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Numerical Behavior of Fingering Phenomenon in a Homogeneous Medium Involving Magnetic Fluid

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Abstract: - In this paper, we have numerically discussed the phenomenon of instabilities in a displacement process involving two immiscible liquids. The phenomenon is considered with the magnetic fluid. Numerical solution of governing non-linear partial differential equation for the phenomenon has been obtained by Finite element techniques. Finite element technique is a numerical method for finding an approximate solution of differential equation in finite region or domain.

Keywords: **Homogeneous Porous Media, Fluid flow, Finite element technique, Magnetic fluid.**

I. INTRODUCTION

If a fluid contained in a porous medium is displaced by another fluid of lesser viscosity, then it is frequently observed that the displacing fluid has a strong tendency to protrude in form of fingers (instabilities) into more viscous fluid. This phenomenon is called fingering.

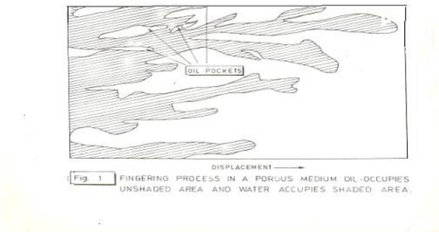

Fig 1

In Figure-1, fingering process has been shown between oil-water flows into a porous medium. In petroleum engineering, the fingering process is a well known phenomenon occurring in displacement of oil by water flooding that is a common oil recovery technique. In the statistical treatment of fingering [1] only average cross-sectional area occupied by the fingers was observed while the size and shape of the individual fingers are neglected as in figure 2. Scheidegger and Johnson [21] discussed the statistical behavior in homogeneous porous media with capillary pressure. Verma [22] has examined the behavior of fingering in a displacement process through heterogeneous porous media.

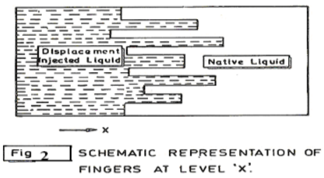

Fig 2

In the present paper, we have obtained a numerical solution of the problem by finite element techniques using Matlab.

II. STATEMENT OF THE PROBLEM

We consider that there is a uniform water injection into an oil saturated porous medium of homogeneous physical characteristics, such that the injecting water cuts through the oil formation and give rise to protuberance. This furnishes a well-developed fingers flow. Since the entire oil at the initial boundary (x=0) is displaced through a small distance due to the water injection. Therefore, we assume, further that complete water saturation exists at the initial boundary.

III. MATHEMATICAL FORMULATION OF THE PROBLEM

Assuming that the flow of two immiscible phases is governed by Darcy's law, We may write the seepage velocity of injected and native fluid as,

$$V_w = -\left(\frac{K_w}{\delta_w}\right) K \left[\frac{\partial P_w}{\partial x}\right] \quad \ldots\ldots\ldots\ldots..(1) \qquad \text{and}$$

$$V_o = -\left(\frac{K_o}{\delta_o}\right) K \left[\frac{\partial P_o}{\partial x}\right] \quad \ldots\ldots\ldots\ldots...(2)$$

where K is the permeability of the homogeneous medium, K_w and K_o are relative permeability of injective and native fluids, which are functions of S_w and S_o (S_w and S_o are the saturation of injected and native fluids) respectively, P_w and P_o are pressure of injected and native fluids, δ_w and δ_o are constant kinematics viscosities, α is the inclination of the bed and g is acceleration due to gravity. If we take the injected liquid containing a thin layer of magnetic fluid where magnetization M is assumed to the directly proportional to the magnetic field intensity H and the microscopic behavior of fingers is governed by a statistical treatment. Then the additional pressure exerted due to presence of a layer of magnetic fluid in the displacing liquid (w) represented by $\left[\frac{\mu_o M + 16\tilde{\lambda}\mu_o M \lambda r^3}{9(l+2)^3}\right]\frac{\partial H}{\partial x}$. Therefore, equation (1), equation of filtration velocities of injected liquid (w), becomes,

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$$V_w=-\left(\frac{K_w}{\delta_w}\right)K\left[\frac{\partial P_w}{\partial x}+\left[\frac{\mu_0 M+16\bar{\lambda}\mu_0 M\lambda r^3}{9(l+2)^3}\right]\frac{\partial H}{\partial x}\right] \quad \text{.........} \; (3)$$

Whereas equation of velocities of native liquid (2) remain same regarding the phase densities to be independent of magnetic field H, and to be constant, the equations of continuity of the two phases are:

$$P\left(\frac{\partial S_w}{\partial t}\right)+\left(\frac{\partial V_w}{\partial x}\right)=0 \quad \text{…………….} \; (4)$$

$$P\left(\frac{\partial S_o}{\partial t}\right)+\left(\frac{\partial V_o}{\partial x}\right)=0 \quad \text{…………….} \; (5)$$

