Comparative Study of MRAC and Fuzzy Control of Two-Tank Interacting Level Process

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Abstract—The industrial process control systems present many challenging control problems due to their non-linear dynamic behavior, uncertain and time varying parameters, constraints on manipulated variable, interaction between manipulated and controlled variables. In this paper, the mathematical model of a two-tank interacting process is obtained using real time data. Proportional Integral (PI) controller, Model Reference Adaptive Controller (MRAC) and Fuzzy Logic Controller (FLC) are designed and implemented for the two-tank interacting process. The PI controller tuning is accomplished using Ziegler Nichols method. Using negative gradient and stability approaches MRAC is design. The Fuzzy logic controller is designed based on Mamdani method. The servo and regulatory responses of the process with these controllers are compared in simulation. It is observed from the results that Fuzzy Logic controller out performs with zero overshoot, faster settling time, better set point tracking and produces lesser Integral square error (ISE) when compared to MRAC and PI controllers.

Keywords—Non-linear, Two-tank interacting process, PI controller, model reference adaptive controller(MRAC) and fuzzy logic controller(FLC).

I. INTRODUCTION

Industries such as petro-chemical industries, paper making industries, waste management and others are the vital industries where liquid level and flow control are essential. Liquids will be processed by chemical or mixing treatment in the tanks, but always the level of fluid in the tanks must be controlled and the flow between tanks must be regulated in the presence of nonlinearity and inexact model description of the plant. PI controllers are popular in industrial applications, as they are easy to install and reasonably robust. However, PI controller wills not giving satisfactory results for non-linear processes. The model based control is very popular nowadays due to the ability of such controllers to handle process effectively. One important type of model based control is model reference adaptive control (MRAC). Adaptive controllers are very effectively handle the unknown parameter variations. Practically system are not precisely linear but may be represented as linearized models around a nominal operating point, the controller parameters tuned at that point may not reflect the real-time system characteristics due to variations in process parameters. So, an adaptive control mechanism is designed for controlling the non-linear tank system [1]. The Fuzzy Logic Controller is well suited for the Level control of two-tank interacting system for which conventional controller is not giving satisfactory result. The fuzzy logic control technology has emerged as one of the most effective nonlinear control technologies used in industrial applications [2]. Fuzzy control has found promising applications for a wide variety of industrial systems based on the universal approximation, many effective fuzzy control scheme have been developed to incorporate with human experts knowledge and information in a systematic way, which can also guarantee various stability and performance criteria [3]. The design of PI, MRAC, FLC are given and the simulation results for servo and regulatory responses are presented in this paper. The paper is organized as follows: Section I discusses about two-tank interacting level process, in Section II and III discusses about model reference adaptive controller (MRAC) and fuzzy logic controller(FLC), respectively. The simulation results are presented in Section IV. The conclusions are given in Section V.

II. TWO-TANK INTERACTING SYSTEM

Fig.1 shows the photograph of the laboratory level process station. It consists of three pumps, two motorized control valves, six process tanks, two overhead tanks, two differential pressure transmitters, five level transmitters and rotameters. Instrumentation panel consists of two PID controllers, main power supply switch, pump switches, motorized control valve switches and auxiliary switches for individual components. Fluid level in the tank is measured by level transmitter (LT). Output of LT is given to the data acquisition setup. It consists of ADC and DAC. The differential pressure level transmitter (DPLT) measures the flow by sensing the difference in level between the tank. The DPLT then transmits a current signal (4-20mA) to the I/V converter. The output of the I/V converter is given to the interfacing hardware associated with the personal computer (PC). A control algorithm is implemented in Lab view software. It compares and takes corrective action on the motorized control valve. Based on the valve opening flow rate is manipulated. Rotameter can visualize the flow rate. The controller compares the controlled variable against set point and generates manipulated variable as current signal (4-20mA). Here the controlled variable is the level (h2) and the manipulated variable is the flow rate(qin).

