ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 4, Issue 1, July 2014

Einstein's Field Equation and Energy

Dipo Mahto, Md Shams Nadeem, Murlidhar Prasad Singh, Krishna Murari Singh & Ashok Prasad Yadav
Department of Physics, Marwari College, Bhagalpur T. M. B. U. Bhagalpur. India
University Department of Physics, T. M. B. University, Bhagalpur, India.

Abstract**—*****This review article discusses the conditional and unconditional validity of the well known mass-energy equivalence relation E=mc*^{2} *as proposed by Albert Einstein with some experimental and theoretical evidences in support.***

Keywords: **Energy, Einstein field equations, conditional validity.**

I. INTRODUCTION

Before the discovery of special theory of relativity as proposed by Albert Einstein in 1905, the mass and energy of any bodies were independent things and there were no any correlation between these two quantities. In 1905, the greatest scientist Albert Einstein proposed a revolutionary idea of mass-energy equivalence relation [1] (E=mc^2) on the basis of variation of mass with velocity correlating with the total energy of the system. At that time, the new ideas like variation of mass with velocity and mass-energy equivalence relation became very popular among the scientists and it gave a large number of applications on the basis of these ideas. In 1905, Einstein proved that the electromagnetic energy is equivalent to mass by showing the photons can be converted into mass [2]. However, Ohanian pointed out that Einstein's proof is incomplete because Einstein had proved the mass-energy equivalence relation (E = mc^2) for the simple special case of slow-moving bodies and he blithely extrapolated this to fast-moving bodies [3]. In 1911, Ohanian claimed that Max von Laue has given the first general proof of E = mc^2, because it is valid for slow-moving and fast-moving bodies [3]. The correctness of Einstein's derivation of $E = mc^2$ was criticized by Max Planck (1907), who argued that it is only valid to first approximation. Another criticism was formulated by Herbert Ives (1952) and Max Jammer (1961), asserting that Einstein's derivation is based on begging the question [4]. On the other hand, John Stachel and Roberto Torretti (1982) argued that Ives' criticism was wrong, and that Einstein's derivation was correct [5]. Hans Ohanian (2008) agreed with Stachel/Torretti's criticism of Ives, though he argued that Einstein's derivation was wrong for other reasons [6].Thus, the famous formula E = mc^2 is actually only conditionally valid for some cases. In 2012, C. Y. Lo had discussed the conditional/un-conditional validity of the mass-energy equivalence relation E=mc^2, by giving a number of theoretical and experimental evidences in this support as proposed by different physicist [7]. In this article, we present the review works regarding the fact that the mass-energy equivalence relation E=mc^2, is conditional valid, but not unconditionally.

II. EINSTEIN FIELD EQUATION AND ENERGY

The Einstein field equations (EFE) or Einstein - Hilbert equations are a set of 10 equations in Albert Einstein's general theory of relativity which describe the fundamental interaction of gravitation as a result of space time being curved by matter and energy [8]. Einstein recognized that the equivalence of inertial and gravitational mass, so called equivalence principle, is the key for the solution of this problem posed in the case of gravitation. The reformulated theory of gravitation is famous for general theory of relativity. According to Einstein theory of gravitation, the distribution of matter changes the Euclidean structure of Minkowski space time[7]. The Reissner-Nordstrom metric for a charge particle is as follows [9]:

$$ds^2 = Adt^2 - A^{-1}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) \text{.......(1)}$$

with $A = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}$ ....(2)

where M, Q and r be the mass, charge and radial distance respectively in terms of the Euclidean-like structure from the particle centre.

With the normalization fixed by comparison with the Newtonian limit, we can present Einstein's field equations for general relativity [8].

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi GT_{\mu\nu} \text{................................(3)}$$

The above equation tells that how the curvature of space-time reacts to the presence of energy-momentum. Einstein thought that the left-hand side was nice and geometrical, while the right-hand side was somewhat less compelling. Einstein field Equation is the most fundamental equation of general relativity. The equation (3) is regarded as a generalization of Poisson's equation for the Newtonian gravitational potential given by

$$\nabla^2\phi = 4\pi G\rho \text{..(4)}$$

In Newtonian gravity, the rest mass generates gravitational effects. From special theory of relativity, however, we have learned that the rest mass is just one form of energy, and that the mass and energy are equivalent. Therefore, we should expect that in the general theory of relativity, all sources of both energy and momentum contribute to generating space-time curvature. This means that in the general theory of

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 4, Issue 1, July 2014

relativity, the energy-momentum tensor $T_{\mu\nu}$ is the source for space-time curvature in the same sense that the mass density ρ is the source for the potential [10] and it expresses how the presence of energy (mass) source curves space time.