Where, P is porosity of the medium. From the definition of phase saturation, it is evident that,
$S_w + S_o = 1$ …..………… (6)
The capillary pressure P_c is defined as,
$P_c=-\beta_o g(S_w)$ ……………. (7)
$P_c = P_o - P_w$ …………….. (8)
Where, β_o is a constant quantity.
At this state, for definiteness of the mathematical analysis, we assume standard relationship due to Scheidegger and Johnson [21], between phase saturation and relative permeability as
$K_w = S_w$ ……….…. (9)
$K_o=S_0= 1 -S_w$ …………. (10)
The equation of motion for saturation can be obtained by substituting the values of V_w and V_o from equations (1) and (2) into the equations (4) and (5) respectively, we get,

$$P\left(\frac{\partial S_w}{\partial t}\right)+\frac{\partial}{\partial x}\left(\left(\frac{K_w}{\delta_w}\right)K\left[\frac{\partial P_w}{\partial x}+\left[\frac{\mu_0 M+16\bar{\lambda}\mu_0 M\lambda r^3}{9(l+2)^3}\right]\frac{\partial H}{\partial x}\right]\right)=0$$

……………. (11)

$$P\left(\frac{\partial S_o}{\partial t}\right)=\frac{\partial}{\partial x}\left[\left(\frac{K_o}{\delta_o}\right)K\left[\frac{\partial P_o}{\partial x}\right]\right] \quad \text{……….} \; (12)$$

Eliminating the time derivative by combining equation (11) and equation (12), using equation (6) and (8). Integrating the resultant equation, then defining the mean pressure, and

$P_o=\frac{P_o+P_w}{2}+\frac{P_o-P_w}{2}=\bar{P}+\frac{1}{2}P_C$ and considering it as constant on the finger tips, we get on performing some simplifications

$$P\left(\frac{\partial S_w}{\partial t}\right)+\frac{\partial}{\partial x}\left(\left(\frac{K_w}{\delta_w}\right)K\left[\frac{1}{2}\left(\frac{dP_C}{dS_w}\right)\left(\frac{\partial S_w}{\partial x}\right)+\frac{\mu_0 M}{}\frac{\partial H}{\partial x}+\frac{16\bar{\lambda}\mu_0 M\lambda r^3}{9(l+2)^3}\frac{\partial H}{\partial x}\right]\right)=0 \quad \text{….} \; (13)$$

Substituting the values of equation (7), (9) & (10) in equation (13) and assuming M = λH, we obtain

$$P\left(\frac{\partial S_w}{\partial t}\right)+\left[\frac{K}{\delta_w}\frac{-\beta_0}{2}\left\{g'(S_w)\left(\frac{\partial S_w}{\partial x}\right)^2+S_w g''(S_w)\left(\frac{\partial S_w}{\partial x}\right)^2+S_w g'(S_w)\frac{\partial^2 S_w}{\partial x^2}\right\}\right.$$
$$\left.+\left(\mu_0\lambda+\frac{16\bar{\lambda}r^3\mu_0\lambda^2}{9(l+2)^3}\right)\left(\frac{\partial S_w}{\partial x}H\frac{\partial H}{\partial x}+S_w\left(\frac{\partial H}{\partial x}\right)^2+\frac{S_w H\partial^2 H}{\partial x^2}\right)\right]=0$$

………….. (14)

This is the desired non linear differential equation of motion for the flow of two immiscible liquids in homogeneous porous medium with effect of magnetic fluid.

Considering the magnetic fluid H in the x-direction only, we may write [24], $H=\frac{\Lambda}{x^n}$ where Λ is a constant parameter and n is an integer. Assuming $g(S_w) = S_w$ and using the value of H for n = -1 in eq. (14), we get,

$$P\left(\frac{\partial S_w}{\partial t}\right)+\frac{-K\beta_0}{2\delta_w}\frac{\partial}{\partial x}\left(S_w\frac{\partial S_w}{\partial x}\right)+\frac{K}{\delta_w}\left(\mu_0\lambda\Lambda^2+\frac{16\bar{\lambda}r^3\mu_0\lambda^2\Lambda^2}{9(l+2)^3}\right)\frac{\partial}{\partial x}(S_w x)=0 \quad \text{….} \; (15)$$

A set of suitable boundary conditions associated to problem (15) are
$S_w(x,0) = 1 \;\; 0 \le x \le L_1$;
$=0; L_1<x\le L$ …….. (16)
$S_w(0,t)=1; S_w(L,t)=0; t\ge 0$ …….. (17)
Equation (15) is reduced to dimensionless form by setting

$$x^*=x/L\,,\quad t^*=\frac{K\beta_o t}{2\delta_w L^2 P},\quad S_w(x,t)=S_w^*(x^*,t^*)$$

