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# Thermo elastic Solution of a Thin Circular Plate due to Partially Distributed Heat Supply

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***Abstract*** ***- In this paper, an attempt has been made to study thermoelastic response of a thin circular plate occupying the space*** $D:0\le r\le a,\ -h\le z\le h$, ***due to heat generation with radiation type boundary conditions. We apply integral transform technique to find the thermoelastic solution.***

***Keywords:*** **Thermo elastic Response, Circular Plate, Marchi-Fasulo transform and Laplace transform.**

## I. INTRODUCTION

The direct and inverse problems of thermoelasticity of thick circular plate are considered by Nowacki [2]. The quasi-static thermal stresses in circular plate subjected to arbitrary initial temperature on the upper face with lower face at zero temperature has determined by Wankhede [5]. Noda et al.[3] has succeeded in determining the quasi-static thermal stresses in a circular plate subjected to transient temperature along the circumference of circular upper face with lower face at zero temperature and the fixed circular edge thermally insulated.

In all aforementioned investigation they have not considered any thermoelastic problem with radiation type boundary conditions. This paper is concerned with transient thermoelastic problem of a thin circular plate occupying the space $D:0\le r\le a,\ -h\le z\le h$, due to heat generation with radiation type boundary conditions.

## II. STATEMENT OF THE PROBLEM

Consider a thin circular plate of thickness 2h occupying the space $D:0\le r\le a,\ -h\le z\le h.$ The differential equation governing the displacement function $U(r,z,t)$ as [2] is,

$$\frac{\partial^2 U}{\partial r^2}+\frac{1}{r}\frac{\partial U}{\partial r}=(1+\nu)a_t T \tag{1}$$

with $\dfrac{\partial U}{\partial r}=0$ at $r=a$ (2)

Where $\nu$ and $a_t$ are the Poisson ratio and the linear coefficient of thermal expansion of the material of the plate and T is the temperature of the plate satisfying the differential equation [**3**]:

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}+\frac{\theta(r,z,t)}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{3}$$

Subject to the initial conditions

$$T(r,z,0)=F(r,z) \tag{4}$$

and the boundary conditions.

$$[T(r,z,t)]_{r=a}=g(z,t)\quad \text{(Unknown)} \tag{5}$$

$$\left[T(r,z,t)+k_1\frac{\partial T(r,z,t)}{\partial z}\right]_{z=h}=f_1(r,t) \tag{6}$$

$$\left[T(r,z,t)+k_2\frac{\partial T(r,z,t)}{\partial z}\right]_{z=-h}=\left(\frac{-Q_o}{\lambda}\right)f_2(r,t) \tag{7}$$

$$[T(r,z,t)]_{r=\xi}=f(z,t)\quad \text{(Known)} \tag{8}$$

Where $k_1$ and $k_2$ are the radiation constants on the two plane surfaces. The stress functions $\sigma_{rr}$ and $\sigma_{\theta\theta}$ are given by [2]

$$\sigma_{rr}=-2\mu\frac{1}{r}\frac{\partial U}{\partial r} \tag{9}$$

$$\sigma_{\theta\theta}=-2\mu\frac{\partial^2 U}{\partial r^2} \tag{10}$$

Where $\mu$ is the Lame's constant, while each of the stress functions $\sigma_{rz}$, $\sigma_{zz}$, $\sigma_{\theta z}$ are zero within the plane in the plane state of stress. The equation (1) to (10) constitute the mathematical formulation of the problem under consideration.

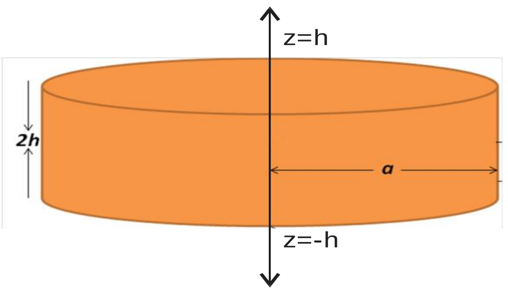


**Fig 1: Shows the geometry of the problem**

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## III. SOLUTION OF THE PROBLEM

Applying finite Marchi-Fasulo integral transform [**1**] and Laplace transform [**4**] to the equations (3) to (5) one obtains

$$\bar{T}^*(r,n,s) = AI_0(qr) + BK_0(qr) + P.I. \qquad (11)$$

Where

$$q^2 = a_n^2 - \frac{1}{k} + \frac{s}{\alpha},$$

$\bar{T}$ denotes the Marchi Fasulo Transform of $T$ and $n$ is

Marchi Fasulo transform parameter.

$A$, $B$ are constants and $I_0, k_0$ are modified Bessel's function of first and second kind of order zero respectively.

As $r \to 0,\ k_0 \to \infty$ but by physical consideration $\bar{T}^*(r,n,s)$ remains finite.

