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# Diffusion Stability of Mechanical Equilibrium in Isothermal Gases

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***Abstract* —this paper is presented stability condition of mechanical equilibrium for diffusion in isothermal gases in a cylindrical domain. We consider two special cases of diffusion in isothermal gases for stable fluxes of the divergent motions when and density depends on conditions $div\,\vec{u}=0$ and $div\vec{u}\neq 0$.**

***Stability criterion of mechanical equilibrium of the gas mixture is proposed by defining continuous dependence of initial date from constructed solution of the Navier-Stokes problem. Introduced stability condition for isothermal gases was predicted in the principal form by the energy conservation law and based on the transport equation for an appropriate converging-diverging turbulent gas motion. With respect to obtained balance equation was defined relationship between external force and pressure distribution in the cylindrical domain.***

*Index Terms*—**stability condition, mechanical equilibrium, energy conservation law, diffusion in isothermal gases, external force and pressure distribution**

## I. INTRODUCTION

Mathematical theory for determining stability condition is fundamentally interesting and practical importance for engineering models of multi-component diffusion with the divergent turbulent effects. The problem of stability condition of multi-component diffusion in gases have received widespread attention and have a great interest in mathematical and physical modeling of turbulent processes in isothermal gases. Experiences in studies of the isothermal diffusion for multi-component gas mixtures have shown that under certain conditions they can have a mechanical stability in which occurs equilibrium and is determined emergence of a subsequent gravitational regime [3]- [4]. Problems which relate to emergence of kinetic phase transitions, convection fluxes are traditionally studied in the framework of the hydrodynamic theory of heat and mass transfer [5]. Determination of regime change, description of the basic regularities of the development of convective perturbation can be implemented in the theory of stability [6]-[7]. Usually are studied convection flows in an environment when the compressibility of the medium is negligible and the perturbations of the phase state of the medium are small compared to their average values [6],[7]. This approach leads to simplification of the basic equations of hydrodynamics and has proved satisfactory for cases describing thermal convection of single-component and binary incompressible media [8]-[10]. In [7]-[10] of formal stability theory has been extended to diffusive mixing isothermal binary and ternary gas mixtures in channels of rectangular and cylindrical shape. Comparison of experimental results of determining the mass transfer regime change is calculated within the experimental errors [10]. One of the main problems of sustainability issues in nonlinear approximation would be determining the conditions of mechanical equilibrium of the gas mixture, as its analysis offer up the characteristic values of the partial concentration gradients. In this paper it is assumed to obtain the stability condition of mechanical equilibrium for multi-component gas mixture in the presence of diffusion and cross effects of the divergent turbulence. The proposed model of stability condition of mechanical equilibrium is based on Lyapunov stability theory in terms uniform continuity for the Navier-Stokes problem where we have a perspective using some alternative approach which different from that is needed for studying stability condition in the general classical issues and deal with model problems for common phenomena of divergent turbulence. Here was defined the velocity vector and stability condition in terms continuity which is indicated mechanics of the turbulent fluxes in isothermal gases.

## II. GOVERNING EQUATION

To illustrate how works stability condition we note, that many problems formally exist for any Reynolds numbers and it can have an exact solution, but not all partial differential equation can describe real-nature phenomenon, therefore we will consider the basic model equations of hydrodynamics that correctly can be solved (existence, uniqueness and stability in terms of continuity). The requirement of stability is caused by the fact that physical evidence is usually determined from experiments and approximately, therefore we must be sure that the determined solution is the stability solution. This requirement of stability in terms of continuity seems to be important. Therefore first of all we must construct Lyapunov theory for the Navier-Stokes problem which will be a powerful determining method for defining the stability or instability domains for the turbulent motion. Under stable flow we will understand continuous motion (Lyapunov stability). With respect to this requirement let us describe the used approach in the proofs of existence and uniqueness of the Navier-Stokes problem under influence divergence and rotation. The key idea of our research is to present existence, uniqueness and stability in terms of continuity. According to this idea we can get an integral representative for determining the velocity vector and the pressure distribution in case when stability criterion of mechanical equilibrium is determined by constructed weak solution in case continuous dependence initial data off defined solution of the Navier-Stokes problem. We involve this method to show that the velocity vector and