So that $\frac{\partial S_w}{\partial t}=\frac{\partial}{\partial x}\left(S_w\frac{\partial S_w}{\partial x}\right)-C_o\frac{\partial}{\partial x}(S_w x)$ ………. (18)

Where, $C_o=\frac{2L^2P}{\beta_o}\left(\mu_o\lambda\Lambda^2+\frac{16\bar{\lambda}r^3\mu_o\lambda^2\Lambda^2}{9(l+2)^3}\right)$

Asterisks are dropped for simplicity.
The initial and boundary conditions (16) & (17) now becomes,
$S_w(x,0) = 1; \; 0 \le x \le L_1$;
$=0; L_1<x\le 1$ ………… (19)
$S_w(0,t)=1; S_w(1,t)=0; t\ge 0$ ...……… (20)
Equation (18) is desired nonlinear differential equation of motion for the flow of two immiscible liquids in homogeneous medium wit effect of magnetic fluid.

A Matlab Code is prepared and executed with $C_o = 1.22694 \times 10^{-11}$, h =1/15 , k = 0.002223 for 225 time levels and $L_1 = 0.5$. The numerical values are shown by table. Curves indicating the behavior of Saturation of injected fluid with respect to various time periods.

IV. FINITE ELEMENT METHOD

We attempt to solve the time dependent one-dimensional problem (18) by the application of finite element technique. We discuss the details of semi-discrete variation formulation of the problem. The domain of the problem, in present case, consists of all points between $x=0$ and $x=1$ (Figure 3(a)). This domain is divided in to set of linear elements (Figure 3(b)).

Fig 3(a)

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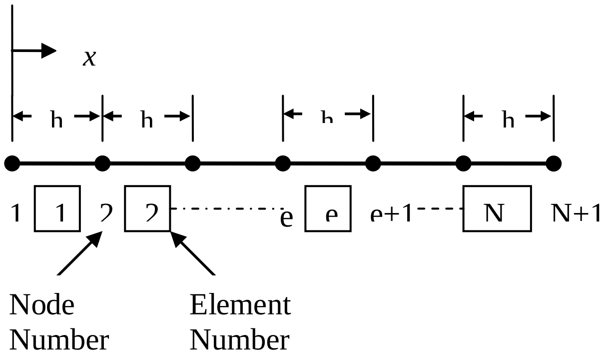

Fig 3(b)

Now, the variational form of given partial differential equation (18) is,

$$J(S_w)=\frac{1}{2}\int_R\left[S_w\left(\frac{\partial S_w}{\partial x}\right)^2+2S_w\frac{\partial S_w}{\partial t}-C_o x S_w\frac{\partial S_w}{\partial x}\right]dx \quad \ldots\ldots(21)$$

Choose an arbitrary linear element $R^{(e)} = [S_1^{(e)}, \; S_2^{(e)}]$ & obtain interpolation function for $R^{(e)}$ using Lagrange interpolation Method such as

$$S^{(e)}(x) = \sum_{j=1}^{2} N_j(x)S_j^{(e)} = N^{(e)}\varphi^{(e)} = \varphi^{(e)T}N^{(e)T}$$

$$where \quad N^{(e)} = [N_1 \; N_2] \;\; \& \;\; \varphi^{(e)} = [S_1 \;\; S_2]^T$$

……………… (22) and N_1 & N_2 are shape functions.

Now, apply Variational Method to $R^{(e)}$,therefore equation (21) becomes,

$$J\left(S^{(e)}\right)=\frac{1}{2}\int_{R^{(e)}}\left[S^{(e)}\left(\frac{\partial S^{(e)}}{\partial x}\right)^2+2S^{(e)}\frac{\partial S^{(e)}}{\partial t}-C_o x S^{(e)}\frac{\partial S^{(e)}}{\partial x}\right]dx$$

………….. (23)

$$As, \quad S^{(e)}(x) = N^{(e)}\varphi^{(e)} = \varphi^{(e)T}N^{(e)T}$$

$$\therefore \quad \frac{\partial S^{(e)}}{\partial x} = \frac{\partial N^{(e)}}{\partial x}\varphi^{(e)} = \varphi^{(e)T}\frac{\partial N^{(e)T}}{\partial x}$$

$$and \left(\frac{\partial S^{(e)}}{\partial x}\right)^2 = \varphi^{(e)T}\frac{\partial N^{(e)T}}{\partial x}\frac{\partial N^{(e)}}{\partial x}\varphi^{(e)}$$

