\nu(x,t)-\nu(\xi,\tau)|\le A(|x-\xi|^\alpha+|t-\tau|^{\alpha/2})$$

for all $(x,t)\in\Omega_T,(\xi,\tau)\in\Omega_T$ in S.

We consider compressible flow given by the general Navier -Stokes problem in the following form

$$\frac{\partial\vec{u}}{\partial t}+(\vec{u}\cdot\nabla)\vec{u}=-\frac{1}{\rho}\nabla p+\nu(x,t)\Delta\vec{u}+c\ (x,t)\nabla div\vec{u}+\vec{f}(x,t)\quad(1)$$

in Ω_T with the initial condition

$$\vec{u}\ |_{t=0}=\vec{u}_0(x)\ \text{ on }\ \Omega \qquad (2)$$

here,

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$$c(x,t)=\frac{4v(x,t)}{3}+\eta(x,t),$$

The vector function

$$\vec{f}(x,t)=f_1(x,t)\vec{i}+f_2(x,t)\vec{j}+f_2(x,t)\vec{k}$$

denotes an external and internal forces, $\nu(x,t)$ is a kinematic viscosity, $\rho(x,t)$ is a fluid density, the symbol ∇ denotes the gradient with respect to the function, the symbol Δ denotes the three dimensional Laplace operator, $\eta(x,t)$ is a dynamic viscosity which is related to the kinematic viscosity by the formula $\eta=\rho\nu$.

There we assume that

$$|\vec{u}|=\sqrt{u_1^2+u_2^2+u_3^2}\to 0 \quad for\ |x|=\sqrt{x_1^2+x_2^2+x_3^2}\quad\to\infty$$

The aim here is to understand turbulent features of the Navier-Stokes problem for compressible flows. The initial value problem (1)-(2) which concerns with the fundamental solution of Poisson and heat conduction equations. Turbulent motion is supported by the subjected power from some external forces and initial velocity. The shape of turbulent region is determined by the property which has shown stability or instability of the velocity motion and the pressure distribution. Stabilizing mechanisms have been advocated to explain features observed in numerical simulations of turbulence.

III. STABLE SOLUTION OF THE NAVIER-STOKES PROBLEM IN THE POTENTIAL FIELD

In this part we consider fluid when characterized by the three-dimensional Navier-Stokes problem with Holder continuous coefficient $\nu(x,t)$ in the following system of equations:

$$\frac{\partial\vec{u}}{\partial t}+(\vec{u}\cdot\nabla)\vec{u}=-\frac{1}{\rho}\nabla p+\nu(x,t)\Delta\vec{u}+\vec{f}(x,t)\ \text{in}\ \Omega_T \qquad (3)$$

$$div\ \vec{u}=0 \quad \text{in}\ \Omega_T \qquad (4)$$

With an initial condition

$$\vec{u}\ \big|_{t=0}=\vec{u}_0(x)\ \text{on}\ \Omega \qquad (5)$$

Using well-known formula of vector analysis

$$\frac{1}{2}grad\ \vec{u}^2=[\vec{u}\times rot\vec{u}]+(\vec{u}\nabla)\vec{u} \qquad (6)$$

we have got

$$\frac{\partial\vec{u}}{\partial t}+grad(\frac{p}{\rho}+\frac{1}{2}\vec{u}^2)=[\vec{u}\times rot\vec{u}]+\nu(x,t)\Delta\vec{u}+\vec{f}(x,t) \qquad (7)$$

Applying the expression

$$grad\ (\frac{u^2}{2}+\frac{p}{\rho})+\vec{f}=0$$

to the Navier-Stokes equations (3)-(5) we obtain the mathematical problem for heat equation

$$\frac{\partial\vec{u}}{\partial t}-\nu(x,t)\Delta\vec{u}=2\vec{f} \qquad (8)$$

$$div\ \vec{u}=0 \qquad (9)$$

With the initial condition

$$\vec{u}\ \big|_{t=0}=\vec{u}_0(x) \qquad (10)$$

Consider the heat equation (5) where the coefficient $\nu(x,t)$ is defined in a cylinder $\overline{D}\times[0,T]=\{(x,t);x\in\overline{D},0\le t\le T\}$, $\overline{D}$ is the closure of a bounded domain $D\in R^3$. Let for all $(x,t)\in\overline{D}\times[0,T]$, $(\xi,\tau)\in\overline{D}\times[0,T]$ coefficient satisfies Holder conditions

