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Solution of Linear Programming Problem by New Approach

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Abstract- In this paper, an alternative method for simplex method is introduced. This method is easy to solve linear programming problem. It is powerful method to reduce number of iterations and save valuable time.

Key words: **Linear programming problem, optimal solution, simplex method, alternative method.**

I. INTRODUCTION

Dantzig's [1] suggestion is to choose that entering vector corresponding to which $z_j - c_j$ is most negative. Khobragade et al. [2, 3, 4] suggested an alternative approach to solve linear programming problem. In this paper, an attempt has been made to solve linear programming problem (LPP) by new method which is an alternative for simplex method. This method is different from Khobragade et al. Method.

II. AN ALTERNATIVE ALGORITHM FOR SIMPLEX METHOD

To find optimal solution of any LPP by an alternative method for simplex method, algorithm is given as follows:

Step 1. Check objective function of LPP is of maximization or minimization type. If it is to be minimization type then convert it into a maximization type by using the result:

$$\text{Min. } Z = -\text{ Max.}(-Z).$$

Step 2. Check whether all b_i (RHS) are non-negative. If any b_i is negative then multiply the corresponding equation of the constraints by -1.

Step 3. Express the given LPP in standard form then obtain initial basic feasible solution.

Step 4. Choose greatest coefficient of decision variables.
(i) If greatest coefficient is unique, then element corresponding to this row and column becomes pivotal (leading) element.

(ii) If greatest coefficient is not unique, then use tie breaking technique.

Step 5. Use usual simplex method for this table and go to next step.

Step 6. Ignore corresponding row and column. Proceed to step 4 for remaining elements and repeat the same procedure until an optimal solution is obtain or there is an indication for unbounded solution.

Step 7. If all rows and columns are ignored, then current solution is an optimal solution.

PROBLEM 1:

Maximize $Z = 4x_1 + 3x_2 + 4x_3 + 6x_4$

Subject to:

$$x_1 + 2x_2 + 2x_3 + 4x_4 \le 80$$
$$2x_1 + \quad 2x_3 + x_4 \le 60$$
$$3x_1 + 3x_2 + x_3 + x_4 \le 80.$$
$$x_1,\ x_2,\ x_3,\ x_4 \ge 0.$$

SOLUTION:

LPP is in standard form:

Maximize $Z =$
$4x_1 + 3x_2 + 4x_3 + 6x_4 + 0s_1 + 0s_2 + 0s_3$

Subject to:

$$x_1 + 2x_2 + 2x_3 + 4x_4 + s_1 = 80$$
$$2x_1 + \quad 2x_3 + x_4 + s_2 = 60$$
$$3x_1 + 3x_2 + x_3 + x_4 + s_3 = 80.$$
$$x_1,\ x_2,\ x_3,\ x_4, s_1,\ s_2,\ s_3 \ge 0.$$

where $s_1,\ s_2, s_3$ are slack variables.

Simplex table:

C_B	BVS	X_B	x_1	x_2	x_3	x_4	s_1	s_2	s_3
0	s_1	80	1	2	2	4	1	0	0
0	s_2	60	2	0	2	1	0	1	0
0	s_3	80	3	3	1	1	0	0	1
6	x_4	20	1/4	1/2	1/2	1	1/2	0	0
0	s_2	40	7/4	-1/2	3/2	0	-1/4	1	0
0	s_3	60	11/4	5/2	1/2	0	-1/4	0	1
6	x_4	160/11	0	3/11	5/11	1	3/11	0	-1/11
0	s_2	20/11	0	-23/11	13/11	0	-1/11	1	-7/11
4	x_1	240/11	1	10/11	2/11	0	-1/11	0	4/11
6	x_4	180/13	0	14/13	0	1	4/13	-5/13	2/13
4	x_3	20/13	0	-23/13	1	0	-1/13	11/13	-7/13
4	x_1	280/13	1	16/13	0	0	-1/13	-2/13	6/13

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Since all rows and column are ignored, hence an optimum solution has been reached. Therefore optimum solution is:

$x_1 = \frac{280}{13},\ x_2 = 0,\ x_3 = \frac{20}{13}, x_4 = \frac{180}{13}$. Max. $Z = \frac{2280}{13}$.

