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Thermal Stresses of a Thin Annular Disc Due to Partially Distributed Heat Supply

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Abstract ***- In this paper, an attempt has been made to determine the temperature distribution, displacement function and thermal stresses, due to partially distributed heat supply*** $\left[-\frac{Q_0}{\lambda} f(r,t)\right]$ ***in a thin annular disc occupying the space*** $a \le r \le b,\ 0 \le z \le h$ ***by applying finite Fourier sine transform and Marchi-Zgrablich transform techniques.***

Keywords: **Thermo elastic solution, annular disc, Thermal Stresses.**

I. INTRODUCTION

Nowacki [4] has determined steady state thermal stresses in a circular edge, respectively. Roy Chaudhary [5] has succeeded in determining the quasi-static thermal stresses in a thin circular plate subjected to transient temperature along the circumference of circular upper face with lower face is at zero temperature and the fixed circular edge thermally insulated. Wankhede [6] has determined the quasi-static thermal stresses in circular plate subjected to arbitrary initial temperature on the upper face with lower face at zero temperature. Deshmukh and Ingle [1] have studied the direct problem of a thin circular plate due to partially distributed the heat supply, where they have determined the temperature (in heating and cooling process), displacement and stress function of the plate defined as $0 \le r \le a,\ 0 \le z \le h.$

In the present paper, the statement of the problem tackle by [1] which is related to the circular plate is reconstructed for an isotropic annular disc defined as $a \le r \le b,\ 0 \le z \le h$ and an attempt is made to study the inverse unsteady state thermoelastic problem to determine the temperature (in heating and cooling process), displacement and stress functions of the disc occupying the space $a \le r \le b,\ 0 \le z \le h$ with the stated boundary condition. Here finite Fourier sine transforms, Marchi-Zgrablich transform have been used to find the solution of problem. The numerical estimate for the temperature has been obtained on upper plane surface of the disc and depicted graphically.

II. STATEMENT OF THE PROBLEM

Consider a thin annular disc occupying the space $D: a \le r \le b,\ 0 \le z \le h$. The initial temperature of the disc is same as the temperature of the surrounding medium which is kept constant for the time $t = 0$ to $t = t_0$. The disc is subjected to a partially distributed axisymmetric heat supply $\left(-\frac{Q_0}{\lambda} f(r,t)\right)$ at point z=0..

After that the heat supply is removed and disc is cooled by surrounding medium.

The derived equation governing the displacement function $U(r,z,t)$ as [**4**] is

$$\frac{\partial^2 U}{\partial r^2} + \frac{1}{r}\frac{\partial U}{\partial r} = (1+\nu) a_t T \tag{1}$$

with $U_r = 0$ at $r = a$ and $r = b$. (2)

where ν and a_t are Poisson's ratio and linear coefficient of thermal expansion of the material of the disc respectively.

The temperature $T(r,z,t)$ is the heating temperature of the disc at time t satisfying the equation:

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{k}\frac{\partial T}{\partial t} \tag{3}$$

Subjected to initial condition

$$T(r,z,t)|_{t=0} = F(r,z) \tag{4}$$

The boundary conditions are

$$\left[T(r,z,t) + k_1 \frac{\partial T(r,z,t)}{\partial r}\right]_{r=a} = f_1(z,t) \tag{5}$$

$$\left[T(r,z,t) + k_2 \frac{\partial T(r,z,t)}{\partial r}\right]_{r=b} = f_2(z,t) \tag{6}$$

$$\left[T(r,z,t)\right]_{z=0} = -\frac{Q_0}{\lambda} f(r,t) \tag{7}$$

$$\left[T(r,z,t)\right]_{z=h} = g(r,t) \tag{8}$$

Where k and λ are the thermal diffusivity and conductivity of the material of the disc respectively, k_1 & k_2 are radiation constants on the curved surface of the disc respectively. The stress function σ_{rr} and $\sigma_{\theta\theta}$ are given by

$$\sigma_{rr} = -2\mu \frac{1}{r}\frac{\partial U}{\partial r} \tag{9}$$

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$$\sigma_{\theta\theta} = -2\mu \frac{\partial^2 U}{\partial r^2} \quad (10)$$

Equations (1) to (10) constitute the mathematical formulation of the problem under consideration.