Therefore equation (23) becomes,

$$J\left(S^{(e)}\right)=\frac{1}{2}\int_{R^{(e)}}\varphi^{(e)T}\left[N^{(e)}\varphi^{(e)}\left(\frac{\partial N^{(e)T}}{\partial x}\frac{\partial N^{(e)}}{\partial x}\right)\varphi^{(e)}+2\left(N^{(e)T}N^{(e)}\right)\frac{\partial \varphi^{(e)}}{\partial t}-C_0 x N^{(e)T}\frac{\partial N^{(e)}}{\partial x}\varphi^{(e)}\right]dx$$

……………….. (24)

For minimization, first differentiate eq. (24) with respect to $\varphi^{(e)}$

$$\frac{\partial J^{(e)}}{\partial \varphi^{(e)}}=\int_{R^{(e)}}\left[N^{(e)}\varphi^{(e)}\left(\frac{\partial N^{(e)T}}{\partial x}\frac{\partial N^{(e)}}{\partial x}\right)\varphi^{(e)}+2\left(N^{(e)T}N^{(e)}\right)\frac{\partial \varphi^{(e)}}{\partial t}-C_0 x N^{(e)T}\frac{\partial N^{(e)}}{\partial x}\varphi^{(e)}\right]dx$$

Now, $\dfrac{\partial J^{(e)}}{\partial \varphi^{(e)}}=0$

Therefore the Element equation is

$$A^{(e)}\frac{\partial \varphi^{(e)}}{\partial T}+B^{(e)}(\varphi^{(e)})\varphi^{(e)}+C^{(e)}\varphi^{(e)}=0 \qquad \ldots\ldots.. (25)$$

Where

$$A^{(e)} = \int_{S_1^{(e)}}^{S_2^{(e)}}\left(N^{(e)T}N^{(e)}\right)dx,$$

$$B^{(e)}(\varphi^{(e)}) = \int_{S_1^{(e)}}^{S_2^{(e)}} N^{(e)}\varphi^{(e)}\left(\frac{\partial N^{(e)T}}{\partial x}\frac{\partial N^{(e)}}{\partial x}\right)dx$$

$$C^{(e)}=-C_o\int_{S_1^{(e)}}^{S_2^{(e)}} x\left(N^{(e)T}\frac{\partial N^{(e)}}{\partial x}\right)dx$$

By Gauss Legendre Quadrature Method, we evaluate these integral. Thus, element matrix transform to

$$A^{(e)} = \int_{-1}^{1}\left(N^{(e)T}N^{(e)}\right)Jdz \approx \sum_{I=1}^{r} A^{(e)}(z_I)W_I$$

$$B^{(e)}(\varphi^{(e)}) = \int_{-1}^{1} N^{(e)}\varphi^{(e)}\left(\frac{1}{J}\frac{\partial N^{(e)T}}{\partial X}\frac{1}{J}\frac{\partial N^{(e)}}{\partial X}\right)Jdz \approx \sum_{I=1}^{r} B^{(e)}(z_I)W_I$$

$$C^{(e)}=-C_0\int_{-1}^{1}\left(\frac{h}{2}(1+z_I)\right)\left(N^{(e)T}\frac{1}{J}\frac{\partial N^{(e)}}{\partial z}\right)Jdz\approx\sum_{I=1}^{r} C^{(e)}(z_I)W_I$$

For $A^{(e)}$ and $C^{(e)}$, degree of polynomial p =2 then r = 2 and For $B^{(e)}$, p = 1 then r =1. z_I and W_I are corresponding gauss points and gauss weights with respect to ‘r’. Then, the element matrix becomes,

$$A^{(e)} = \frac{h^{(e)}}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix},$$

$$B^{(e)}(\varphi^{(e)})=\frac{1}{2h^{(e)}}\begin{bmatrix}S_1+S_2 & -S_1-S_2\\ -S_1-S_2 & S_1+S_2\end{bmatrix}, \qquad \ldots\ldots\ldots\ldots.. (26)$$

$$C^{(e)} = \frac{-C_o h^{(e)}}{6}\begin{bmatrix}-1 & -2\\ 1 & 2\end{bmatrix}$$

IV.I. Assembling of elements

In deriving the element equations, we isolated a typical element (the eth element) from the mesh and formulated the variational problem and developed its finite element model. To obtain the finite element equations of the total problem, we must put the elements back into their original positions. The assembly of linear elements is carried out by imposing the following two conditions:

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(i)The continuity of primary variable requires $S_n^e = S_1^{e+1}$

(ii) The balance of secondary variables at connecting nodes requires

$$S_n^e + S_1^{e+1} = \begin{cases} 0 & \text{if no external point surce is applied} \\ S_1 & \text{if an external point surce of magnitude } S_1 \text{ is applied} \end{cases}$$

$$S_1^1 = S_1, \quad S_2^1 = S_1^2 = S_2, \quad S_2^2 = S_1^3 = S_3, \ldots\ldots,$$
$$S_2^{N-1} = S_1^N = S_N, \quad S_2^N = S_{N+1}$$