In the presence of cosmological constant Λ, Einstein field equation is given by

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 0 \quad \text{.............................(5)}$$

Einstein's original motivation for introducing the cosmological constant Λ was that it became clear that there were no solutions to his equations representing a static cosmology with non-zero matter content.

III. NEW GRAVITATIONAL FIELD EQUATIONS

Tian Ma and Shouhong Wang (2014) discovered new gravitational field equations [11] with scalar potential under the postulate that the energy momentum tensor T_{ij} needs not to be divergence-free due to the presence of dark energy and dark matter as given by

$$R_{ij} - \frac{1}{2} gijR = -\frac{8\pi G}{c^4} T_{ij} D_i D_j \varphi \quad \text{.......................(6)}$$

This equation is still obeying the three basic principles of the general theory of relativity: the principle of equivalence, the principle of general relativity and the principle of Lagrangian dynamics. The first two principles tell us that the spatial and temporal world is a 4-dimensional Riemannian manifold (M; g_{ij}), where the metric φ represents gravitational potential, and the third principle determines that the Riemannian metric g_{ij} is an extremum point of the Lagrangian action, which is the Einstein-Hilbert functional. This suggests us two important postulates for deriving the new gravitational field equations: The energy-momentum tensor of matter need not to be divergence-free due to the presence of dark energy and dark matter and the field equation obeys the Euler-Lagrange equation of the Einstein-Hilbert functional under the natural divergence-free constraint.

IV. CONDITIONAL VALIDITY OF EQUIVALENCE MASS-ENERGY RELATION

According to the Newtonian mechanics, the principle of conservation of the mass and energy are independent of each other and both have separate laws. In 1905, Albert Einstein proved that the mass and energy are the manifestation of the same things by his famous mass-energy equivalence relation ($E=mc^2$). We know that the mechanical energy is the sum of its potential energy and kinetic energy. This sum of both energies remains constant. When friction is involved, heat energy is accounted for. Because for any given amount of heat produced by friction, an exact proportional amount of energy has to be expanded, we obtain the principle of the equivalence of work and heat ($W \alpha Q$). Thus the principles of conservation of mechanical and thermal energies were merged into one. The conservation principle for both the mass and energy may be extended in chemical and electromagnetic processes and found that in our physical system, the sum of total energies remain constant through all the change. Mass of any body is defined as the resistance that a body opposes to its acceleration (inertial mass). According to this, any interaction would not change the total mass of body; however, special theory of relativity suggests that the rest energy E_0 of a particle is m_0c^2, where m_0 is the rest mass of the particle. For a particle moving with velocity v, the energy of the particle is given by [1]

$$E = mc^2 \quad \text{...(7)}$$

Differentiating, we have

$$dE = dm.c^2 \quad \text{......................................(8)}$$

And the variation of mass with velocity is given by

$$m = \frac{m_0}{\left[1 - \frac{v^2}{c^2}\right]^{\frac{1}{2}}} \quad \text{..................................(9)}$$

$$m = m_0 (1 - v^2/c^2)^{-\frac{1}{2}} \quad \text{.............................(10)}$$

For higher value of the velocity (v), eq^n (10) can be written as

$$m = m_0 (1 + \frac{1}{2} v^2/c^2) \quad \text{..............................(11)}$$

Differentiating with respect to v, we have

$$dm = 2m_0 vdv \;\; / c^2 \quad \text{...............................(12)}$$

Putting the above equation in eq^n(8) and solving, we have

$$dE/dv = m_0 v \quad \text{....................................(13)}$$

The above equation shows that the change in energy of a body with respect to v increases with increasing the velocity and due to increase in velocity, the mass will increase. This justifies the mass-energy equivalence relation.

Eq^n(13) can be written as

$$dE = m_0 vdv \quad \text{....................................(14)}$$

Integrating with proper limit, we have

$$\int dE = \int_0^c m_0 vdv \quad \text{..............................(15)}$$

Solving, we have

$$E = \frac{1}{2} m_0 c^2 \quad \text{....................................(16)}$$

But we know that

$$E = mc^2 \quad \text{...(17)}$$

Equating (16) & (17) and solving, we have

$$m_0 = 2m \quad \text{..(18)}$$

This means that the mass of body becomes half of the rest mass, when it acquires the velocity of light. This contradicts the eq^n (9) and hence eq^n (7).