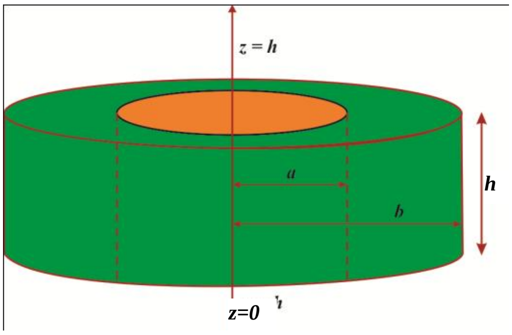

Fig 1.The geometry of the problem

III. DETERMINATION OF HEATING TEMPERATURE

Applying finite Marchi-Zgrablich transform and finite Fourier sine transform to the equation (3), one obtains

$$\bar{\bar{T}}^*(n,m,t) = e^{-kp^2t}\left[\int_0^t (Q_1\bar{\bar{g}}(n,t') - Q_2\bar{f}(n,t') + \psi_1)e^{kp^2t'}dt' + \bar{\bar{F}}^*(n,m)\right] \quad (11)$$

Where

$$p^2 = \mu_n^2 + q_m^2,\ q_m = \frac{m\pi}{h},\ Q_1 = \frac{m\pi}{h}(-1)^{m+1}k,\ Q_2 = \frac{kq_mQ_0}{\lambda},\ \psi_1 = k\psi^*$$

$$\psi = \frac{b}{k_2}S_0(k_1,k_2,\mu_n b)f_2 - \frac{a}{k_1}S_0(k_1,k_2,\mu_n a)f_1$$

Applying inverse Fourier sine transform and inverse Marchi-Zgrablich transform to the equation (11) we get

$$T(r,z,t) = \frac{2}{h}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\sin q_m z}{c_n}e^{-kp^2t}$$
$$\left[\int_0^t (Q_1\bar{\bar{g}}(n,t') - Q_2\bar{f}(n,t') + \psi_1)e^{kp^2t'}dt' + \bar{\bar{F}}^*(n,m)\right]$$
$$\times S_0(k_1,k_2,\mu_n r) \quad (12)$$

IV. STATEMENT OF THE PROBLEM (COOLING PROCESS)

The temperature change $T'(r,z,t)$ for the cooling process satisfies the differential equation:

$$\frac{\partial^2 T'}{\partial r^2} + \frac{1}{r}\frac{\partial T'}{\partial r} + \frac{\partial^2 T'}{\partial z^2} = \frac{1}{k}\frac{\partial T'}{\partial t} \quad (13)$$

Subject to initial condition

$$[T'(r,z,t)]_{t=t_0} = T'(r,z,t_0) \quad (14)$$

The boundary conditions are

$$\left[T'(r,z,t) + k_1\frac{\partial T'}{\partial r}(r,z,t)\right]_{r=a} = f_1(z,t) \quad (15)$$

$$\left[T'(r,z,t) + k_2\frac{\partial T'}{\partial r}(r,z,t)\right]_{r=b} = f_2(z,t) \quad (16)$$

$$[T(r,z,t)]_{z=0} = f(r,t) \quad (17)$$

$$[T'(r,z,t)]_{z=h} = g(r,t) \quad (18)$$

Where $T'(r,z,t)$ is the cooling temperature of the disc at time *t*.

V. DETERMINATION OF TEMPERATURE (COOLING PROCESS)

Applying finite Marchi-Zgrablich transform and finite Fourier sine transform to the equation (13), one obtains

$$\bar{\bar{T}}'(n,m,t) = e^{-kp^2t}\left(\int_0^t (Q_1'\bar{\bar{g}}(n,t') + Q_2'\bar{f}(n,t') + \psi_1)e^{kp^2t'}dt' + \bar{\bar{F}}^*(n,m)\right) \quad (19)$$

Where $Q_2' = kq_m$

Applying inverse Fourier sine transform and inverse Marchi-Zgrablich transform to the equation (19), we get

$$T'(r,z,t) = \frac{2}{h}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\sin q_m z}{c_n}e^{-kp^2t}\left[\int_0^t (Q_1'\bar{\bar{g}}(n,t') + Q_2'\bar{f}(n,t') + \psi_1)e^{kp^2t'}dt' + \bar{\bar{F}}^*(n,m)\right]$$
$$\times S_0(k_1,k_2,\mu_n r) \quad (20)$$

Which is the required solution.