The inter-element continuity of primary variable can be imposed by simply renaming the variables of all elements connected to common node. For example, for a mesh of N linear finite element connected in series, we have For a uniform mesh of N elements, by equation (25) & equation (26), the assembled equation becomes,

$$A\frac{\partial\varphi}{\partial T} - B(\varphi)\varphi + C\varphi = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (27)$$

where,
$$A = \frac{h}{6}\begin{bmatrix} 2 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 1 & (2+2) & 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & (2+2) & 0 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & \cdots & (2+2) & 1 \\ 0 & 0 & 0 & 0 & \cdots & 1 & 2 \end{bmatrix},$$

$$B(\varphi) = \frac{1}{2h}\begin{bmatrix} S_1+S_2 & -S_1-S_2 & 0 & 0 & \cdots & 0 & 0 \\ -S_1-S_2 & S_1+2S_2+S_3 & -S_2-S_3 & 0 & \cdots & 0 & 0 \\ 0 & -S_2-S_3 & S_2+2S_3+S_4 & -S_3-S_4 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & \cdots & S_{14}+2S_{15}+S_{16} & -S_{15}-S_{16} \\ 0 & 0 & 0 & 0 & \cdots & -S_{15}-S_{16} & S_{15}+S_{16} \end{bmatrix}$$ and

$$C = \frac{-C_o h}{6}\begin{bmatrix} -1 & -2 & 0 & 0 & \cdots & 0 & 0 \\ 1 & 1 & -2 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 1 & 0 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & \cdots & 1 & -2 \\ 0 & 0 & 0 & 0 & \cdots & 1 & 2 \end{bmatrix}, \quad \varphi = [S_1 \quad S_2 \quad \cdots \quad \cdots \quad S_{16}]^T \qquad (28)$$

Equation (27) represents the assembled equation.

IV.II. Time approximation

We have obtained the finite element equation in the global form, which represent a system of simultaneous ordinary differential equation. We now introduced δ family of approximations which approximates weighted average of a dependent variable of two consecutive time steps by linear interpolation of the values of the variable at the two time steps such as,

$S_j = \delta S_j^{n+1} + (1-\delta)S_j^n$ ………………. (29) The time derivatives $\dot{\theta}_j$ are replaced by Forward finite difference formula such as $\dot{S}_j = \frac{S_j^{n+1} - S_j^n}{k}$ ………………. (30)

In view of (29) and (30), equation (25) written as,

$[A + \delta k (B(\phi^{(n+1)}) + C)]\phi^{(n+1)} = [A - (1-\delta)k(B(\phi^{(n)})+C)]\phi^{(n)}$ ………………… (31) where **δ =1/2** and n = 0,1,2,……

For a uniform mesh of N elements, by equation (28), the global equation (31) takes the form,

$$\left[K(\varphi^{(n+1)})\right]\varphi^{(n+1)} = \left[F_1(\varphi^{(n)})\right]\varphi^{(n)} = F(\varphi^{(n)}) \quad \ldots\ldots\ldots. (32)$$

where,

$$\varphi^{(n)} = \begin{bmatrix} S_1^{(n)} \\ S_2^{(n)} \\ S_3^{(n)} \\ \vdots \\ \vdots \\ S_{15}^{(n)} \\ S_{16}^{(n)} \end{bmatrix},$$

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Volume 4, Issue 1, July 2014