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an external force with respect to the pressure function exist and satisfy the energy conservation law. It has been found that a stability criterion is very useful in simplifying the processing and analysis of the experimental date. We use this method to show that the velocity vector and an external force with respect to the pressure function exist and satisfy the energy conservation law. Recalling from previous studies [11]-[12] idea of constructing stability condition was divided into two steps. In the first step, the pressure function is excluded from the the Navier-Stokes equation by using rotor operator. According to this transformation we can get an integral representative for determining the velocity vector and the pressure distribution. We claim that we may assume that

$$div\ \vec{u} = 0$$

and

$$rot\ \vec{f} = 0,\ rot\vec{u}_0 = 0$$

Then we will get the following condition

$$grad\ \left(\frac{u^2}{2} + \frac{p}{\rho} - \Phi\right) = 0$$

where

$$grad\Phi(x,t) = -\vec{f}(x,t)$$

Due to this assertion we can find a weak solution and influence of divergence from rotation for the velocity vector in the Navier-Stokes problem. It is proved that under dependence of the energy conservation law from the external force there exists a unique velocity vector given by the integral representation. Due to appropriate a priory estimate here we get a stable solution for the Navier-Stokes problem which can be seen from "a priory" estimations of the Navier-Stokes problem. In the second step we assume that $div\vec{u} \neq 0$. In this case under identical dependence of the energy conservation law from the external force we have got the integral representation. for the Navier-Stokes problem. Under above assumption $div\vec{u} \neq 0$ there exists a unique stable solution with the appropriate properties. This mathematical concept links with the identical energy conservation law and characterizes steady behavior of a converging-diverging turbulent gas motion. The shape of turbulent region is determined by the properties which have shown stability of the velocity motion and the pressure distribution. Stabilizing mechanisms can be used to explain features observed in numerical simulations of turbulence. In the papers [11]-[12] have been noted that instability behavior of the velocity vector and the pressure distribution depends on conditions of the initial data

$$rot\ \vec{f} = 0 \text{ or } rot\vec{u}_0 = 0.$$

We will exclude these cases and deal only with the stable turbulent motion.

Let us suppose that

$$\Omega = \{(r,\varphi,z): 0 < r < \infty, 0 < \varphi < 2\pi, 0 < z < h,\},\ \Omega_T = \Omega \times (0 < t < \infty)$$

and consider the Navier -Stokes problem in the cylindrical coordinates $r,\varphi,z)$ ( Fig. 1) in the following form

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u}\cdot\nabla)\vec{u} = -\frac{1}{\rho}\nabla p + \nu\Delta\vec{u} + (\frac{\nu}{3}+\eta)\nabla div\vec{u} + \vec{f}(r,\varphi,z,t) \qquad (1)$$

$$\vec{u}(x,t) = \begin{cases} \vec{u}_\nu(x,t) \ \ if\ div\vec{u}(x,t) = 0 \\ \vec{u}_\alpha(x,t) \ \ if\ div\vec{u}(x,t) \neq 0 \end{cases}$$

in $\Omega_T$ with the initial conditions

$$\vec{u}\ \big|_{t=0} = \vec{u}_0(r,\varphi,z) \ \text{ on } \Omega \qquad (2)$$

and boundary conditions when the body does not react with the surrounding medium

$$\left[\frac{\partial\vec{u}}{\partial z} - h_1\vec{u}\right]_{z=0} = 0 \qquad (3)$$

$$\left[\frac{\partial\vec{u}}{\partial z} + h_2\vec{u}\right]_{z=h} = 0 \qquad (4)$$

$$\left[\frac{\partial\vec{u}}{\partial r} + h_3\vec{u}\right]_{r=r_0} = 0 \qquad (5)$$