PROBLEM 2:

Minimize $Z = 3x_1 - 7x_2 + 5x_3$

Subject to:
$$5x_1 - x_2 + 4x_3 \leq 15$$
$$-3x_1 + 4x_2 \leq 8$$
$$4x_1 + 3x_2 - 8x_3 \leq 31.$$
$$x_1,\ x_2,\ x_3 \geq 0.$$

SOLUTION:

LPP is in standard form:

Min Z = - Max.$(-Z)$, $-Z = Z^*$

Max. $Z^* = -3x_1 + 7x_2 - 5x_3 + 0s_1 + 0s_2 + 0s_3$

Subject to:
$$5x_1 - x_2 + 4x_3 + s_1 = 15$$
$$-3x_1 + 4x_2 + s_2 = 8$$
$$4x_1 + 3x_2 - 8x_3 + s_3 = 31.$$
$$x_1,\ x_2,\ x_3,\ x_4, s_1,\ s_2,\ s_3 \geq 0.$$

where $s_1,\ s_2, s_3$ are slack variables.

Simplex table:

C_B	BVS	X_B	x_1	x_2	x_3	s_1	s_2	s_3
0	s_1	15	5	-1	4	1	0	0
0	s_2	8	-3	4	0	0	1	0
0	s_3	31	4	3	-8	0	0	1
-3	x_1	3	1	-1/5	4/5	1/5	0	0
0	s_2	17	0	17/5	0	3/5	1	0
0	s_3	19	0	19/5	-56/5	-4/5	0	1
-3	x_1	4	1	0	4/19	3/19	0	1/19
0	s_2	0	0	0	952/95	25/19	1	-17/19
7	x_2	5	0	1	-56/19	-4/19	0	5/19
-3	x_1	4	1	0	0	124/952	-20/952	68/952
-5	x_3	0	0	0	1	125/952	95/952	-85/952
7	x_2	5	0	1	0	168/952	280/952	-4760/19(952)

Since all rows and column are ignored, hence an optimum solution has been reached. Therefore optimum solution is:

$x_1 = 4,\ x_2 = 5,\ x_3 = 0$. Min. $Z = -23$.

PROBLEM 3:

Maximize $Z = 2x_1 + 5x_2 + 3x_3$

Subject to:
$$-x_1 + 2x_2 + x_3 \leq 30$$
$$3x_1 + 4x_2 + 2x_3 \leq 60$$
$$2x_1 + x_2 + 2x_3 \leq 40$$
$$x_1 + 7x_2 + 2x_3 \leq 80$$
$$-2x_1 - 2x_2 + 2x_3 \leq 20.$$
$$x_1,\ x_2,\ x_3 \geq 0.$$

SOLUTION:

LPP is in standard form:

Maximize $Z = 2x_1 + 5x_2 + 3x_3 + 0s_1 + 0s_2 + 0s_3 + 0s_4 + 0s_5$

Subject to:
$$-x_1 + 2x_2 + x_3 + s_1 = 30$$
$$3x_1 + 4x_2 + 2x_3 + s_2 = 60$$
$$2x_1 + x_2 + 2x_3 + s_3 = 40$$
$$x_1 + 7x_2 + 2x_3 + s_4 = 80$$
$$-2x_1 - 2x_2 + 2x_3 + s_5 = 20.$$
$$x_1,\ x_2,\ x_3,\ s_1,\ s_2,\ s_3,\ s_4,\ s_5 \geq 0.$$

where $s_1,\ s_2, s_3,\ s_4, s_5$ are slack variables.