$$|\nu(x,t)-\nu(\xi,\tau)|\le|x-\xi|^{\alpha}+|t-\tau|^{\alpha/2}$$

Following the classical procedure [6] we can get solutions for the problem (8)-(10) in the integral sum of the parabolic potentials

$$\vec{u}(x,t)=\int_{R^3}\vec{u}_0(\xi)G(x-\xi,t)d\xi+ \qquad (11)$$

$$+2\int_0^t d\tau\int_{R^3}\vec{f}(\xi,\tau)G(x-\xi,t-\tau)d\xi$$

Fundamental solution $G\ (x-\xi,t)$ was obtained from integral equation (12)

$$G\ (x-\xi,t)=\frac{e^{-\frac{(x-\xi)^2}{4\nu t}}}{\left(2\sqrt{\pi\nu t}\right)^3}+$$

$$+\int_0^T\int_{R^3}g(x-\eta;t-\sigma)\Phi_\nu(\eta,-\xi,\sigma)d\eta d\sigma$$

Using the equation (8) was found unknown function $\Phi_\nu(x,t;\xi,\tau)$ as solution of the following Volterra-Fredholm integral equation

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$$\Phi_\nu(x,t;\xi,\tau) = LZ_\nu(x,t;\xi,\tau) + \int_{)}^{T}\int_{R^3} LZ_\nu(x,t;\xi,\tau)\Phi_\nu(x,t;\xi,\tau)d\xi d\tau$$

$$LZ_\nu(x,t;\xi,\tau) = [\nu(x,t) - \nu(\xi,\tau)]\Delta_x Z_\nu(x,t;\xi,\tau)$$

by using successive approximation method

$$\Phi_\nu(x,t;\xi,\tau) = \sum_{n=1}^{\infty}(LZ_\nu)_n(x,t;\xi,\tau)$$

Where

$$(LZ_\nu)_{n+1}(x,t;\xi,\tau) = \int_{)}^{T}\int_{R^3}(LZ_\nu(x,t;\xi_1,\tau_1))(LZ_\nu)_n(\xi,\tau;\xi_1,\tau_1)d\xi_1 d\tau_1$$

Fundamental solution $G(x,\xi,t)$ has estimations for their derivations

$$\left|\frac{\partial}{\partial x_i}G(x-\xi,t)\right| \le \frac{e^{-\frac{(x-\xi)^2}{8\nu t}}}{(\sqrt{\pi})^3\nu^2 t^2}$$

$$(\ i = 1,2,3)$$

Using properties of the fundamental solution $G(x,\xi,t)$ and its derivative evaluations we have got a uniqueness and stable solution (11) satisfying following estimation

$$\|\vec{u}\|_{H^{(2,1)}_{\Omega_T}} \le M_0(\|\vec{u}_0\|_{H^{(1)}_{\Omega}} + 2t\|\vec{f}\|_{L_2})$$

Condition for the scalar pressure function $p(x,t)$

$$\frac{P}{\rho} + \frac{u^2}{2} - div\vec{f} * \frac{1}{4\pi|x-\xi|} = 0 \qquad (13)$$

Predicts a steady feature which introduces a stable turbulent motion when

$$rot\ \vec{f} = 0,\ rot\vec{u}_0 = 0$$

For pressure function $p(x,t)$ we have got the following estimation

$$\|p\|_{H^{(1,0)}_{\Omega_T}} \le M_0(\|\vec{u}_0\|_{H^{(1)}_{\Omega}} + \|\vec{f}\|_{L_2})$$

Consequently, we see that a stability of the turbulent flow depends on the condition (13).