We will look for the stable velocity vector

$$\vec{u}(r,\varphi,z,t) = u_r(r,\varphi,z,t)\vec{i} + u_\varphi(r,\varphi,z,t)\vec{j} + u_z(r,\varphi,z,t)\vec{k}$$

and the gas pressure field $p(r,\varphi,z,)$ when

$$\vec{f}(r,\varphi,z,t) = f_r(r,\varphi,z,t)\vec{i} + f_\varphi(r,\varphi,z,t)\vec{j} + f_z(r,\varphi,z,t)\vec{k}$$

is the known vector function of an external force, $\nu$ is a kinematic viscosity, $\rho$ is a gas density, $\eta$ is a dynamic viscosity which is related to the kinematic viscosity by $\eta = \rho\nu$, the symbol $\nabla$ denotes the gradient with respect to the function , the symbol $\Delta$ denotes the three dimensional Laplace operator .

The initial- boundary volume problem (1)-(6) is concerned with the fundamental solutions for Poisson and heat conduction equations in the considered domain. Particular attention is paid to the integral representation of solutions with their initial values for the turbulent flux which is the basis of hydrodynamics.

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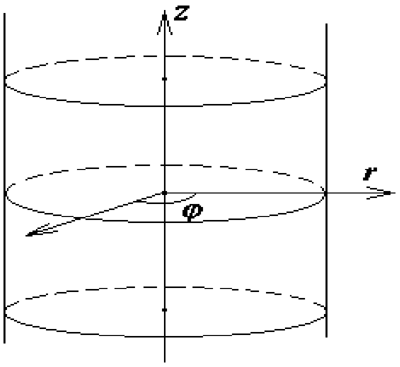


Fig. 1: Geometry of the problem

## III. STABLE SOLUTION FOR THE NAVIER-STOKES PROBLEM

In this part we consider flux characterized by the three -dimensional Navier-Stokes problem in the following system of equations:

$$\frac{\partial \vec{u}}{\partial t}+(\vec{u}\cdot\nabla)\vec{u}=-\frac{1}{\rho}\nabla p+\nu\Delta\vec{u}+\vec{f}(x,t)\ \text{in}\,\Omega_T \qquad (7)$$

$$div\ \vec{u}=0 \quad \text{in} \quad \Omega_T \qquad (8)$$

with an initial conditions

$$\vec{u}\,\big|_{t=0}=\vec{u}_0(x) \quad \text{on} \quad \Omega \qquad (9)$$

and boundary conditions

$$\left[\frac{\partial \vec{u}}{\partial z}-h_1\vec{u}\right]_{z=0}=0 \qquad (10)$$

$$\left[\frac{\partial \vec{u}}{\partial z}+h_2\vec{u}\right]_{z=h}=0 \qquad (11)$$

$$\left[\frac{\partial \vec{u}}{\partial r}+h_3\vec{u}\right]_{r=r_0}=0 \qquad (12)$$

Using well-known formula of vector analysis

$$\frac{1}{2}grad\ \vec{u}^{\,2}=[\vec{u}\times rot\vec{u}]+(\vec{u}\nabla)\vec{u} \qquad (13)$$

and operator $rot\ \vec{u}=\nabla\times\vec{u}$ which is the determinant of the third order

$$rot\ \vec{u}=\nabla\times\vec{u}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ \frac{\partial}{\partial x_1} & \frac{\partial}{\partial x_1} & \frac{\partial}{\partial x_2}\\ u_1 & u_2 & u_3\end{vmatrix}$$

we have got the following equation

$$\frac{\partial \vec{u}}{\partial t}+grad(\frac{p}{\rho}+\frac{1}{2}\vec{u}^2)=[\vec{u}\times rot\vec{u}]+\nu\Delta\vec{u}+\vec{f}(x,t) \qquad (14)$$

Considering the function

$$grad\Phi(r,z,t)=-\vec{f}(r,z,t) \qquad (15)$$

which represents potential energy and using the divergence operator we can get an important expression for potential energy

$$\Phi(r,z,t)=div\vec{f}*G_0(r-r_1,z-z_1)$$

Here symbol $*$ is a convolution between two functions:

$$div\vec{f} \quad \text{and} \quad G_0(r-r_1,z-z_1)$$

where $G_\nu(r,z,r_1,z_1,t)$ is the Green's function for the three dimensional finite domain $\Omega_T=\Omega\times(0<t<\infty)$ where $\Omega=\{(r,\varphi,z):0<r<\infty,0<\varphi<2\pi,0<z<h,\}$.