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 4, Issue 1, July 2014

Experimentally, the conversion of m to $\Delta E = \Delta mc^2$ does occur in radioactive disintegration [1]. However, the reverse formula $\Delta m = \Delta E/c^2$ has never been generally proven [7].

Putting eq^n (18) in the eq^n (9), we have

$$m = \frac{2m}{\left[1-\frac{v^2}{c^2}\right]^{\frac{1}{2}}} \quad \ldots\ldots(19)$$

$$1 = \frac{2}{\left[1-\frac{v^2}{c^2}\right]^{\frac{1}{2}}} \quad \ldots\ldots(20)$$

$$1-\frac{v^2}{c^2} = 4 \quad \ldots\ldots(21)$$

$$\frac{v^2}{c^2} = -3 \quad \ldots\ldots(22)$$

$$\frac{v}{c} = \pm\sqrt{3}i \quad \ldots\ldots(23)$$

$$v = \pm\sqrt{3}ci \quad \ldots\ldots(24)$$

The above equation indicates that the velocity of a particle may be imaginary, when $m_0 = 2m$. In my opinion, the eq^n (24) should be invalid, because it is impossible.

The conditional validity of $E = mc^2$ exposes two misconceptions namely [7]:

1. Gravity would always be attractive to mass since masses attract each other. Such a belief of attractiveness is at the foundation of the theories of black holes.
2. The coupling constants must have the same sign. The unique sign for couplings is the crucial physical assumption for the space-time singularity theorems of Hawking and Penrose.

V. RESULTS AND DISCUSSION

There are some phenomena like clock paradox, advance of perihelion of Mercury planet, deflection of light rays due to gravitational field, red shift of spectral lines etc could not be explained by the special theory of relativity. To explain the above phenomena, Einstein extended this theory to generalize the special theory of relativity with respect to gravitational phenomenon called general theory of relativity.

After the discovery of special and general theory of relativity, a large number of researchers started their works in this direction and discovered some facts which do not agree theoretically and experimentally with mass-energy equivalence relation. Regarding these works, they gave so many theoretical and experimental evidences which proves that the mass-energy equivalence relation ($E=mc^2$) has conditional validity but not unconditional. We have also discussed in brief new gravitational field equations with scalar potential under the postulate that the energy momentum tensor T_{ij} needs not to be divergence-free due to the presence of dark energy and dark matter. Based on mass- energy equivalence relation ($E=mc^2$), Einstein pointed out that an increase of energy E, in amount of energy must be accompanied by an increase E/c^2 in the mass and thus the increased temperature would lead to an increased weight. However, based on the recently discovered charge-mass interaction, it is predicted instead that a heated up matter would have a reduced weight. The reduction in weight never is mean that the mass is certainly reduced or increased. Fan et al. regarded these weight reductions as a result of modifying the mass in Newtonian gravity. When we extended the formula for the variation of mass with velocity with proper operation and certain limit as discussed from equation (7) to (17), we obtain $m_0 = 2m$. This shows that the mass of body becomes half of the rest mass, when it acquires the velocity of light and gives the contradictory result. Further when we have used eq^n (18) is in eq^n (9), then proper operation gives that velocity of a body may be imaginary, if it acquires the velocity of light. This also gives the contradictory result. Hence from the above discussion as proposed by different physicist, we come in conclusion that the mass-energy equivalence relation has conditional validity and unconditional validity is contradictory either by misinterpretation or exact interpretation, because Einstein's field equation is so complicated that the most of the solutions out of 10 set of equations are unsolved. Scientists are trying to discover the problems in these directions. Further research works regarding this will give exact answer.