Therefore $B$ must be zero.

Using equation

$$\left[\bar{T}^*(r,n,s)\right]_{r=\xi} = \bar{f}^*(n,s) \qquad (12)$$

And equation (11) one obtains the value of A and B. Substituting the values of $A$ and $B$ in (11) and taking inversion of Laplace transform and finite Marchi Fasulo integral transform leads to

$$T(r,z,t) = \frac{2k}{\xi}\sum_{n=1}^{\infty}\frac{P_n(z)}{\lambda_n}\sum_{m=1}^{\infty}\frac{\lambda_m J_0(\lambda_m r)}{J_1(\lambda_m \xi)} \times \int_0^t [\bar{f}(n,t') - \psi(a)]\, e^{-k(\lambda_m^2 + a_n^2)(t-t')} dt' + \Phi \qquad (13)$$

$$g(z,t) = \frac{2k}{\xi}\sum_{n=1}^{\infty}\frac{P_n(z)}{z}\sum_{m=1}^{\infty}\frac{\lambda_m J_0(\lambda_m a)}{J_1(\lambda_m \xi)} \times \int_0^t [\bar{f}(n,t') - \psi(a)]\, e^{-k(\lambda_m^2 + a_n^2)(t-t')} dt' + \Phi \qquad (14)$$

Where

$$\Phi = \sum_{n=1}^{\infty}\frac{P_n(z)}{\lambda_n}\,\phi(r,z,t),$$

$\lambda_m$ are positive roots of transcendental equation $J_0(\lambda_m a) = 0$ and

$$\psi(a) = P.I,\quad \phi(r,z,t) = L^{-1}(P.I)$$

$$P_n(z) = Q_n\cos(a_n z) - W_n\sin(a_n z)$$

$$Q_n = a_n(\alpha_1 + \alpha_2)\cos(a_n h) + (\beta_1 - \beta_2)\sin(a_n h)$$

$$W_n = (\beta_1 + \beta_2)\cos(a_n h) + (\alpha_2 - \alpha_1)a_n\sin(a_n h)$$

Equations (13) and (14) are the desired solutions of the given problem with $\beta_1 = \beta_2 = 1$ and $\alpha_1 = k_1,\ \alpha_2 = k_2$.

## IV. THERMOELASTIC DISPLACEMENT FUNCTION

Substituting the value of $T(r,z,t)$ from equation (13) in equation (1) one obtains the thermoelastic displacement function $U(r,z,t)$ as

$$U(r,z,t) = (1+\nu)a_t\frac{2k}{\xi}\sum_{n=1}^{\infty}\frac{P_n(z)}{\lambda_n}\sum_{m=1}^{\infty}\frac{J_0(\lambda_m r)}{\lambda_m J_1'(\lambda_m \xi)} \times \int_0^t [\bar{f}(n,t') - \psi(a)]\, e^{-k(\lambda_m^2 + a_n^2)(t-t')} dt' + \Phi \qquad (15)$$

## V. DETERMINATIONOF STRESS FUNCTION

Using equation (15) in equations (9) and (10), the stress functions are obtained as

$$\sigma_{rr} = -(1+\nu)a_t\frac{4\mu k}{r\xi}\sum_{n=1}^{\infty}\frac{P_n(z)}{\lambda_n}\sum_{m=1}^{\infty}\frac{J_0'(\lambda_m r)}{J_1(\lambda_m \xi)} \times \int_0^t [\bar{f}(n,t') - \psi(a)]\, e^{-k(\lambda_m^2 + a_n^2)(t-t')} dt' + \Phi' \qquad (16)$$

$$\sigma_{\theta\theta} = -(1+\nu)a_t\frac{4\mu k}{\xi}\sum_{n=1}^{\infty}\frac{P_n(z)}{\lambda_n}\sum_{m=1}^{\infty}\frac{\lambda_m J_0''(\lambda_m r)}{J_1(\lambda_m \xi)} \times \int_0^t [\bar{f}(n,t') - \psi(a)]\, e^{-k(\lambda_m^2 + a_n^2)(t-t')} dt' + \Phi'' \qquad (17)$$