By utilizing $G_\nu(r,z,r_1,z_1,t)$ function in case $t=0$ we have got Green's function for Laplace operator in the following form

$$G_0(r-r_1,z-z_1)=\sum_{k=0}^{\infty}\sum_{m=-\infty}^{\infty}B_{k,m}\delta(z-z_1-2\lambda_m\pi))\delta(r-r_1)$$

$$\sum_{k=0}^{\infty}\sum_{m=-\infty}^{\infty}B_{k,m}<\infty$$

$\delta(u-a)$ is the Dirac Delta function satisfying the following integral

$$\int_{-\infty}^{t}\delta(u-a)\,du=\begin{cases}0 & \text{if } t<a\\ 1 & \text{if } t>a\end{cases}$$

Considering the problem (7)-(12) and assuming that

$$rot\vec{u}=0 \qquad (16)$$

we have got

$$\frac{\partial \vec{u}}{\partial t}+grad(\frac{p}{\rho}+\frac{1}{2}u^2)=\nu\Delta\vec{u}+\vec{f}(x,t) \qquad (17)$$

Using condition (16) for the expression we obtain the balance equation

$$grad(\frac{p}{\rho}+\frac{1}{2}\vec{u}^2-\Phi)=0 \qquad (18)$$

Using divergence operator for the expression (18) we have

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$$\frac{p}{\rho}+\frac{u^2}{2}-div\vec{f}*G_0(r-r_1,z-z_1)=0 \quad (19)$$

This expression (19) represents the energy conservation law.

Applying the balance expression (18) to the Navier-Stokes equation (17) we obtain the linear mathematical problem

$$\frac{\partial\vec{u}}{\partial t}-\nu\Delta\vec{u}=2\vec{f} \quad (20)$$

$$div\ \vec{u}=0 \quad (21)$$

with an initial condition

$$\vec{u}\,\big|_{t=0}=\vec{u}_0(x) \quad (22)$$

and boundary conditions

$$\left[\frac{\partial\vec{u}}{\partial z}-h_1\vec{u}\right]_{z=0}=0 \quad (23)$$

$$\left[\frac{\partial\vec{u}}{\partial z}+h_2\vec{u}\right]_{z=h}=0 \quad (24)$$

$$\left[\frac{\partial\vec{u}}{\partial r}+h_3\vec{u}\right]_{r=r_0}=0 \quad (25)$$

Following the classical procedure of partial differential equation we can get solutions for the problem (20)-(25) in the integral sum of the parabolic potentials

$$\vec{u}(r,z,t)=\int_{\Omega}\vec{u}_0(r_1,z_1)G_\nu(r_1,z_1;r,z,t)d\Omega+$$
$$+2\int_0^t d\tau\int_{\Omega}\vec{f}(r_1,z_1,\tau)G_\nu(r_1,z_1;r,z,t-\tau)d\Omega \quad (26)$$

Where $G(r,z;r_1,z_1,t)$ is the Green's function for the three dimensional finite domain $\Omega_T=\Omega\times(0<t<\infty)$ where $\Omega=\{(r,\varphi,z):0<r<\infty,0<\varphi<2\pi,0<z<h,\}$ which was defined in the following form

$$G_\nu(r,z,r_1,z_1,t)=\sum_{k,m=0}^{\infty}\frac{Z_{k,m}(z,z_1,t)}{2\beta t}e^{-\frac{(r^2+r_1^2)}{4\beta t}}I_0\left(\frac{rr_1}{2\beta t}\right) \quad (27)$$

$$\beta=\frac{\nu(\mu_k^2+r_0^2\lambda_m^2)}{r_0^2}$$

where

$$Z_{k,m}(z,z_1,t)=A_{k,m}\,e^{-\lambda_m\nu t}\sin(\lambda_m z_1+z_m)\sin\lambda_m(z+z_m)$$

$$A_{k,m}=\frac{1}{r_0^2J_0^2(\mu_k)\pi\left(h+\frac{(h_1h_2+\lambda_m^2)(h_1+h_2)}{(h_1^2+\lambda_m^2)(h_2^2+\lambda_m^2)}\right)\left(1+\frac{r_0^2h_3^2-h_3^2}{\mu_k^2}\right)}$$