The Control valve gives restriction to the flow through the pipeline and hence the desired level is achieved.
A. Mathematical modeling of a two-tank interacting process

Consider the process consisting of two interacting liquid tanks in the Fig.2. The volumetric flow into tank1 is \( q_{in} \) (cm\(^3\)/min), the volumetric flow rate from tank1 to tank2 is \( q_1 \) (cm\(^3\)/min), and the volumetric flow rate from tank2 is \( q_2 \) (cm\(^3\)/min). The height of the liquid level is \( h_1 \) (cm) in tank1 and \( h_2 \) in tank2 (cm). Both tanks have the same cross sectional area denotes the area of tank1 is \( A_1 \) (cm\(^2\)) and area of tank2 is \( A_2 \) (cm\(^2\)). \( q_{in} \) is the inflow of tank1 as load disturbance (cm\(^3\)/min) and \( q_1 \) is the inflow of tank2 as load disturbance (cm\(^3\)/min) [4]. The differential equations for tank1 and tank 2 are given in equations (1) and (2).

For tank 1

\[
A_1 \frac{dh_1}{dt} = q_{in} - q_1 \quad (1)
\]

\[
A_1 \frac{dh_1}{dt} = q_1 - \frac{h_1 - h_2}{R_1} \quad (2)
\]

For tank 2

\[
A_2 \frac{dh_2}{dt} = q_1 - q_o \quad (3)
\]

\[
A_2 \frac{dh_2}{dt} = h_1 - h_2 - \frac{h_2}{R_1} - \frac{h_2}{R_2} \quad (4)
\]

\[
\frac{h_2(s)}{q_{in}(s)} = \frac{R_2}{\tau_1 \tau_2 s^2 + s(\tau_1 + \tau_1 R_2 + \tau_2) + 1} \quad (5)
\]

where, \( \tau_1, \tau_2 \) are time constants and \( R_1 \) and \( R_2 \) are restriction of manual valves for tank1 and tank2. \( \tau_1 = A_1 R_1 \), \( \tau_2 = A_2 R_2 \).

Fig.3 shows the experimental open loop response for a step change of 50-60% in flow rate \( q_{in} \). The level of the tank2 is maintained around 13.7 cm initially and raised and reached steady state at 15.6 cm.

From the open loop response the hydraulic resistances \( R_1 \) and \( R_2 \) values are calculated [5]. The hydraulic resistances of tank1 and tank2 for various operating conditions are given in Table I.

<table>
<thead>
<tr>
<th>Operating conditions</th>
<th>Area (cm(^2))</th>
<th>Hydraulic resistance ( (R_1) ) min/cm(^3)</th>
<th>Hydraulic resistance ( (R_2) ) min/cm(^3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30-40%</td>
<td>113.0973</td>
<td>0.004</td>
<td>0.01</td>
</tr>
<tr>
<td>50-60%</td>
<td>113.0973</td>
<td>0.004</td>
<td>0.01</td>
</tr>
</tbody>
</table>

B. Open loop responses for two-tank interacting process

Fig.4 shows the simulated open loop response of interacting process. The level \( h_2 \) changes from 0 to 8.3 cm, when applying a step input in \( q_{in} \). The level \( h_1 \) changes from 0 to 11.6 cm due to interaction. The simulated process reaction curve (PRC) of \( h_2 \) for step change in \( q_{in} \) for \( \pm 499.8 \) cm\(^3\)/min is shown in Fig.5.
Fig. 4. Simulated open loop response of h₁ and h₂ of interacting process.

Fig. 5. Simulated PRC of h₂ for step change in qᵢₐ for ±499.8 cm³/min.

The transfer functions are obtained and tabulated in Table II. From the average transfer function, the controller parameters are obtained using Z-N tuning rule [6]. For two-tank interacting process the PI controller parameters are tabulated in Table III.

III. DESIGN OF MODEL REFERENCE ADAPTIVE CONTROLLER

Model Reference Adaptive Control (MRAC) is a non-identifier technique based on the assumption of the upper bound on the degree of the controlled system. The upper bound on the degree is needed to determine the order and structure of the adaptive controller with sufficient freedom to achieve exact model matching. An adaptive control scheme, with a known relative degree of the process is developed, using only input output measurements. The objective for the design of this method is to track the trajectory of the reference model with Mean Square Error (MSE) minimization [8].

A. Adaptive MIT algorithm

The MRAC with adaptive MIT controller scheme is presented in Fig. 6. It consists of process, reference model, controller and adjustment mechanism.