VI. DETERMINATION OF DISPLACEMENT FUNCTION

Substituting value of $T(r,z,t)$ from equation (12) in equation (1) one obtains thermoelastic displacement function $U(r,z,t)$ as

$$U(r,z,t) = \frac{-2(1+\nu)a_t}{h} \times \sum_m\sum_n \frac{\sin q_m z}{c_n}e^{-kp^2t}$$
$$\left[\int_0^t (Q_1\bar{\bar{g}}(n,t') - Q_2\bar{f}(n,t') + \psi_1)e^{kp^2t'}dt' + \bar{\bar{F}}^*(n,m)\right]$$
$$\times S_0(k_1,k_2,\mu_n r) \quad (21)$$

VII. DETERMINATION OF STRESS FUNCTION

Substituting the value of equation (21) in equation (9) and (10)

$$\sigma_{rr} = \frac{4\mu(1+\nu)a_t}{h} \times \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\mu_n \sin q_m z}{c_n}e^{-kp^2t}$$

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$$\left[\int_0^t (Q_1\bar{g}(n,t') - Q_2\bar{f}(n,t') + \psi_1)e^{kp^2t'}dt' + \bar{F}^*(n,m)\right]$$

$$\times \frac{S_0'(k_1,k_2,\mu_n r)}{r} \quad (22)$$

$$\sigma_{\theta\theta} = \frac{4\mu(1+v)a_t}{h} \times \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \frac{\mu_n^2 \sin q_m z}{c_n} e^{-kp^2t}$$

$$\left[\int_0^t (Q_1\bar{g}(n,t') - Q_2\bar{f}(n,t') + \psi_1)e^{kp^2t'}dt' + \bar{F}^*(n,m)\right]$$

$$\times S_0''(k_1,k_2,\mu_n r) \quad (23)$$

VIII. SPECIAL CASE

Set $F(r,z) = -\dfrac{Q_0}{\lambda} e^z e^{-r}$ (24)

Applying Marchi-Zgrablich transform to above equation we get

$$\bar{F}(n,z) = -\frac{Q_0}{\lambda} e^z \int_a^b r e^{-r} S_0(k_1,k_2,\mu_n r)dr = -\frac{Q_0}{\lambda} e^z e^{-a} \mu_n D_n \quad (25)$$

Where $D_n = J_0(\mu_n a)y_0(\mu_n a) + y_0(\mu_n b)J_0(\mu_n b)$

$$S_p(k_1,k_2,\mu_n r) = J_p(\mu_n r)[y_p(k_1,\mu_n a) + y_p(k_2,\mu_n b)] - y_p(\mu_n r)[J_p(k_1,\mu_n a) + J_p(k_2,\mu_n b)]$$

And $J_p(\mu r)$, $y_p(\mu r)$ are Bessel's function of 1st and 2nd kind respectively. The eigen values μ_n are the positive roots of the characteristic equation

$$J_0(k_1,\mu a) + y_0(k_2,\mu b)] - J_0(k_2,\mu b)y_0(k_1,\mu a)] = 0$$

Applying Fourier sine transform to above equation we get

$$\bar{F}^*(n,m) = -\frac{Q_0\pi h}{\lambda} e^{-a} \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{m\mu_n D_n}{h^2 + m^2\pi^2}\right)(1-(-1)^m e^h) \quad (26)$$

which is the required solution.

Now set $g(r,t) = -\dfrac{Q_0}{\lambda} e^{-t} e^{-r} e^h$,

$$f(r,t) = -\frac{Q_0}{\lambda} e^{-t} e^{-r}$$

Applying Marchi-Zgrablich transform to above equation, we get

$$\bar{g}(n,t') = -\frac{Q_0}{\lambda} e^{-t'} e^h e^{-a} \mu_n D_n \quad (27)$$

$$\bar{f}(n,t') = -\frac{Q_0}{\lambda} e^{-t'} e^{-a} \mu_n D_n \quad (28)$$

Therefore the final expression for temperature distribution is

$$T(r,z,t) = \frac{2}{h}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \frac{\sin q_m z}{c_n} e^{-kp^2t} \times S_0(k_1,k_2,\mu_n r)$$

$$\times\left[\int_0^t \left(Q_1\left(\frac{Q_0}{\lambda} e^{-t'} e^h e^{-a} \mu_n D_n\right) - Q_2\left(\frac{Q_0}{\lambda} e^{-t'} e^{-a} \mu_n D_n\right) + \psi_1\right) e^{kp^2t'}dt'\right.$$

$$\left. + \left\{-\frac{Q_0\pi h e^{-a}}{\lambda} \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \frac{m\mu_n D_n}{h^2 + m^2\pi^2}(1-(-1)^m e^h)\right\}\right] \quad (29)$$