Simplex table:

C_B	BVS	X_B	x_1	x_2	x_3	s_1	s_2	s_3	s_4	s_5
0	s_1	30	-1	2	1	1	0	0	0	0
0	s_2	60	3	4	2	0	1	0	0	0
0	s_3	40	2	1	2	0	0	1	0	0
0	s_4	80	1	7	2	0	0	0	1	0
0	s_5	20	-2	-2	2	0	0	0	0	1
0	s_1	50/7	-9/7	0	3/7	1	0	0	-2/7	0
0	s_2	100/7	17/7	0	6/7	0	1	0	-4/7	0
0	s_3	200/7	13/7	0	12/7	0	0	1	-1/7	0
5	x_2	80/7	1/7	1	2/7	0	0	0	1/7	0
0	s_5	300/7	-12/7	0	18/7	0	0	0	2/7	1
0	s_1	0	-1	0	0	1	0	0	-1/3	-1/6
0	s_2	0	3	0	0	0	1	0	-2/3	-1/3
0	s_3	0	3	0	0	0	0	1	-1/3	-2/3
5	x_2	20/3	1/3	1	0	0	0	0	1/9	-1/9
3	x_3	50/3	-2/3	0	1	0	0	0	1/9	7/18
0	s_1	0	0	0	0	1	0	1/3	-4/9	-7/18
0	s_2	0	0	0	0	0	1	-1	-1/3	1/3
2	x_1	0	1	0	0	0	0	1/3	-1/9	-2/9
5	x_2	20/3	0	1	0	0	0	-1/9	4/27	-1/27
3	x_3	50/3	0	0	1	0	0	2/9	1/27	13/54

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Since all rows and column are ignored, hence an optimum solution has been reached. Therefore optimum solution is:

$x_1 = 0,\ x_2 = \frac{20}{3},\ x_3 = \frac{50}{3}$. Max. $Z = \frac{250}{3}$.

IV. ALTERNATIVE ALGORITHM FOR BIG-M METHOD

To find optimal solution of any LPP by an alternative method for Big-M method, algorithm is given as follows:

Step 1. Check objective function of LPP is of maximization or minimization type. If it is to be minimization type then convert it into a maximization type by using the result:

$$\text{Min. } Z = -\text{ Max.}(-Z).$$

Step 2. Check whether all b_i (RHS) are non-negative. If any b_i is negative then multiply the corresponding equation of the constraints by -1.

Step 3. Express the given LPP in standard form then obtain initial basic feasible solution.

If basic solution is non-feasible due to the constraints of the type $\geq$ and $=$ then we add artificial variable to the corresponding constraint in standard form. Assign to very large value $+M$ for maximization and $-M$ for minimization in objective function.

Step 4. Choose greatest coefficient of decision variables. (i) If greatest coefficient is unique, then variable corresponding to this column becomes incoming variable.

(ii) If greatest coefficient is not unique, then use tie breaking technique.

Step 5. Compute the ratio with X_B. Choose minimum ratio, then variable corresponding to this row is outgoing variable. The element corresponding to incoming variable and outgoing variable becomes pivotal (leading) element.

Step 6. Use usual simplex method for this table and go to next step.

Step 7. Ignore corresponding row and column. Proceed to step 4 for remaining elements and repeat the same procedure until an optimal solution is obtain or there is an indication for unbounded solution.

Step 8: If all rows and columns are ignored, then current solution is an optimal solution.

PROBLEM 4:

Maximize $Z = 5x_1 - 2x_2 + 3x_3$

Subject to: $2x_1 + 2x_2 - x_3 \geq 2$

$3x_1 - 4x_2 \leq 3$

$x_2 + 2x_3 \leq 5.$

$x_1,\ x_2,\ x_3 \geq 0.$

SOLUTION:

LPP is in standard form:

Maximize Z = $5x_1 - 2x_2 + 3x_3 + 3x_3$

$Z = 5x_1 - 2x_2 + 3x_3 + 0s_1 + 0\,s_2 + 0s_3 - Ma_1$

Subject to: $2x_1 + 2x_2 - x_3 - s_1 + a_1 = 2$

$3x_1 - 4x_2 + s_2 = 3$

$x_2 + 3x_3 + s_3 = 5$

$x_1,\ x_2,\ x_3,\ s_1,\ s_2,\ s_3 \geq 0.$

where $s_1,\ s_2, s_3$ are slack variables and a_1 is artificial variable.