![Block diagram of MRAC with Adaptive MIT controller](image)

The Adaptive controller has two loops. The inner loop consists of the process and an ordinary feedback controller. The outer loop adjusts the controller parameters in such a way that the error, which is the difference between the process output y and model output yₘ is small [7] [8]. The Reference Model is used to specify the ideal response of the adaptive control system to external command, r to the plant output y is equal to the dynamics of the reference model, yₘ. Matching the plant and the reference model dynamics guarantees the convergence of the modeling error to zero for any given command signal r [9]. Controller is usually parameterized by a number of adjustable parameters. In this paper two parameters θ₁ and θ₂ are used to define the controller law. The control law is linear in terms of the adjustable parameters. To obtain the adaptation mechanism with guaranteed stability and tracking convergence. Adaptation Mechanism is used to adjust the parameters in the control law. To minimize J, the
parameters can be changed in the direction of negative gradient of J. The adaptive MIT algorithms
\[
\frac{d\theta}{dt} = -\gamma \frac{dJ}{d\theta} - \gamma^2 \frac{\partial e}{\partial \theta}
\]  
(6)
where, \(\frac{\partial e}{\partial \theta}\) sensitivity derivative of the error with respect to controller parameter and \(\gamma\) determines the adaptation rate. Based on the a priory knowledge, the reference model is described by equation (7).
\[
\frac{d^2 y_m}{dt^2} + A_1 \frac{dy_m}{dt} + A_2 y_m = K_m r
\]  
(7)
where, \(K_m, A_1\) and \(A_2\) are reference model parameters.
\[
\frac{d^2 y}{dt^2} + a_1 \frac{dy}{dt} + a_2 y = K_p u
\]  
(8)
where, \(K_p, a_1\) and \(a_2\) are the process parameters. By using the control law is given
\[
u = \theta_1 r - \theta_2 y
\]  
(9)
For perfect model following,
\[
\theta_1 = \frac{K_m}{K_p}, \quad \theta_2 = \frac{A_s + A_2 - a_1 s - a_2}{K_p}
\]
The following equations are obtained for updating the controller parameters \(\theta_1\) and \(\theta_2\).
\[
\theta_1 = -\frac{\gamma}{s} e \left(1 \over s^2 + A_1 s + A_2\right) r
\]  
(10)
\[
\theta_2 = \frac{\gamma}{s} e \left(1 \over s^2 + A_1 s + A_2\right) y
\]  
(11)

B. Adaptive Lyapunov Algorithm

The MRAC with adaptive Lyapunov(ALYAP) controller scheme is presented in Fig.7. The Lyapunov stability theory can be used to describe the algorithms for adjusting parameters in Model Reference Adaptive control system. The drawback of MIT rule based MRAS design is that there is no guarantee that the resulting closed loop system will be stable but the Lyapunov theory based MRAS can be designed, which ensures that the resulting closed loop system is stable. There is no filtering of the input and output signals used in the Lyapunov controller [9]. The nonlinear system, the reference model and the control law for this algorithm are described by Equations (7), (8) and (9).

\[
\nabla(e, \theta_1, \theta_2) = \left[\frac{1}{b'y} \left(\frac{a_2 + \theta_2 K_p}{a_1} - \frac{A_2}{A_1}\right)^2 + 1 \left(\frac{K_p \theta_1 - K_M}{A_1}\right)^2 \right]
\]  
(12)
For perfect model following,
\[
\theta_1 = \frac{K_m}{K_p}, \quad \theta_2 = \frac{A_s - a_2}{K_p}
\]
\[
\theta_1 = -\gamma^2 e u_c
\]  
(13)

The controller parameters \(\theta_1\) and \(\theta_2\) are updated the equations (11) and (12).
\[
G_M(s) = \frac{Y_m}{r} = \frac{1}{s^2 + 1.4s + 1}
\]  
(14)
The damping ratio and the natural frequency for the reference model are 0.7 and 1. The adaptation gain \(\gamma\) is chosen as 0.6.

IV. FUZZY LOGIC CONTROLLER

The Fuzzy Logic Controller for a two-tank interacting process is shown in Fig.8.

The fuzzy logic controller are designed with two input variables error and change in error, one output variable. The mamdani based fuzzy inference system uses linear membership function for both inputs and outputs [2]. Triangular membership functions are used for input and output variable [3]. The universe of discourse of error, change in error output are [-12 12]cm, [-6 6]cm and [-1333 1333]cm³/min. The membership function for error, change in error and change in controller output consist of negative big (NB), negative small (NS), zero (Z), positive small (PS)and positive big (PB). The membership diagram for error, change
in error and change in controller output are shown in Fig.9 (a), 9(b) and 9(c), respectively.