IX. NUMERICAL RESULTS

Set
$k_1 = 0.2$, $k_2 = 0.2$, $h = 0.25\,ft$, $\pi = 3.14$, $a = 1.5\,ft$, $b = 2\,ft$, t in seconds, $k = 0.5$ in equation (29) we get

$$T(r,z,t) = \frac{2}{h}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \frac{\sin q_m z}{c_n} e^{-0.5p^2t} \times S_0(0.2,0.2,\mu_n r)$$

$$\times\left[\int_0^t \left(Q_1\left(\frac{Q_0}{\lambda} e^{-t'} e^h e^{-1.5} \mu_n D_n\right) - Q_2\left(\frac{Q_0}{\lambda} e^{-t'} e^{-1.5} \mu_n D_n\right) + \psi_1\right) e^{kp^2t'}dt'\right.$$

$$\left. + \left\{-\frac{Q_0(3.14)he^{-1.5}}{\lambda} \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{m\mu_n D_n}{h^2 + m^2(3.14)^2}\right)(1-(-1)^m e^h)\right\}\right]$$

(30)

X. MATERIAL PROPERTIES

The numerical calculation has been carried out for an Aluminum (Pure) annular disc with the material properties as,

Density ρ = 169 lb/ft3

Specific heat = 0.208 Btu/ lb0F

Thermal conductivity k = 117 Btu/(hr.ft0F)

Thermal diffusivity α = 3.33 ft2/hr

Poisson ratio ν = 0.35

Coefficient of linear thermal expansion α_t = 12.84 $\times 10^{-6}$ 1/F

Lame constant μ = 26.67

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XI. CONCLUSION

The temperature distribution, displacement function and thermal stresses at any point of the disc have been determined when the other boundary condition are known with the aids of finite Fourier sine transform and Marchi Zgrablich transform techniques. The expressions are obtained in the form of infinite series and are represented graphically. Any particular case of special interest can be derived by assigning suitable values to the parameters and functions in the equation. The results presented here will be more useful in engineering problems particularly in the determination of the state of strain in the disc constituting the foundation of container for hot gases or liquid in foundation for furnaces etc.

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AUTHOR BIOGRAPHY

Dr. N.W. Khobragade for being M.Sc in statistics and Maths he attained Ph.D. He has been teaching since 1986 for 27 years at PGTD of Maths, RTM Nagpur University, Nagpur and successfully handled different capacities. At present he is working as Professor. Achieved excellent experiences in Research for 15 years in the area of Boundary value problems and its application. Published more than 180 research papers in reputed journals. Fourteen students awarded Ph.D Degree and four students submitted their thesis in University for award of Ph.D Degree under their guidance.

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APPENDIX

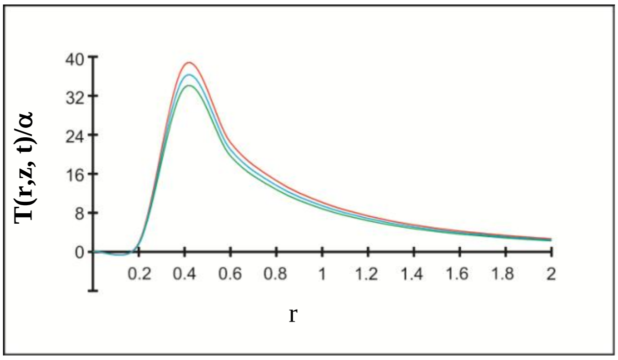

Fig. 1: Graph of r versus T(r, z, t)/α

Fig 1: shows that for some value of r (=0.4), temperature T increases and then goes on decreeing gradually, this is because of partially distributed heat supply.