IV. VELOCITY COMPONENTS AND FUNCTION OF PRESSURE FOR TURBULENT SWIRLING MOTION

Assume that

$$grad\ (\frac{u^2}{2} + \frac{p}{\rho} - \Phi) \neq 0, \qquad (14)$$

then the Navier-Stokes equation (3) can be written as :

$$\frac{\partial\vec{u}}{\partial t} - \nu\Delta\vec{u} - [\vec{u}\times rot\ \vec{u}] + grad\ (\frac{u^2}{2} + \frac{p}{\rho} - \Phi) = \vec{f}^* + 2f \quad (15)$$

There vector function $\vec{f}^*$ is a convolution between vector and matrix

$$\vec{f}^8 = \begin{pmatrix} -\frac{\partial^2 f_1}{\partial x_2^2} - \frac{\partial^2 f_1}{\partial x_3^2} + \frac{\partial^2 f_2}{\partial x_1\partial x_2} + \frac{\partial^2 f_3}{\partial x_1\partial x_3} \\ \frac{\partial^2 f_1}{\partial x_1\partial x_2} - \frac{\partial^2 f_2}{\partial x_1^2} - \frac{\partial^2 f_2}{\partial x_3^2} + \frac{\partial^2 f_3}{\partial x_2\partial x_3} \\ \frac{\partial^2 f_1}{\partial x_1\partial x_3} + \frac{\partial^2 f_3}{\partial x_1\partial x_3} - \frac{\partial^2 f_3}{\partial x_1^2} - \frac{\partial^2 f_3}{\partial x_2^2} \end{pmatrix} * \begin{pmatrix} \frac{1}{4\pi|x-\xi|} & 0 & 0 \\ 0 & \frac{1}{4\pi|x-\xi|} & 0 \\ 0 & 0 & \frac{1}{4\pi|x-\xi|} \end{pmatrix}$$

Considering condition $rot\ \vec{f} \neq 0$ and using rotor operator we obtain equation

$$rot\left[\frac{\partial}{\partial t}\ \vec{u} - [\vec{u}\ \times rot\ \vec{u}] - \nu\Delta\vec{u}\right] = rot\ \vec{f}^* \qquad (15)$$

Donate that

$$\vec{g} = \frac{\partial}{\partial t}\ \vec{u} - [\vec{u}\ \times rot\ \vec{u}] - \nu\Delta\vec{u} \qquad (16)$$

$$\vec{z} = rot\ \vec{f}^*$$

With respect to (16) we have got vector equation

$$rot\ \vec{g} = \vec{z} \qquad (17)$$

Using representation

$$\tilde{Z}_0(f_1, f_2, f_3) = \begin{cases} \tilde{z}_1 = is_2\tilde{f}_3^* - is_3\tilde{f}_2^* \\ \tilde{z}_2 = is_3\tilde{f}_1^* - is_1\tilde{f}_3^* \\ 0 \end{cases}$$

we obtain following vector equation

$$\frac{\partial\vec{u}}{\partial t} - \nu\Delta\vec{u} - [\vec{u}\times rot\ \vec{u}] = \vec{b}(x,t)$$

Where vector

$$\vec{b}(x,t) = \frac{1}{4\pi}\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \qquad (18)$$

has components

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$$b_1 = (\frac{\partial^3}{\partial x_3 \partial^2 x_2} + \frac{\partial^3}{\partial^3 x_3}) \int_{R^3} \frac{1}{|x-\xi|} \int_{-\infty}^{\infty} \theta(\xi_3 - \varsigma_3) f_1^*(\xi_1, \xi_2, \varsigma_3, t) d\varsigma_3 d\xi -$$

$$-(\frac{\partial^3}{\partial x_3 \partial x_2 \partial x_1} + \frac{\partial^3}{\partial x_1 \partial^2 x_2} + \frac{\partial^3}{\partial x_1 \partial^2 x_3}) \int_{R^3} \frac{1}{|x-\xi|} \int_{-\infty}^{\infty} \theta(\xi_3 - \varsigma_3) f_2^*(\xi_1, \xi_2, \varsigma_3, t) d\varsigma_3 d\xi +$$

$$+ \frac{\partial^3}{\partial x_3 \partial x_2 \partial x_1} \int_{R^3} \frac{1}{|x-\xi|} \int_{-\infty}^{\infty} \theta(\xi_3 - \varsigma_3) f_3^*(\xi_1, \xi_2, \varsigma_3, t) d\varsigma_3 d\xi$$