$$K(\phi^{(n+1)})=$$

$$\begin{bmatrix}
\frac{h}{3}+\delta k\left(\frac{1}{2h}\left(S_1^{(n+1)}+S_2^{(n+1)}\right)-\frac{C_0 h}{6}\right) & \frac{h}{6}+\delta k\left(\frac{1}{2h}(-S_1^{(n+1)}-S_2^{(n+1)})-\frac{C_0 h}{3}\right) & 0 & \cdots\ \cdots & 0 & 0 \\
\frac{h}{6}+\delta k\left(\frac{1}{2h}(-S_1^{(n+1)}-S_2^{(n+1)})+\frac{C_0 h}{6}\right) & \frac{2h}{3}+\delta k\left(\frac{1}{2h}(S_1^{(n+1)}+2S_2^{(n+1)}+S_3^{(n+1)})+\frac{C_0 h}{6}\right) & \frac{h}{6}+\delta k\left(\frac{1}{2h}(-S_2^{(n+1)}-S_3^{(n+1)})-\frac{C_0 h}{3}\right) & 0\ \cdots & 0 & 0 \\
0 & \frac{h}{6}+\delta k\left(\frac{1}{2h}(-S_2^{(n+1)}-S_3^{(n+1)})+\frac{C_0 h}{6}\right) & \frac{2h}{3}+\delta k\left(\frac{1}{2h}(S_2^{(n+1)}+2S_3^{(n+1)}+S_4^{(n+1)})+\frac{C_0 h}{6}\right) & \cdots\ \cdots & 0 & 0 \\
\cdots & \cdots & \cdots & \cdots\ \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots\ \cdots & \cdots & \cdots \\
0 & 0 & 0 & 0\ \cdots & \frac{2h}{3}+\delta k\left(\frac{1}{2h}(S_{14}^{(n+1)}+2S_{15}^{(n+1)}+S_{16}^{(n+1)})+\frac{C_0 h}{6}\right) & \frac{h}{6}+\delta k\left(\frac{1}{2h}(-S_{15}^{(n+1)}-S_{16}^{(n+1)})-\frac{C_0 h}{3}\right) \\
0 & 0 & 0 & 0\ \cdots & \frac{h}{6}+\delta k\left(\frac{1}{2h}(-S_{15}^{(n+1)}-S_{16}^{(n+1)})+\frac{C_0 h}{6}\right) & \frac{h}{6}+\delta k\left(\frac{1}{2h}(+S_{15}^{(n+1)}+S_{16}^{(n+1)})+\frac{C_0 h}{3}\right)
\end{bmatrix}$$

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$$F_1\left(\varphi^{(n)}\right)=$$

$$\begin{bmatrix}
\frac{h}{3}-(1-\delta)k\left(\frac{1}{2h}(S_1^{(n)}+S_2^{(n)})-\frac{C_0h}{6}\right) & \frac{h}{6}-(1-\delta)k\left(\frac{1}{2h}(-S_1^{(n)}-S_2^{(n)})-\frac{C_0h}{3}\right) & 0 & 0 & \cdots & 0 & 0\\
\frac{h}{6}-(1-\delta)k\left(\frac{1}{2h}(-S_1^{(n)}-S_2^{(n)})+\frac{C_0h}{6}\right) & \frac{2h}{3}-(1-\delta)k\left(\frac{1}{2h}(S_1^{(n)}+2S_2^{(n)}+S_3^{(n)})+\frac{C_0h}{6}\right) & \frac{h}{6}-(1-\delta)k\left(\frac{1}{2h}(-S_1^{(n)}-S_2^{(n)})-\frac{C_0h}{3}\right) & 0 & \cdots & 0 & 0\\
0 & \frac{h}{6}-(1-\delta)k\left(\frac{1}{2h}(-S_2^{(n)}-S_3^{(n)})+\frac{C_0h}{6}\right) & \frac{2h}{3}-(1-\delta)k\left(\frac{1}{2h}(S_2^{(n)}+2S_3^{(n)}+S_4^{(n)})+\frac{C_0h}{6}\right) & \cdots & \cdots & 0 & 0\\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots\\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots\\
0 & 0 & 0 & 0 & \cdots & \frac{2h}{3}-(1-\delta)k\left(\frac{1}{2h}(S_{14}^{(n)}+2S_{15}^{(n)}+S_{16}^{(n)})+\frac{C_0h}{6}\right) & \frac{h}{6}-(1-\delta)k\left(\frac{1}{2h}(-S_{15}^{(n)}-S_{16}^{(n)})-\frac{C_0h}{3}\right)\\
0 & 0 & 0 & 0 & \cdots & \frac{h}{6}-(1-\delta)k\left(\frac{1}{2h}(-S_{15}^{(n)}-S_{16}^{(n)})+\frac{C_0h}{6}\right) & \frac{h}{6}-(1-\delta)k\left(\frac{1}{2h}(S_{15}^{(n)}+S_{16}^{(n)})+\frac{C_0h}{3}\right)
\end{bmatrix}$$