$J_0(y)$ - Bessel function

$\mu_k$ is a positive solution of the transcendent equation

$$\mu_kJ_o^{/}(\mu_k)+r_0h_3J_o^{/}(\mu_k)=0$$

$\lambda_m$ is a positive solution of the transcendent equation

$$cth\lambda_m h=\frac{\lambda_m^2-h_1h_2}{\lambda_m(h_1+h_2)}$$

Parameter $z_m$ is defined as

$$z_m=artg\frac{\lambda_m}{h_3}$$

The Green's function $G(r,z;r_1,z_1,t)$ has estimation in the following view

$$|G(r,z;r_1,z_1,t|\le\frac{e^{-\frac{r^2+r_1^2r+(z-z_1)^2}{8\nu_0t}}}{\left(\sqrt{\pi}\right)^3\nu_0^{3/2}t^{3/2}}I_0\left(\frac{rr_1}{4\nu_0t}\right)$$

with its partial derivative in the following forms

$$\left|\frac{\partial}{\partial r}G_\nu\right|\le\frac{e^{-\frac{r^2+r_1^2r+(z-z_1)^2}{8\nu_0t}}}{\left(\sqrt{\pi}\right)^3\nu_0^2t^2}I_0\left(\frac{rr_1}{4\nu_0t}\right)$$

$$\left|\frac{\partial}{\partial z}G_\nu\right|\le\frac{e^{-\frac{r^2+r_1^2r+(z-z_1)^2}{8\nu_0t}}}{\left(\sqrt{\pi}\right)^3\nu_0^2t^2}I_0\left(\frac{rr_1}{4\nu_0t}\right)$$

$$(\ i=1,2,3)$$

$I_0(y)$ is the modified zero order Bessel function *of* first kind.

Notice that stability in terms continuity is closely related with the following conditions

$$rot\ \vec{f}=0,\ rot\vec{u}_0=0$$

Using properties of the Green's function $G(r,z;r_1,z_1,t)$ and its derivative evaluations we have got a uniqueness and stable solution (26) satisfying following estimations

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$$\|\vec{u}\|_{L_2(\Omega_T)} \leq M_0(\|\vec{u}_0\|_{L_2(\Omega)} + 2t\|\vec{f}\|_{L_2(\Omega_T)})$$

$$\|\vec{u}\|_{H^{(2,1)}_{\Omega T}} \leq M_0(\|\vec{u}_0\|_{H^{(1)}_{\Omega}} + 2\|\vec{f}\|_{L_2})$$

for positive constant $M_0$.

Condition (19) for the scalar pressure function $p(x,t)$ predicts a steady feature which introduces the balance equation for a stable turbulent motion. This condition is the energy conservation law and characterizes steady behavior for the turbulent motion that can be main property for the stability turbulent flows.

After using the Navier-Stokes equation (17) and estimation (27) has been obtained the evaluation with norm on the Hilbert spaces for the pressure function $p(x,t)$

$$\|p\|_{H^{(1,0)}_{\Omega T}} \leq M_1(\|\vec{u}_0\|_{H^{(1)}_{\Omega}} + \|\vec{f}\|_{L_2})$$

with positive constant $M_1$.

In this case new obtained condition admits solution that can be predicted stable process in case absence of rotation of the velocity vector. Consequently, we see from estimation (27) that stability of the turbulent flow depends on the condition (19).

## IV. STABLE DIVERGENT MOTION

In this part we introduce the mathematical description for compressible fluid given by governing equations: momentum conservation and energy conservation. The development of our idea is the most challenging task due to behavior of expression

$$(\frac{\nu}{3}+\eta)\nabla div\vec{u}$$

in the Navier-Stockes problem (1)-(5) which can change physical properties and characteristics of turbulent interaction effects.