Simplex table:

C_B	BVS	X_B	x_1	x_2	x_3	s_1	s_2	s_3	a_1	Ratio
-M	a_1	2	**2**	2	-1	-1	0	0	1	1 →
0	s_2	3	3	-4	0	0	1	0	0	1
0	s_3	5	0	1	3	0	0	1	0	5
5	x_1	1	1	1	-1/2	-1/2	0	0	1/2	
0	s_2	0	0	-7	**3/2**	3/2	1	0	-3/2	0 →
0	s_3	5	0	1	3	0	0	1	0	5/3
5	x_1	1	1	-4/3	0	0	1/3	0	0	
3	x_3	0	0	-14/3	1	1	2/3	0	-1	
-2	s_3	5	0	15	0	-3	-2	1	3	1/3 →
5	x_1	13/9	1	0	0	-4/15	7/45	4/45	4/15	
0	x_3	14/9	0	0	1	1/15	2/45	14/45	-1/15	70/3
-2	x_2	1/3	0	1	0	-1/15	-2/45	1/15	1/5	
5	x_1	23/3	1	0	4	0	1/3	4/3	0	
0	s_1	70/3	0	0	15	1	2/3	14/3	-1	
-2	x_2	5	0	1	3	0	0	1	2/15	

Since all rows and column are ignored, hence an optimum solution has been reached. Therefore optimum solution is:

$x_1 = \frac{23}{3},\ x_2 = 5,\ x_3 = 0.$ Max. $Z = \frac{85}{3}$.

PROBLEM 5:

Maximize $Z = x_1 + 2x_2 + 3x_3 - x_4$

Subject to: $x_1 + 2x_2 + 3x_3 = 15$

$2x_1 + x_2 + 5x_3 = 20$

$x_1 + 2x_2 + 2x_3 + x_4 = 10.$

$x_1,\ x_2,\ x_3,\ x_4 \geq 0.$

SOLUTION:

LPP is in standard form:

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Maximize
$Z = x_1 + 2x_2 + 3x_3 - x_4 - Ma_1 - Ma_2$

Subject to: $x_1 + 2x_2 + 3x_3 + a_1 = 15$
$2x_1 + x_2 + 5x_3 + a_2 = 20$
$x_1 + 2x_2 + 2x_3 + x_4 = 10.$
$x_1, x_2, x_3, x_4, a_1, a_2 \geq 0.$

where a_1, a_2 are artificial variables.

Simplex table:

C_B	BVS	X_B	x_1	x_2	x_3	x_4	a_1	a_2	Ratio
-M	a_1	15	1	2	3	0	1	0	5
-M	a_2	20	2	1	**5**	0	0	1	4 →
-1	x_4	10	3	2	1	1	0	0	10
-M	a_1	3	-1/5	**7/5**	0	0	1	-3/5	15/7 →
3	x_3	4	2/5	1/5	1	0	0	1/5	
-1	x_4	6	3/5	9/5	0	1	0	-1/5	10
2	x_2	15/7	-1/7	1	0	0	5/7	-3/7	
3	x_3	25/7	3/7	0	1	0	-1/7	2/7	
-1	x_4	15/7	**6/7**	0	0	1	-9/7	4/7	5/2 →
2	x_2	5/2	0	1	0	1/6	1/2	-1/3	
3	x_3	5/2	0	0	1	1/2	1/2	0	
1	x_1	5/2	1	0	0	-7/6	-3/2	2/3	

Since all rows and column are ignored, hence an optimum solution has been reached. Therefore optimum solution is:

$x_1 = x_2 = x_3 = \frac{5}{2}$. Max. $Z = 15$.