$$b_2 = -\Big[\frac{\partial^3}{\partial x_3 \partial x_2 \partial x_1} \int_{R^3} \frac{1}{|x-\xi|} \int_{-\infty}^{\infty} \theta(\xi_3 - \varsigma_3) f_1^*(\xi_1, \xi_2, \varsigma_3, t) d\varsigma_3 d\xi +$$

$$+(\frac{\partial^3}{\partial x_3 \partial^2 x_1} + \frac{\partial^3}{\partial x_2 \partial^2 x_3} + \frac{\partial^3}{\partial x_2 \partial^2 x_1}) \int_{R^3} \frac{1}{|x-\xi|} \int_{-\infty}^{\infty} \theta(\xi_3 - \varsigma_3) f_2^*(\xi_1, \xi_2, \varsigma_3, t) d\varsigma_3 d\xi +$$

$$+(\frac{\partial^3}{\partial x_2 \partial^2 x_1} + \frac{\partial^3}{\partial x_3 \partial^2 x_2}) \int_{R^3} \frac{1}{|x-\xi|} \int_{-\infty}^{\infty} \theta(\xi_3 - \varsigma_3) f_3^*(\xi_1, \xi_2, \varsigma_3, t) d\varsigma_3 d\xi$$

$$b_3 = -\Big[\frac{\partial^3}{\partial x_3 \partial x_2 \partial x_1} \int_{R^3} \frac{1}{|x-\xi|} \int_{-\infty}^{\infty} \theta(\xi_3 - \varsigma_3) f_1^*(\xi_1, \xi_2, \varsigma_3, t) d\varsigma_3) d\xi -$$

$$-(\frac{\partial^3}{\partial x_2 \partial^2 x_3} - \frac{\partial^3}{\partial x_3 \partial^2 x_1}) \int_{R^3} \frac{1}{|x-\xi|} \int_{-\infty}^{\infty} \theta(\xi_3 - \varsigma_3) f_2^*(\xi_1, \xi_2, \varsigma_3, t) d\varsigma_3 d\xi -$$

$$- \frac{\partial^3}{\partial x_3 \partial^2 x_2} \int_{R^3} \frac{1}{|x-\xi|} \int_{-\infty}^{\infty} \theta(\xi_3 - \varsigma_3) f_3^*(\xi_1, \xi_2, \varsigma_3, t) d\varsigma_3 d\xi \Big]$$

Due to this fact we have the unique solution of the problem (3)-(5)

$$\vec{u} = \int_0^t d\tau \int_\Omega R\big[A(\vec{F}(\xi,\tau))\Gamma(x-\xi,t-\tau)\big] d\Omega + \vec{F}(x,t) \quad (19)$$

Where

$$A(F) = \begin{pmatrix} \frac{1}{2}(F_1^2+F_2^2+F_3^2) & -F_1F_2 & -F_1F_3 \\ -F_1F_2 & \frac{1}{2}(F_1^2+F_2^2+F_3^2) & -F_2F_3 \\ -F_1F_3 & -F_2F_3 & \frac{1}{2}(F_1^2+F_2^2+F_3^2) \end{pmatrix}$$

$$\vec{F} = \vec{b} * G + u_0 * G$$

$$\Gamma(x-\xi,t) = \begin{pmatrix} \frac{\partial G(x-\xi,t)}{\partial x_1} \\ \frac{\partial G(x-\xi,t)}{\partial x_2} \\ \frac{\partial G(x-\xi,t)}{\partial x_3} \end{pmatrix}$$

$$R[\cdot] = F(x,t) + \int_0^t d\tau \int_{R^3} A[F(\xi,\tau)]\Gamma(x-\xi,t-\tau)d\xi +$$

$$+ \int_0^t d\tau \int_{R^3} A\left[\int_0^\tau d\tau_1 \int_{R^3} A[F(\xi,\tau)]\Gamma(\xi-\zeta,\tau-\tau_1)d\zeta\right]\Gamma(x-\xi,t-\tau)d\xi +$$

$$+ \int_0^t d\tau \int_{R^3} A\left[\ldots\left[\int_0^\tau d\tau_1 \int_{R^3} A[F(\xi,\tau)]\Gamma(\xi-\zeta,\tau-\tau_1)d\zeta\right]\ldots\right]\Gamma(x-\xi,t-\tau)d\xi + \ldots$$