## VI. SPECIAL CASE AND NUMERICAL RESULTS

Set $f(z,t) = e^{t+z}z^2$

By applying Marchi-Fasulo transform, we get

$$\bar{f}(z,t) = \frac{2e^{t+\xi}}{a_n^2}(a_n^2 z - 2z - 2)\sin(2a_n h)$$

Substituting this value in equation (14) we obtain

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$$g(z,t)=\frac{2k}{\xi}\sum_{n=1}^{\infty}\frac{P_n(z)}{\lambda_n}\sum_{m=1}^{\infty}\frac{\lambda_m J_0(\lambda_m a)}{J_1(\lambda_m \xi)}$$

$$\times\int_0^t\left[\frac{2e^{t+\xi}}{a_n^2}(a_n^2-2z-2)\sin(2a_n h)-\psi\right]e^{-k(\lambda_m^2+a_n^2)(t-t')}dt'$$

$$+\Phi \qquad (18)$$

Put $\frac{2k}{\xi}=\alpha, a=2, h=1, t=1\sec$, we get

$$\frac{g(z,t)}{\alpha}=\sum_{n=1}^{\infty}\frac{P_n(z)}{\lambda_n}\sum_{m=1}^{\infty}\frac{\lambda_m J_0(2\lambda_m)}{J_1(\lambda_m \xi)}$$

$$\times\int_0^1\left[\frac{2e^{1+\xi}}{a_n^2}(a_n^2-2z-2)\sin(2a_n)-\psi\right]e^{-k(\lambda_m^2+a_n^2)(t-t')}dt'$$

$$+\frac{1}{\alpha}\sum_{n=1}^{\infty}\frac{P_n(z)}{\lambda_n}\phi(r,z,t) \qquad (19)$$

## VII. CONCLUSION

The temperature distribution, unknown temperature gradient, displacement function and thermal stresses of a thin circular plate have been investigated with known boundary conditions. Finite integral transform techniques have been used to obtain numerical results. Any particular cases of special interest can be assigned to the parameters and functions in expressions. The results that are obtained can be useful to the design of structure or machines in engineering applications.

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## APPENDIX

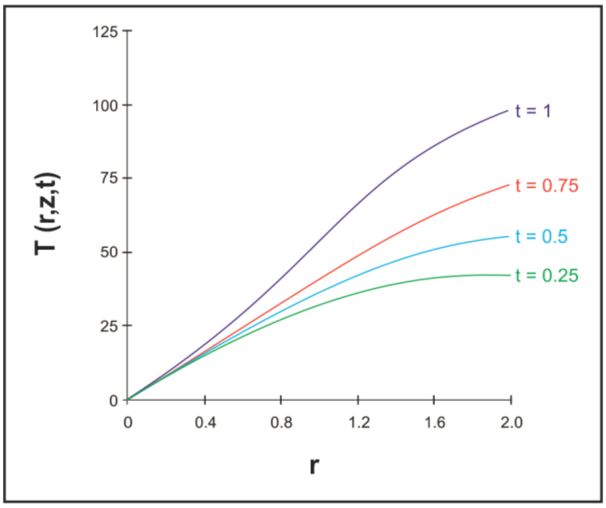


**Fig. 1: Graph of equation (13)**

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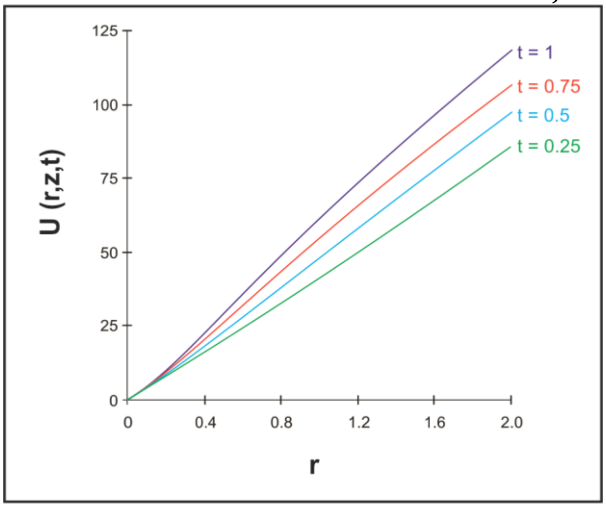


**Fig. 2: Graph of equation (4.1)**

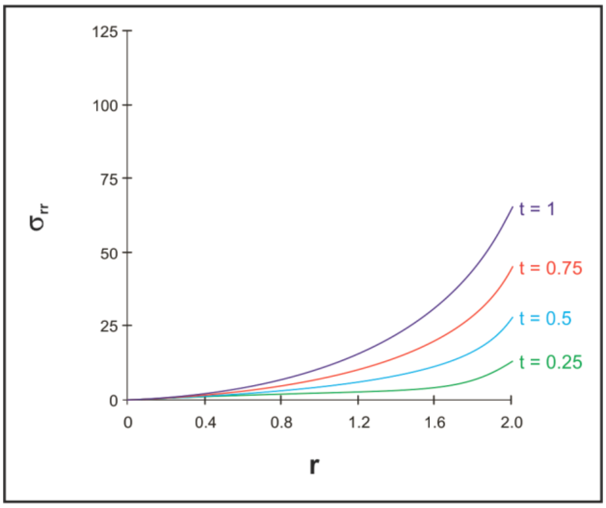


**Fig. 3 : Graph of equation (5.1)**