Problem (1)-(2) deals with flows having the vector velocity with condition

$$div\vec{u} \neq 0 \qquad (28)$$

Assuming

$$rot\vec{u} = 0$$

then the equation (26) can be written as

$$\frac{\partial \vec{u}}{\partial t} = \nabla(\frac{1}{\rho}p + \frac{u^2}{2}) + \nu\Delta\vec{u} + (\frac{\nu}{3}+\eta)\nabla div\vec{u} + \vec{f}(x,t) \qquad (29)$$

Denote that

$$\vec{U} = div\vec{u}$$

we have got some analogies for the heat problem

$$\frac{\partial \vec{U}}{\partial t} = (\frac{4\nu}{3}+\eta)\Delta\vec{U} + 2div\vec{f}(x,t) \qquad (30)$$

with the initial conditions

$$\vec{U}\,\big|_{t=0} = div\vec{u}_0(x) \qquad (31)$$

and the boundary conditions

$$\left[\frac{\partial \vec{U}}{\partial z} - h_1\vec{U}\right]_{z=0} = 0 \qquad (32)$$

$$\left[\frac{\partial \vec{U}}{\partial z} + h_2\vec{U}\right]_{z=h} = 0 \qquad (33)$$

$$\left[\frac{\partial \vec{U}}{\partial r} + h_3\vec{U}\right]_{r=r_0} = 0 \qquad (34)$$

There exists a unique stable solution for the problem (30)-(34) in the following form

$$\vec{U} = div\vec{u}_0 * G^*_\alpha + 2div\vec{f} * G^*_\alpha \qquad (35)$$

where

$$G_\alpha(r,z,r_1,z_1,t) = G_\nu(r,z,r_1,z_1,t)\,\big|_{\nu=\alpha}$$

$$\alpha = \frac{4\nu}{3}+\eta$$

Using solution (35) and properties of the Green's function $G_\alpha(x,\xi,t)$ we have got solution for the problem (30)-(34)

$$\vec{u} = grad\{div\vec{u}_0 * G^*_\alpha + 2div\vec{f} * G^*_\alpha\} \qquad (36)$$

In the form of the sum of potentials we have got

$$\vec{u}_\alpha(r,z,t) = \int_\Omega \vec{u}_0(r_1,z_1)G^*_\alpha(r_1,z_1;r,z,t)d\Omega + \\ + 2\int_0^t d\tau \int_\Omega \vec{f}(r_1,z_1,\tau)G^*_\alpha(r_1,z_1;r,z,t-\tau)d\Omega \qquad (37)$$

Where

$$G^*_\alpha(r,z,t) = G_\alpha(r_1,z_1,t) * G_0(r-r_1,z-z_1)$$

$$G_\alpha(r,z,r_1,z_1,t) = G_\nu(r,z,r_1,z_1,t)\,\big|_{\nu=\alpha}$$

Using properties of the Green's function $G^*_\alpha(x,\xi,t)$ and its derivative evaluations we have got uniqueness and stable solution (37) satisfying following estimations

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$$\|\vec{u}\|_{L_2(\Omega_T)} \le M_0(\|\vec{u}_0\|_{L_2(\Omega)} + 2\sqrt{t}\|\vec{f}\|_{L_2(\Omega_T)}) \quad (38)$$

$$\|\vec{u}\|_{H^{(2,1)}_{\Omega_T}} \le M_0(\|\vec{u}_0\|_{H^{(1)}_{\Omega}} + 2\|\vec{f}\|_{H^{(1,0)}_{\Omega_T}})$$

$$\|p\|_{H^{(1,0)}_{\Omega_T}} \le M_0(\|\vec{u}_0\|_{H^{(1)}_{\Omega}} + 2\|\vec{f}\|_{H^{(1,0)}_{\Omega_T}})$$

Here $M_0$ is a positive constant.

In this case new obtained same condition which can be predicted specific stable process in case absence of rotation of the velocity vector. Consequently, we see from estimation (38) that stability of the turbulent flow depends on the condition (19).