![Membership diagram for error](image)

![Membership diagram for change in controller output](image)

**TABLE IV. FAM(CONTROL RULES)**

<table>
<thead>
<tr>
<th>((\Delta e))</th>
<th>NB</th>
<th>NS</th>
<th>Z</th>
<th>PS</th>
<th>PB</th>
</tr>
</thead>
<tbody>
<tr>
<td>NB</td>
<td>NB</td>
<td>NB</td>
<td>NS</td>
<td>NS</td>
<td>Z</td>
</tr>
<tr>
<td>NS</td>
<td>NB</td>
<td>NS</td>
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<tr>
<td>Z</td>
<td>NS</td>
<td>NS</td>
<td>Z</td>
<td>PS</td>
<td>PS</td>
</tr>
<tr>
<td>PS</td>
<td>NS</td>
<td>Z</td>
<td>PS</td>
<td>PS</td>
<td>PB</td>
</tr>
<tr>
<td>PB</td>
<td>Z</td>
<td>PS</td>
<td>PS</td>
<td>PB</td>
<td>PB</td>
</tr>
</tbody>
</table>

**V. SIMULATION RESULTS**

**A. Servo responses of levels with PI, adaptive lyapunov, adaptive MIT and FLC**

Fig.10 shows the set point tracking for level \((h_2)\) with PI, adaptive Lyapunov(ALYAP), adaptive MIT(AMIT) and FLC from 8 to 12cm, 12 to 8cm, 8 to 4cm and 4 to 8cm. The level \(h_1\) also increases from 10.4 to 16cm, 16 to 10.4cm, 10.4 to 4.8cm and 4.8 to 10.4cm due to interaction as shown in Fig.11. Also corresponding controller output \(q_{in}\) is shown in Fig.12. The adaptation of ALYAP and AMIT Controller parameters and modeling error for servo responses are shown in Fig.13, 14. It is observed from figures that the PI, ALYAP, AMIT controller takes more settling time for the level \((h_2)\) and maximum integral square error. The fuzzy logic controller takes less settling time for the level \((h_2)\), better set-point tracking, no overshoot and thereby producing minimum integral square error. The performance measures are tabulated in Table V.
B. Regulatory response of levels with PI, ALYAP, AMIT and FLC (+6% load disturbance from $q_{L2}$)

A sudden load disturbance of +6% is given in inlet flow rate of tank2 at 100th sample from $q_{L2}$ as shown in Fig.2. Due to this level in $h_2$ increases from 8 to 9.5cm(referring Fig.15) and controllers takes necessary action to reduce the flow rate, i.e from 600 to 450cm$^3$/min(referring Fig.17) thereby decreasing $h_1$ from 10.4 to 8.3cm(referring Fig.16). Also corresponding adaptation of controller parameters and modeling error for ALYAP and AMIT controllers are shown in Fig.18, 19. The performance measures are tabulated in Table VI.

**TABLE V**

<table>
<thead>
<tr>
<th>Controller</th>
<th>Servo Response $h_2$ (2-8)cm</th>
<th>Modelling error $h_1$ (cm)</th>
<th>$q_{in}$ (cm$^3$/min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PI</td>
<td>38</td>
<td>168</td>
<td>38</td>
</tr>
<tr>
<td>AMIT</td>
<td>50</td>
<td>464.1</td>
<td>50</td>
</tr>
<tr>
<td>ALYAP</td>
<td>50</td>
<td>463.5</td>
<td>50</td>
</tr>
<tr>
<td>FLC</td>
<td>10</td>
<td>33.64</td>
<td>10</td>
</tr>
</tbody>
</table>