and

$$F(\varphi^{(n)})=$$

$$\begin{bmatrix}
\frac{h}{3}-(1-\delta)k\left(\frac{1}{2h}(S_1^{(n)}+S_2^{(n)})-\frac{C_0h}{6}\right)S_1^{(n)}+\frac{h}{6}-(1-\delta)k\left(\frac{1}{2h}(-S_1^{(n)}-S_2^{(n)})-\frac{C_0h}{3}\right)S_2^{(n)}\\
\frac{h}{6}-(1-\delta)k\left(\frac{1}{2h}(-S_1^{(n)}-S_2^{(n)})+\frac{C_0h}{6}\right)S_1^{(n)}+\frac{2h}{3}-(1-\delta)k\left(\frac{1}{2h}(S_1^{(n)}+2S_2^{(n)}+S_3^{(n)})+\frac{C_0h}{6}\right)S_2^{(n)}+\frac{h}{6}-(1-\delta)k\left(\frac{1}{2h}(-S_2^{(n)}-S_3^{(n)})-\frac{C_0h}{3}\right)S_3^{(n)}\\
\vdots\\
\vdots\\
\vdots\\
\frac{2h}{3}-(1-\delta)k\left(\frac{1}{2h}(S_{14}^{(n)}+2S_{15}^{(n)}+S_{16}^{(n)})+\frac{C_0h}{6}\right)S_{15}^{(n)}+\frac{h}{6}-(1-\delta)k\left(\frac{1}{2h}(-S_{15}^{(n)}-S_{16}^{(n)})-\frac{C_0h}{3}\right)S_{16}^{(n)}\\
\frac{h}{6}-(1-\delta)k\left(\frac{1}{2h}(-S_{15}^{(n)}-S_{16}^{(n)})+\frac{C_0h}{6}\right)S_{15}^{(n)}+\frac{h}{3}-(1-\delta)k\left(\frac{1}{2h}(S_{15}^{(n)}+S_{16}^{(n)})+\frac{C_0h}{3}\right)S_{16}^{(n)}
\end{bmatrix}$$

Where K is called global stiffness matrix and F is called global generalized force vector and N+1 is total number of global nodes.

IV.III. Imposing boundary conditions

We now apply the boundary condition (20) to the global equation (32) of the problem and simplifying, we get

$$[K(\varphi^{(n+1)})]\varphi^{(n+1)}=F \qquad \text{……………………..} (33)$$

Where,

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ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 4, Issue 1, July 2014

$$K(\phi^{(n+1)})=$$

$$\begin{bmatrix} 1 & 0 & 0 & \cdots\ \cdots & 0 & 0 \\ \frac{h}{6}+\delta k\left(\frac{1}{2h}(-S_1^{(n+1)}-S_2^{(n+1)})+\frac{C_0h}{6}\right) & \frac{2h}{3}+\delta k\left(\frac{1}{2h}(S_1^{(n+1)}+2S_2^{(n+1)}+S_3^{(n+1)})+\frac{C_0h}{6}\right) & \frac{h}{6}+\delta k\left(\frac{1}{2h}(-S_2^{(n+1)}-S_3^{(n+1)})-\frac{C_0h}{3}\right) & 0\ \cdots & 0 & 0 \\ 0 & \frac{h}{6}+\delta k\left(\frac{1}{2h}(-S_2^{(n+1)}-S_3^{(n+1)})+\frac{C_0h}{6}\right) & \frac{2h}{3}+\delta k\left(\frac{1}{2h}(S_2^{(n+1)}+2S_3^{(n+1)}+S_4^{(n+1)})+\frac{C_0h}{6}\right) & \cdots\ \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots\ \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots\ \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0\ \cdots & \frac{2h}{3}+\delta k\left(\frac{1}{2h}(S_{14}^{(n+1)}+2S_{15}^{(n+1)}+S_{16}^{(n+1)})+\frac{C_0h}{6}\right) & \frac{h}{6}+\delta k\left(\frac{1}{2h}(-S_{15}^{(n+1)}-S_{16}^{(n+1)})-\frac{C_0h}{3}\right) \\ 0 & 0 & 0 & 0\ \cdots & 0 & 1 \end{bmatrix}$$

$$F(\varphi^{(n)})=\begin{bmatrix} 1 \\ \frac{h}{6}-(1-\delta)k\left(\frac{1}{2h}(-S_1^{(n)}-S_2^{(n)})+\frac{C_0h}{6}\right)S_1^{(n)}+\frac{2h}{3}-(1-\delta)k\left(\frac{1}{2h}(S_1^{(n)}+2S_2^{(n)}+S_3^{(n)})+\frac{C_0h}{6}\right)S_2^{(n)}+\frac{h}{6}-(1-\delta)k\left(\frac{1}{2h}(-S_2^{(n)}-S_3^{(n)})-\frac{C_0h}{3}\right)S_3^{(n)} \\ \vdots \\ \vdots \\ \vdots \\ \frac{2h}{3}-(1-\delta)k\left(\frac{1}{2h}(S_{14}^{(n)}+2S_{15}^{(n)}+S_{16}^{(n)})+\frac{C_0h}{6}\right)S_{15}^{(n)}+\frac{h}{6}-(1-\delta)k\left(\frac{1}{2h}(-S_{15}^{(n)}-S_{16}^{(n)})-\frac{C_0h}{3}\right)S_{16}^{(n)} \\ 0 \end{bmatrix}$$

Thus, equation (33) is the resulting system of nonlinear algebraic equation.