PROBLEM 6:

Minimize $Z = 4x_1 + 2x_2$

Subject to: $3x_1 + 2x_2 \geq 27$
$x_1 + x_2 \geq 21$
$x_1 + 2x_2 \geq 30.$
$x_1, x_2 \geq 0.$

SOLUTION:

Min Z = - Max.$(-Z)$, $-Z = Z^*$

LPP is in standard form:

Maximize $Z^* = -4x_1 - 2x_2 + 0s_1 + 0s_2 + 0s_3 - Ma_1 - Ma_2 - Ma_3$

Subject to: $3x_1 + 2x_2 - s_1 + a_1 = 27$
$x_1 + x_2 - s_2 + a_2 = 21$
$x_1 + 2x_2 - s_3 + a_3 = 30.$
$x_1, x_2, x_3, x_4, a_1, a_2 \geq 0.$

where s_1, s_2, s_3 are slack variables and a_1, a_2 are artificial variables.

Simplex table:

C_B	BVS	X_B	x_1	x_2	s_1	s_2	s_3	a_1	a_2	a_3	Ratio
-M	a_1	27	**3**	1	-1	0	0	1	0	0	9 →
-M	a_2	21	1	1	0	-1	0	0	1	0	21
-M	a_3	30	1	2	0	0	-1	0	0	1	30
-4	x_1	9	1	1/3	-1/3	0	0	1/3	0	0	
-M	a_2	12	0	2/3	1/3	-1	0	-1/3	1	0	18
-M	a_3	21	0	**5/3**	1/3	0	-1	-1/3	0	1	63/5 →
-4	x_1	24/5	1	0	-2/5	0	1/5	2/5	0	-1/5	
-M	a_2	18/5	0	0	1/5	-1	**2/5**	-1/5	1	-2/5	9 →
-2	x_2	63/5	0	1	1/5	0	-3/5	-1/5	0	3/5	
-4	x_1	3	1	0	-1/2	1/2	0	1/2	-1/2	0	
0	s_3	9	0	0	1/2	-5/2	1	-1/2	5/2	-1	
-2	x_2	18	0	1	1/2	-3/2	0	-1/2	3/2	0	

Since all rows and column are ignored, hence an optimum solution has been reached. Therefore optimum solution is:

$x_1 = 3, \; x_2 = 18$. Min. $Z = 48$.

V. ALTERNATIVE ALGORITHM FOR DUAL SIMPLEX METHOD

To find optimal solution of any LPP by an alternative method for dual simplex method, algorithm is given as follows:

Step 1. The objective function of the LPP must be maximize. If it is minimize then convert it into maximize by using the result:

$$\text{Min. } Z = -\text{ Max.}(-Z).$$

Step 2. Convert all $\geq$ constraints into $\leq$ by multiplying the corresponding equation of the constraints by -1.

Step 3. Convert inequality constraints into equality by addition of slack variables and obtain an initial basic

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solution. Express the above information in the form of a table known as dual simplex table.

Step 4. Choose most negative X_B , then variable corresponding to this row becomes outgoing variable. Select the most negative coefficient of decision variables in that row, and then variable corresponding to this column becomes incoming variable. The element corresponding to incoming variable and outgoing variable is pivotal (leading) element.

Step 5. Use usual simplex method for this table and go to next step.

Step 6. Ignore corresponding row and column. Proceed to step 4 for remaining elements and repeat the same procedure until an optimal solution is obtained in finite number steps or there is an indication of the non-existence of a feasible solution.

Step 7: If all rows and columns are ignored, then current solution is an optimal solution.