## V. RESULTS AND DISCUSSION

Let us gather and formulate main results about properties of the vector velocity and the scalar function of pressure.

**Theorem1.** Let $\vec{u}_0(x,t) \in H^{(1)}(\Omega)$ and $\vec{f}(x,t) \in L_2(\Omega_T)$ be periodic functions which satisfy conditions $rot\ \vec{f} = 0$, $rot\vec{u}_0 = 0$. Then there for the Navier-Stokes problem (1) - (5) exists a unique stable solutions in the form

$$\vec{u}(x,t) = \begin{cases} \vec{u}_\nu(x,t) = \vec{u}_0 * G_\nu + 2\vec{f} * G_\nu \text{ if } div\vec{u}(x,t) = 0 \\ \vec{u}_\alpha(x,t) = grad\{div\vec{u}_0 * G^*_\alpha + 2div\vec{f} * G^*_\alpha\} \text{ if } div\vec{u}(x,t) \ne 0 \end{cases}$$

$$G_\alpha(r,z,r_1,z_1,t) = G_\nu(r,z,r_1,z_1,t)\,\big|_{\nu=\alpha}$$
$$G^*_\alpha(r,z,t) = G_\alpha(r_1,z_1,t) * G_0(r-r_1,z-z_1)$$

and a unique scalar function of pressure $p(x,t)$ which satisfies energy conservation law

$$\frac{p}{\rho} + \frac{u^2}{2} - div\vec{f} * G_0(r-r_1,z-z_1) = 0$$

where

$$u^2 = u_r^2 + u_\varphi^2 + u_z^2$$

$$G_0(r-r_1,z-z_1) = G_\nu(r,z,r_1,z_1,t)\,\big|_{t=0}$$

Moreover, there exists positive constant $M_0$ such that for all functions $\vec{u}(x,t) \in H^{(2,1)}(\Omega_T)$ and $p(x,t) \in H^{(1,0)}(\Omega_T)$ satisfy the following estimates for the velocity vector

$$\|\vec{u}\|_{H^{(2,1)}_{\Omega_T}} \le \begin{cases} M_0(\|\vec{u}_0\|_{H^{(1)}_{\Omega}} + 2\|\vec{f}\|_{L_2}) \text{ if } div\vec{u}(x,t) = 0 \\ M_0(\|\vec{u}_0\|_{H^{(1)}_{\Omega}} + 2\|\vec{f}\|_{H^{(1,0)}_{\Omega_T}}) \text{ if } div\vec{u}(x,t) \ne 0 \end{cases}$$

for the pressure function

$$\|p\|_{H^{(1,0)}_{\Omega_T}} \le \begin{cases} M_0(\|\vec{u}_0\|_{H^{(1)}_{\Omega}} + 2\|\vec{f}\|_{L_2}) \text{ if } div\vec{u}(x,t) = 0 \\ M_0(\|\vec{u}_0\|_{H^{(1)}_{\Omega}} + 2\|\vec{f}\|_{H^{(1,0)}_{\Omega_T}}) \text{ if } div\vec{u}(x,t) \ne 0 \end{cases}$$

We can formulize this simple result that Bernoulli's equation is an consequence of the formula (19). Assume that $rot\ \vec{f} = 0$, $rot\vec{u}_0 = 0$ are satisfied. If $\vec{f} = C\vec{x} + \vec{d}$, where $C$ cab be chosen as matrix

$$C = \begin{pmatrix} \frac{c_1}{m} & 0 & 0 \\ 0 & \frac{c_2}{m} & 0 \\ 0 & 0 & -gh \end{pmatrix} \quad (39)$$

$\vec{d}$ - a numerical vector, $m$ - a body's mass, $c_1, c_2$ are independent constants which satisfy the condition $c_1 + c_2 \ge 0$, $g$ is the acceleration of gravity, $h$ is the height. Then fluid flow can be considered to be an incompressible flow which satisfies Bernoulli's equation

$$\frac{mp}{\rho} + \frac{mu^2}{2} + mgh = c \quad (40)$$

Here $c = c_1 + c_2$, $\frac{mp}{\rho}$ is a binding energy of the mass elements, $\frac{mu^2}{2}$ is a kinetic energy , $mgh$ is a potential energy .