Vector function $\vec{F}(x,t)$ satisfies following estimation

$$\|\vec{F}(x,t)\| \le C(\|\vec{u}_0\| + t\|\vec{b}\|)$$

Using the well-known properties of Green's functions we have got estimation for the vector velocity

$$\|\vec{u}\|_{L_2}^2 \le (\|\vec{u}_0\|_{L_2}^2 + t\|\vec{b}\|_{L_2}^2)\Big[\ 1 +$$

$$+ M_0(\|\vec{u}_0\|_{L_2}^2 + t\|\vec{b}\|_{L_2}^2)e^{(\|\vec{u}_0\|_{L_2}^2 + t\|\vec{b}\|_{L_2}^2)^2}\Big] \quad (20)$$

in the functional space $L_2(R^3 \times [0,T])$ .Following the classical procedure we get the uniqueness and stability of solution for the problem (3)-(5). Also we obtain equation for the pressure function

$$\frac{u^2}{2} + \frac{p}{\rho} - div\vec{f} * \frac{1}{4\pi|x-\xi|} - div f^{**} * \frac{1}{4\pi|x-\xi|} = 0 \quad (21)$$

where

$$\vec{f}^{**} = \vec{b} - \vec{f}^*$$

$$\|p\|_{L_2(\Omega_T)} \le C_0(\|\vec{u}_0\|_{L_2(\Omega)} + t\|\vec{\Psi}\|_{H_{\Omega T}^{(1,0)}})\Big[\ 1 +$$

$$+C_1\sqrt{t}(\|\vec{u}_0\|_{L_2(\Omega)}+t\|\vec{\Psi}\|_{H^{(1,0)}_{\Omega_T}})e^{C_2t(\|\vec{u}_0\|_{L_2(\Omega_T)}+t\|\vec{\Phi}\|_{H^{(1,0)}_{\Omega_T}})^2}\Bigg]$$

$$\|\Psi\|_{L_2}=\sqrt{(\Psi_1)^2+(\Psi_2)^2+(\Psi_3)^2}$$

$$\vec{\Psi}(x,t)=\begin{pmatrix}\Psi_1\\ \Psi_2\\ \Psi_3\end{pmatrix}\equiv\begin{pmatrix}\int_{-\infty}^{\infty}\theta(x_1-\xi_1)f_1(\xi_1,x_2,x_3,t)d\xi_1\\ \int_{-\infty}^{\infty}\theta(x_2-\xi_2)f_2(x_1,\xi_2,x_3,t)d\xi_2\\ \int_{-\infty}^{\infty}\theta(x_3-\xi_3)f_3(x_1,x_2,\xi_3,t)d\xi_3\end{pmatrix}$$

The problem (1)-(2) when $div\vec{u}(x,t)=0$ is Clay Institute's Millennium problem which was formulated by Fefferman [3]. When $rot\ \vec{f}\neq 0$ is defined solution which has non-continuous Heaviside step function $\vec{\Psi}(x,t)$ therefore in general case can be determined only weak solution which satisfies the obtained balance equation (21). Due to this obtained balance equation for the pressure distribution were defined significant properties of the transient incompressible flow which provide a description of the constitutive relationships between three physical quantities: the velocity vector, the external and internal forces, the scalar pressure distribution. The Navier-Stockes problem (1)-(2) in the general when $div\vec{u}(x,t)\neq 0$ is applicable to real turbulent processes which represent an average departure from the different points of the space and we have different combinations of the conditions

$$div\vec{u}(x,t)<0,\quad div\vec{u}(x,t)>0,\quad div\vec{u}(x,t)=0$$

Combining these conditions for the vector velocity we can explain transfer mechanisms of divergent-convergent flows.