**Fig.13.** Adaptation of ALYAP and AMIT Controller parameters for servo response.

**Fig.14.** Response of ALYAP and AMIT controller Modelling error for servo response.

**Fig.15.** Regulatory response of $h_2$ with PI, ALYAP, AMIT and FLC due to load variation in +6% from $q_{L2}$.

**Fig.16.** Regulatory response of $h_1$ with PI, ALYAP, AMIT and FLC due to load variation in +6% from $q_{L2}$.

**Fig.17.** Response of PI, ALYAP, AMIT and FLC output $q_{in}$ for load variation in +6% from $q_{L2}$.

**Fig.18.** Adaptation of ALYAP and AMIT Controller parameters for load variation in +6% from $q_{L2}$. 

B. Regulatory response of levels with PI, ALYAP, AMIT and FLC (+6% load disturbance from $q_{L2}$)

A sudden load disturbance of +6% is given in inlet flow rate of tank2 at 100th sample from $q_{L2}$ as shown in Fig.2. Due to this level in $h_2$ increases from 8 to 9.5cm(referring Fig.15) and controllers takes necessary action to reduce the flow rate, i.e from 600 to 450cm$^3$/min(referring Fig.17) thereby decreasing $h_1$ from 10.4 to 8.3cm(referring Fig.16). Also corresponding adaptation of controller parameters and modeling error for ALYAP and AMIT controllers are shown in Fig.18, 19. The performance measures are tabulated in Table VI.
C. Regulatory responses of levels with PI, ALYAP, AMIT and FLC (-6% load disturbance from $q_{L2}$)

A sudden load disturbance of -6% is given in inlet flow rate of tank2 at 100th sample from $q_{L2}$ as shown in Fig.2. Due to this level $h_2$ decreases from 8 to 6.5 cm (referring Fig.20) and controller takes necessary action to increase the flow rate, i.e from 600 to 750 cm³/min (referring Fig.22) thereby increasing $h_1$ from 10.4 to 12.5 cm (referring Fig.21). Also corresponding adaptation of controller parameters and modeling error for ALYAP and AMIT controllers are shown in Fig.23, 24. The performance measures are tabulated in Table VI.

To check the speed of the closed loop and the convergence rate of the controllers, the adaptation gains are raised and reduced to 1 and 0.2 for both ALYAP and AMIT controllers. The closed loop with adaptation gain 1 follows the reference model earlier compared to the control loops with adaptation gains 0.2 and 0.6. The responses of $h_2$ with AMIT controller for various adaptation gains are shown in Fig.25. Also corresponding the level $h_1$, controller output, controller parameters and modeling error are shown in Fig.26, 27, 28 and 29. Fig.28 shows convergence rate of the controller parameters is more when adaptation gain increases.
Similarly, the responses $h_2$ with ALYAP controller for various adaptation gains are shown in Fig.30. Also corresponding the level $h_1$, controller output, adaptation of controller parameters and modelling error are shown in Fig.31, 32, 33 and 34. It is observed that when adaptation gain increases $h_2$ settled earlier.
earlier. A major drawback in the AMIT rule is that it does not guarantee the stability of the system, but the Lyapunov rule assures the stability of the closed loop system. When comparing the performance of PI, ALYAP and AMIT and fuzzy logic controller, it is observed that fuzzy logic controller gives better performance than PI, ALYAP and AMIT controllers for both servo and regulatory problems in terms of less integral square error, faster settling time and better set-point tracking. Therefore the fuzzy logic controller is working properly for both servo and regulatory problems of two-tank interacting process.

REFERENCES


VI. CONCLUSION

In this work, the conventional PI, ALYAP, AMIT and fuzzy logic controllers are developed for a two-tank interacting process. It is inferred that both ALYAP and AMIT controllers when the adaptation gain increases the convergence of the controller parameters reaches the steady state value earlier and the process and the model output settles

For two-tank interacting process the closed loop with adaptation gain 1 follows the reference model earlier when compared to the closed loop with adaptation gain 0.2 and 0.6 for both ALYAP and AMIT controllers.

VI. COMPARISON OF PERFORMANCE MEASURES OF LEVEL WITH PI, ALYAP, AMIT AND FLC FOR REGULATORY RESPONSE

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<tr>
<th>Controller</th>
<th>Regulatory response</th>
<th>$h_2(2-8)\text{cm}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$t_e$ (sec)</td>
<td>ISE</td>
</tr>
<tr>
<td>PI</td>
<td>20</td>
<td>10.5</td>
</tr>
<tr>
<td>ALYAP</td>
<td>30</td>
<td>134.1</td>
</tr>
<tr>
<td>AMIT</td>
<td>30</td>
<td>134.8</td>
</tr>
<tr>
<td>FLC</td>
<td>9</td>
<td>1.782</td>
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Dr. D. Rathikanu received her B.E., Instrumentation and Control from Govt. College of Technology, Coimbatore in 1988, M.E., Instrumentation from MIT, campus, Anna University in 1998 and Ph.D. in Process control and instrumentation from Anna University in 2011. Currently working as a Professor in the Dept. of Instrumentation Engg., Annamalai University. Her area of interests includes Digital watermarking, process control, adaptive control.