IV.IV. Solution of non-algebraic equation

In the previous section, we obtained the assembled equation which is nonlinear. The assembled nonlinear equation after imposing boundary conditions is given by equation (33). We seek an approximate solution by the linearization which based on scheme

$$\left[K(\varphi^{(n)})\right]\varphi^{(n+1)}=F \qquad \text{.....................} \ (34)$$

Where $\varphi^{(n)}$ denotes the solution of the n iteration. Thus, the coefficient K_{ij} are evaluated using the solution $\varphi^{(n)}$ from the previous iteration and the solution at the $(n+1)^{th}$ iteration can be obtained by solving equation (34) using Gauss Elimination Method. At the beginning of the iteration (i.e. n=0), we assume the solution $\phi^{(0)}$ from initial condition which requires to have $S_1^{(0)}= S_2^{(0)}=\ldots\ldots\ldots\ldots\ldots= S_{N+1}^{(0)}= 0$.

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 4, Issue 1, July 2014

V. GRAPHICAL REPRESENTATION AND INTERPRETATION

A Matlab Code is prepared for 15 elements model and resulting equation (34) for N = 15 is solved by Gauss Elimination method. Saturation of injected liquid at time t = 0.1, t = 0.2, t = 0.3, t = 0.4 and t = 0.5 seconds are

1.0000e+000	1.0000e+000	1.0000e+000	1.0000e+000	1.0000e+000
9.5834e-001	9.5744e-001	9.6027e-001	9.6273e-001	9.6426e-001
9.1438e-001	9.1301e-001	9.1896e-001	9.2405e-001	9.2720e-001
8.6738e-001	8.6656e-001	8.7601e-001	8.8389e-001	8.8871e-001
8.1655e-001	8.1793e-001	8.3134e-001	8.4214e-001	8.4866e-001
7.6102e-001	7.6694e-001	7.8486e-001	7.9868e-001	8.0689e-001
6.9992e-001	7.1336e-001	7.3646e-001	7.5335e-001	7.6319e-001
6.3240e-001	6.5699e-001	6.8601e-001	7.0592e-001	7.1727e-001
5.5760e-001	5.9759e-001	6.3331e-001	6.5609e-001	6.6874e-001
4.7475e-001	5.3492e-001	5.7805e-001	6.0338e-001	6.1706e-001
3.8323e-001	4.6876e-001	5.1979e-001	5.4709e-001	5.6142e-001
2.8266e-001	3.9892e-001	4.5770e-001	4.8606e-001	5.0051e-001
1.7155e-001	3.2523e-001	3.9024e-001	4.1826e-001	4.3213e-001
3.4351e-002	2.4740e-001	3.1400e-001	3.3956e-001	3.5188e-001
7.4980e-003	1.6283e-001	2.1943e-001	2.3905e-001	2.4831e-001
0	0	0	0	0

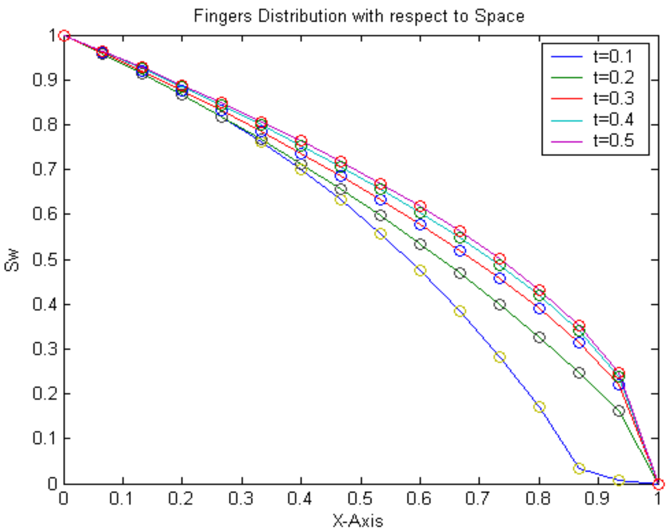

Fig4

In above graphs, X-axis represents the saturation of injected liquid (S_w) of porous media of length one. Y-axis represents the time 't' in seconds. Solution is obtained with $C_o = 1.22694 \times 10^{-11}$, h =1/15 , k = 0.002223 for 225 time levels and $L_1 = 0.5$. It is clear from graph that there is full injected liquid saturation (i.e. $S_w = 1$) at injected face x = 0 & there is no saturation of injected liquid at other end (x =1) irrespective of time. It is clear from graph that, for each value of T, Saturation S_w has a decreasing tendency along the space co-ordinate axis. Also, for each point of X, the Saturation increases as time increases but the rate at which it rises at each point in observed region slows down with increase in time. This shows that the stabilization of the fingers is truly possible with the assumptions made for capillary pressure and water saturation.

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