V. THE NAVIER-STOKES PROBLEM FOR THE DIVERGENT POTENTIAL FIELD

Let investigate the behavior of the Navier-Stockes problem (1)-(2) in the general case when $div\vec{u}(x,t)\neq 0$.

$$\frac{\partial\vec{u}}{\partial t}+(\vec{u}\cdot\nabla)\vec{u}=-\frac{1}{\rho}\nabla p+\nu(x,t)\Delta\vec{u}+\alpha(x,t)\nabla div\vec{u}+\vec{f}(x,t)\quad(1)$$

in Ω_T with the initial condition

$$\vec{u}\,|_{t=0}=\vec{u}_0(x)\quad\text{on}\quad\Omega\qquad(2)$$

For problem (1)-(2) in the general case we have got second order nonlinear Volterra-Fredholm integral equation in a matrix form satisfying

$$\begin{pmatrix}u_1\\u_2\\u_3\end{pmatrix}=\begin{pmatrix}\frac{(u_1-u_2-u_3)^2}{2}&0&0\\0&\frac{(u_2-u_1-u_3)^2}{2}&0\\0&0&\frac{(u_3-u_1-u_2)^2}{2}\end{pmatrix}*\begin{pmatrix}\frac{\partial G_\nu}{\partial x_1}\\ \frac{\partial G_\nu}{\partial x_2}\\ \frac{\partial G_\nu}{\partial x_3}\end{pmatrix}+$$

$$+\begin{pmatrix}u_1\cdot D^*&0&0\\0&u_2\cdot D^*&0\\0&0&u_3\cdot D^*\end{pmatrix}*\begin{pmatrix}G_\nu\\G_\nu\\G_\nu\end{pmatrix}+\begin{pmatrix}F_1^*\\F_2^*\\F_3^*\end{pmatrix}\quad(22)$$

(47)

Where

$$\vec{F}^*=\vec{b}^**G_\nu+u_0*G_\nu$$

Successive approximation method can be successfully applied to solve the nonlinear Volterra-Fredholm matrix integral equation (22). Considering (22) in the operator form we have equation

$$\vec{u}=K_2[\vec{u}]+K_1[\vec{u}]+\vec{F}\qquad(23)$$

Here

$$K_2[\vec{u}]=\begin{pmatrix}\frac{(u_1-u_2-u_3)^2}{2}&0&0\\0&\frac{(u_2-u_1-u_3)^2}{2}&0\\0&0&\frac{(u_3-u_1-u_2)^2}{2}\end{pmatrix}*\begin{pmatrix}\frac{\partial G_\nu}{\partial x_1}\\ \frac{\partial G_\nu}{\partial x_2}\\ \frac{\partial G_\nu}{\partial x_3}\end{pmatrix}$$

$$K_1[\vec{u}]=\begin{pmatrix}u_1\cdot D^*&0&0\\0&u_2\cdot D^*&0\\0&0&u_3\cdot D^*\end{pmatrix}*\begin{pmatrix}G_\nu\\G_\nu\\G_\nu\end{pmatrix}$$

Using inverse operators R_i for the operators K_i ($i=1,2$) we have got

$$\vec{u}=R_2(K_1[\vec{u}])+K_1[\vec{u}]+R_2(\vec{F})+\vec{F}$$

or

$$\vec{u}=R_1(R_2[R_2(\vec{F}^*)+\vec{F}^*])+R_1[R_2(\vec{F}^*)+\vec{F}^*]+R_2(\vec{F}^*)+\vec{F}^*$$

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$$\vec{F}^* = \vec{d} * G_\nu + \vec{b} * G_\nu + u_0 * G_\nu$$

Here

$$\vec{d} = (\frac{\nu}{3} + \eta) grad D^*(x,t)$$

$$\vec{u} = R_1(R_2[R_2(\vec{F}^*) + \vec{F}^*]) + R_1[R_2(\vec{F}^*) + \vec{F}^*] + R_2(\vec{F}^*) + \vec{F}^*$$

$$\vec{u}_\alpha(x,t) = \vec{u}_\alpha^{\,*}(x,t) + \vec{u}_\nu(x,t) \qquad (24)$$

$$\vec{u}_\alpha^{\,*}(x,t) = R_1[R_2[R_2(\vec{F}^*) + 2\vec{F}^*]] + R_1[\vec{F}^*] + R_2(\vec{d} * G_\nu) + \vec{d} * G_\nu$$