When initial density is a constant, divergence of the initial velocity is a constant and divergence of the external force depends on a given time $t$:

$$\rho_{0i} = const,\ div\vec{u}_{0i} = const,\ div\vec{f}_i = div\vec{f}_i(t)$$

Using properties of the Green's function

$$G_\alpha(r,z,r_1,z_1,t) = G_\nu(r,z,r_1,z_1,t)\,\big|_{\nu=\alpha}$$

and species conservation law for $n$ species we have got density for every species $i = 1,2,\ldots,n$ in the following form

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$$\rho_i(t) = \rho_{0i}\, e^{-\int_0^t \left(div\vec{u}_{0i} + 2\int_0^\tau div\vec{f}_i(\tau_1)d\tau_1\right)d\tau}$$

Similar comparative trend can be observed (Fig. 2) dependence of divergence from density.

Total flux density can be found as sum

$$\rho(t) = \frac{1}{n}\sum_{i=0}^{n} \rho_{0i}\, e^{-\int_0^t \left(div\vec{u}_{0i} + 2\int_0^\tau div\vec{f}_i(\tau_1)d\tau_1\right)d\tau} \tag{41}$$

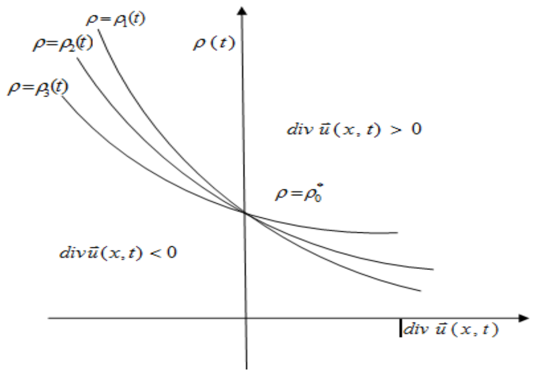


**Fig. 2: Dependence of the divergence from the density**

Using formula (42) we have got following figure (Fig. 2) which is described dependence of divergence from density.

## VI. CONCLUSION

Assume that a mathematical model of turbulent flow was written as an initial value problem we have presented new analytic method which can be classified by stability balance condition resulted from the velocity vector and external force which are expected to exist in finite domain. There are two unknown independent thermodynamic parameters (the velocity vector and the scalar function of pressure) which play a prominent role in the obtained integral representation of the velocity distribution for the description of the turbulent behavior of fluid motion.The Navier-Stokes equations have been the basis for description of turbulent phenomena where experimental selection of the regime turbulent fluctuation is costly and sometimes not always realizable process, therefore important argument for analytic research of the Navier-Stokes equation. There is presented an mathematical conception which is based on the Green's function and required a good deal with the parabolic and elliptic potential theories. In processes dealing with governing equations the main point stressed that the velocity vector and the pressure function satisfy their balance criteria of stability motion which is the energy conservation law. In this research is submitted convenient procedure to investigate the Navier-Stokes equations which allows to use 'a priori' estimates for proof existence and uniqueness of weak solution. Weak formulation for the Navier-Stokes problem is based on the extension of idea to the case where the energy falls in the critical domain, due to the pressure transition. There we use this essential feature of the Navier-Stokes equations. Moreover, basic concept our research is based on the weak formulation for the turbulent flows and introduced technique has been investigated in the Hilbert space. When we have stable solution then the weak formulation coincides with the classical formulation, so we can say under the stability condition we have the classical solution of the considered Navier Stokes problem. This research can be applied to engineering models for demonstrating technological applications of new analytic approach for modeling multi component fluxes characterizing turbulent motion and playing a key role in the mass-heat transfer of the turbulent motion. Introduced approach leads to the conclusion that this submitted analytic solution would have been used for visualization the basic mechanism and the significant physical structure on the turbulence effects for turbulent influence of the pressure distribution from external force in the considered areas.

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