VI. CONCLUSION

In the study of present review work, we can draw the following conclusion:

1. The extension of formula for the variation out of 10 set of equations are of mass with velocity with proper operation gives imaginary values of the velocity which is not possible.
2. The conditional and unconditional validity of mass-energy equivalence relation is contradictory either by misinterpretation or exact interpretation.
3. Mass-energy equivalence relation is universally correct, but few limitations are exceptional case.
4. Out of 10 set of Einstein field equations, most parts are complicated and unsolved.
5. Both the Einstein field equations and mass-energy equivalence relation are the matter of further research.
6. The new gravitational field equations suggest that the energy-momentum tensor of matter need not to be divergence-free due to the presence of dark energy and dark matter

ACKNOWLEDGEMENT

Authors are grateful to the referee for pointing out the technical errors in the original manuscript and making constructive suggestions. Authors are also grateful to Dr. Neeraj Pant, Associate Professor, Department of

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 4, Issue 1, July 2014

Mathematics, N.D.A. Khadakwasala, Pune for his kindly inspiration.

REFERENCES

[1] Bergmann, PG: Introduction to the Theory of Relativity, Prentice Hall, New Delhi, 1976.

[2] Veltman, MJG: From weak interaction to gravitation, Nobel lecture, December, 1999.

[3] Einstein, A: The Foundation of the General Theory of Relativity. Annalen der Physik**354** (7): 769, 1916.

[4] Ives, Herbert E: Derivation of the mass–energy relation, Journal of the Optical Society of America **42** (8): 540–543, 1952.

[5] Stachel, John; Torretti, Roberto: Einstein's first derivation of mass–energy equivalence, American Journal of Physics **50** (8): 760–763, 1982.

[6] Ohanian, Hans: Did Einstein prove $E=mc^2$?. Studies in History and Philosophy of Science Part B **40** (2): 167–173, 2008.

[7] C. Y. Lo, “On the weight reduction of metals due to temperature increments”, Volume 12, Issue 7, Version 1.0 Year 2012

[8] Peer, A. : Einstein’s field equation, Physics Dep., University College Cork February 17, 2014

[9] Herrera L, Santos NO. and Skea JEF.:Active Gravitational Mass and The Invariant Characterization of Reissner-Nordstrom Space time, February 7, 2008.

[10] http://www.nicadd.niubterz/phys652/Astrophys.

[11] Ma, T and Wang, S: Gravitational field equation and theory of dark matter and dark energy, Discrete and continuous dynamical system, Volume 34, Number 2, February 2014 pp. 335-366.

AUTHOR BIOGRAPHY

Dr. Dipo Mahto (1965): He is assistant Professor in the Department of Physics, Marwari College, Bhagalpur under T.M.B.U. Bhagalpur from 2003. He has passed M.Sc. in 1990 and awarded by PhD degree in 2005 on the topic-Contribution of Mathematical Physics in the development of theory of relativity with special reference to rotating media under the supervision of Dr. Gopi Kant Jha, Former Head & Professor, University Department of Physics of L.N. Mithila University, Darbhanga, India. He is also reviewers of many International Journals. He is invited as resource person for special classes in Physics. His research work is associated with relativity and Astrophysics, especially black holes. There are forty two research papers credit to his name published in repute national and international journals.

Md Shams Nadeem (1982): He is a research scholar in the University Department of Physics, T.M.B.U. Bhagalpur under the supervision of first author. He has passed M. Sc. in 2008 from Magadh University, Bihar, India. His research work is associated with relativity and Astrophysics, specially the gravitational force of super dense stars. There are five papers published in repute international Journals.

Dr. Murlidhar Prasad Singh (1965): He is Assistant Professor in the Department of Physics, B. N. M. College, Barahiya, under T.M.B.U. Bhagalpur, India. He has passed M. Sc. in 1988 from B.R.A. University, Muzaffarpur and awarded by PhD degree in 2013 from the same university. There are three papers published in repute international journals.

Dr. K.M. Singh (1957): He is associate Professor in the Department of Physics, Marwari College, Bhagalpur from 1983 under T.M.B.U. Bhagalpur. He has passed M.Sc. in 1979 from T. M. B. U. Bhagalpur, India and awarded by PhD degree in 1998 on the topic-Multi photon ionization in intense electromagnetic field from Magadh University, Bodh Gaya, India. He is Gold Medalist in B.Sc(H) and M.Sc. His research work is associated with relativity and Astrophysics, especially black holes. There are twenty four research papers published in repute national and international Journals.

Ashok Prasad Yadav (1963): He is a guest lecturer of Physics in G. B. College, in Naugachia, T. M. B. University Bhagalpur India- 812 007. He has passed M. Sc. Physics in 1994 and awarded by Ph. D. degree in 2002 from B. N. Mandal University, Madhepura, Bihar India. He has started his research work with corresponding author.