Vector function $\vec{F}^*(x,t)$ satisfies the following estimation

$$\|\vec{F}^*(x,t)\| \le C(\|\vec{u}_0\| + \sqrt{t}\|\Psi\|)$$

Using the properties of Green's functions we have got estimation for the vector velocity in the space $L_2(R^3 \times [0,T]$

$$\|\vec{u}_\alpha\|_{L_2} \le (c_0\|\vec{y}\| + c_1\|\vec{y}\|^2 e^{c_2 t\|\vec{y}\|^2})(1 + c^*\|\vec{y}\|^2 e^{c_4 t\|\vec{y}\|^2})$$

$$\|\vec{y}\| = \|\vec{u}_0\|^2_{L_2(\Omega)} + t\|\vec{\Psi}\|^2_{H^{(1,0)}(\Omega_T)}$$

$$\|\vec{u}_\alpha\|_{H^{(2,1)}(\Omega_T)} \le (C_0\|\vec{y}^*\| + C_1\|\vec{y}^*\|^2 e^{C_2 t\|\vec{y}^*\|^2})(1 + C^*\|\vec{y}^*\|^2 e^{C_3 t\|\vec{y}^*\|^2})$$

$$\|p\|_{H^{(1,0)}(\Omega_T)} \le (C_0\|\vec{y}^*\| + C_1\|\vec{y}^*\|^2 e^{C_2 t\|\vec{y}^*\|^2})(1 + C^*\|\vec{y}^*\|^2 e^{C_3 t\|\vec{y}^*\|^2})$$

Where

$$\|\vec{y}^*\| = \|\vec{u}_0\|^2_{H^{(1,0)}(\Omega)} + \|\vec{\Psi}\|^2_{H^{(2,1)}(\Omega_T)}$$

Using the Navier-Stokes equation (1) we have a unique scalar function of pressure $p(x,t)$ which satisfies

$$\frac{u^2}{2} + \frac{p}{\rho} - div\vec{f} * \frac{1}{4\pi|x-\xi|} - div\vec{f}^{**} * \frac{1}{4\pi|x-\xi|} = 0 \quad (21)$$

where

$$\vec{f}^{**} = \vec{b}^* - \vec{f}^*$$

VI. RESULTS AND DISCUSSION

From previous studies [5]-[6] in cases $div\vec{u}(x,t) = 0$ and $div\vec{u}(x,t) \neq 0$ for unstable motions we have the nonlinear Folterra -Fredholm matrix equation which give some simple consequences of the theorems given in [6]. This result are shown analogical conditions of existence and uniqueness for the three dimensional Navier-Stokes equations with Holder continuous coefficients in the Cartesian coordinate systems which depend on the energy conservation law and provide behavior of motion for compressible flows. There Navier-Stokes equations in the general case represent the evolution of the governing distribution functions, which depend on the velocity vector in the position of particles as a result of thermal excitation at any finite turbulent energy.

VII. CONCLUSION

This paper is presented convenient procedure in solving the Navier-Stokes problem for compressible flows which allows to prove the uniqueness and regularity of the solutions in the general case. In the considered case when weak solution of the Navier-Stokes problem (1)- (2) satisfies the energy conversation law (13) we have sufficiently regular solution which means $\vec{u}(x,t) \in C^{(2,1)}(\Omega_T) \cap C^{(1,0)}(\overline{\Omega}_T)$ and satisfies classical formulation in this considered case. In view of this fact, the obtained balance relation between components of the velocity vector and the pressure function in terms on the energy conversation law (13) provides equivalence of the strong and weak solution. In case when $rot\ \vec{f} \neq 0$ the Navier-Stokes problem (1)- (2) for compressible flows satisfies weak formulation. Conversely, unsteady behavior represents a departure from the average energy of the fluid known as eddy energy. involving fundamental properties of the turbulent flows which demonstrate technology and principal importance at the forefront of classical approach where the expression

$$div\vec{f}^{**} * \frac{1}{4\pi|x-\xi|}$$

of the turbulent fluid energy is not regular and does not allow to use classical formulation. In this way some difficulties arise in solving of the Navier-Stokes problem which was encountered in studying turbulent behavior for unstable motion by using weak formulation. The similarity of the balance equations (13) and (21) for the external force and the pressure distribution between the turbulent divergent and non divergent flows demonstrate significant and principal importance of the energy conservation law for the compressible turbulent flows.

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