PROBLEM 7:

Minimize $Z = 80x_1 + 60x_2 + 80x_3$

Subject to: $x_1 + 2x_2 + 3x_3 \geq 4$

$2x_1 + \quad 3x_3 \geq 3$

$2x_1 + 2x_2 + x_3 \geq 4$

$4x_1 + x_2 + x_3 \geq 6.$

$x_1, x_2, x_3 \geq 0.$

SOLUTION:

Min. Z = - Max.$(-Z)$, $-Z = Z^*$

Max. $Z^* = -80x_1 - 60x_2 - 80x_3$

Subject to: $-x_1 - 2x_2 - 3x_3 \leq -4$

$-2x_1 \quad - 3x_3 \leq -3$

$-2x_1 - 2x_2 - x_3 \leq -4$

$-4x_1 - x_2 - x_3 \leq -6.$

$x_1, x_2, x_3 \geq 0.$

LPP in standard form:

Max. $Z^* = -80x_1 - 60x_2 - 80x_3 + 0s_1 + 0s_2 + 0s_3 + 0s_3 + 0s_4$

Subject to: $-x_1 - 2x_2 - 3x_3 + s_1 = -4$

$-2x_1 \quad - 3x_3 + s_2 = -3$

$-2x_1 - 2x_2 - x_3 + s_3 = -4$

$-4x_1 - x_2 - x_3 + s_4 = -6.$

$x_1, x_2, x_3, s_1, s_2, s_3, s_4 \geq 0.$

where s_1, s_2, s_3, s_4 are slack variables.

Simplex table:

C_B	BVS	X_B	x_1	x_2	x_3	s_1	s_2	s_3	s_4
0	s_1	-4	-1	-2	-3	1	0	0	0
0	s_2	-3	-2	0	-3	0	1	0	0
0	s_3	-4	-2	-2	-1	0	0	1	0
0	s_4	-6	-4	-1	-1	0	0	0	1
0	s_1	-5/2	0	-7/4	-11/4	1	0	0	1/4
0	s_2	0	0	1/2	-5/2	0	1	0	1/2
0	s_3	-1	0	-3/2	-1/2	0	0	0	1/2
80	x_1	3/2	1	1/4	1/4	0	0	1	1/4
80	x_3	10/11	0	7/11	1	-4/11	0	0	-1/11
0	s_2	25/11	0	23/11	0	-10/11	1	0	3/11
0	s_3	-6/11	0	-13/11	0	-2/11	0	1	5/11
80	x_1	14/11	1	1/11	0	1/11	0	0	3/11
80	x_3	8/13	0	0	1	-6/13	0	7/13	2/13
0	s_2	17/13	0	0	0	-16/13	1	23/13	14/13
60	x_2	6/13	0	1	0	2/13	0	-11/13	-5/13
80	x_1	16/13	1	0	0	1/13	0	1/13	4/13

Since all X_B are positive, current solution is an optimal solution.

$x_1 = \frac{16}{13}, \; x_2 = \frac{6}{13}, \; x_3 = \frac{8}{13}$. Min. $Z = \frac{2280}{13}$.

PROBLEM 8:

Minimize $Z = 6x_1 + 7x_2 + 3x_3 + 5x_4$

Subject to: $5x_1 + 6x_2 - 3x_3 + 4x_4 \geq 12$

$x_2 + 5x_3 - 6x_4 \geq 10$

$2x_1 + 5x_2 + x_3 + x_4 \geq 8.$

$x_1, x_2, x_3, x_4 \geq 0.$

SOLUTION: Min. Z = - Max.$(-Z)$, $-Z = Z^*$

Max. $Z^* = -6x_1 - 7x_2 - 3x_3 - 5x_4$

Subject to: $-5x_1 - 6x_2 + 3x_3 - 4x_4 \leq -12$

$-x_2 - 5x_3 + 6x_4 \leq -10$

$-2x_1 - 5x_2 - x_3 - x_4 \leq -8.$

$x_1, x_2, x_3 \geq 0.$

LPP in standard form:

Max. $Z^* = -6x_1 - 7x_2 - 3x_3 - 5x_4 + 0s_1 + 0s_2 + 0s_3 + 0s_4$

Subject to: $-5x_1 - 6x_2 + 3x_3 - 4x_4 + s_1 = -12$

$-x_2 - 5x_3 + 6x_4 + s_2 = -10$

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$$-2x_1 - 5x_2 - x_3 - x_4 + s_3 = -8.$$
$$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0.$$

where s_1, s_2, s_3 are slack variables.
Simplex table:

C_B	BVS	X_B	x_1	x_2	x_3	x_4	s_1	s_2	s_4
0	s_1	-12	-5	-6	3	-4	1	0	0
0	s_2	-10	0	-1	-5	6	0	1	0
0	s_3	-8	-2	-5	-1	1	0	0	1
-7	x_2	2	5/6	1	-1/2	2/3	-1/6	0	0
0	s_2	-8	5/6	0	-11/2	20/3	-1/6	1	0
0	s_3	2	13/6	0	-7/2	7/3	-1/6	0	1
-7	x_2	30/11	25/33	1	0	2/33	-5/33	-1/11	0
-3	x_3	16/11	-5/33	0	1	-40/33	1/33	-2/11	0
0	s_3	78/11	18/33	0	0	-21/33	-8/33	-7/11	1

Since all X_B are positive, current solution is an optimal solution.

$x_1 = 0,\ x_2 = \frac{30}{11},\ x_3 = \frac{16}{11},\ x_4 = 0.$ Min. $Z = \frac{258}{11}.$

PROBLEM 9: Maximize $Z = -5x_1 - 3x_2$

Subject to: $x_1 + x_2 \geq 1$
$$x_1 + x_2 \leq 7$$
$$x_1 + 2x_2 \geq 10$$
$$x_2 \leq 3.$$
$$x_1, x_2 \geq 0.$$

SOLUTION: Maximize $Z = -5x_1 - 3x_2$

Subject to: $-x_1 - x_2 \leq -1$
$$x_1 + x_2 \leq 7$$
$$-x_1 - 2x_2 \leq -10$$
$$x_2 \leq 3.$$
$$x_1, x_2 \geq 0.$$

LPP in standard form:

Maximize

$$Z = -5x_1 - 3x_2 + 0s_1 + 0s_2 + 0s_3 + 0s_4$$

Subject to: $-x_1 - x_2 + s_1 = -1$
$$x_1 + x_2 + s_2 = 7$$
$$-x_1 - 2x_2 + s_3 = -10$$
$$x_2 + s_4 = 3.$$
$$x_1, x_2, s_1, s_2, s_3, s_4 \geq 0.$$

where s_1, s_2, s_3, s_4 are slack variables.
Simplex table:

C_B	BVS	X_B	x_1	x_2	s_1	s_2	s_3	s_4
0	s_1	-1	-1	-1	1	0	0	0
0	s_2	7	1	1	0	1	0	0
0	s_3	-10	-1	-2	0	0	1	0
0	s_4	3	0	1	0	0	0	1
0	s_1	4	-1/2	0	1	0	-1/2	0
0	s_2	2	1/2	0	0	1	1/2	0
-3	x_2	5	1/2	1	0	0	-1/2	0
0	s_4	-2	-1/2	0	0	0	1/2	1
0	s_1	6	0	0	1	0	-1	-1
0	s_2	0	0	0	0	1	1	1
-3	x_2	3	0	1	0	0	-3/4	1
-5	x_1	4	1	0	0	0	-1	-2

Since all X_B are positive, current solution is an optimal solution. $x_1 = 4, x_2 = 3$. Max. $Z = -29$.

IX. CONCLUSION

An alternative method for dual simplex method to obtain the solution of linear programming problem has been derived. The proposed algorithm has simplicity and ease of understanding. This reduces number of iterations and save valuable time.

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