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Stochastic Modeling of Galileo E1 and E5a Signals

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Abstract*— In order to take full advantage of the new Galileo signals E1 and E5a, it is essential that their stochastic characteristics are rigorously modelled. In this paper, various sets of GPS and Galileo measurements, collected at two stations with short separation, were used to study the stochastic characteristics of Galileo E1 and E5a signals. As a by-product, the stochastic characteristics of the legacy GPS P1 code was obtained and then used to verify the developed stochastic model of the Galileo signal. It is shown that the developed stochastic models improve the accuracy of precise point positioning solution and its convergence time.

***Index Terms*—Galileo, GNSS, GPS, Stochastic Characteristics, PPP.**

I. INTRODUCTION

GPS precise point positioning (PPP) technique has attracted many users due to its lower cost and comparable precision to that of the differential technique. A drawback of a single GNSS system such as GPS, however, is the availability of sufficient number of visible satellites in urban areas. With the launch of the new Galileo satellites, a PPP solution based on the combined GPS/Galileo measurements is feasible. Combining GPS and Galileo systems offers more visible satellites to users, which in turn is expected to enhance the satellite geometry and the overall positioning solution [5]. Generally, the mathematical model for GNSS PPP consists of two parts, namely functional and stochastic models. The functional part describes the physical or geometrical characteristics of the parameters of the PPP model, while the stochastic part describes the statistical (or stochastic) properties of the residual components in the functional model. The stochastic model is represented by the observations variances-covariance parameters (i.e., the covariance matrix). The functional models related to the GNSS observables have been extensively studied by many researchers. This, however, is not the case with the more complex stochastic models. Often, a simplified empirical stochastic model is used in GNSS positioning, which assumes that all the GNSS observables are statistically independent and of the same quality. This, however, can lead to an overestimation of the PPP parameters. This research aims to improve the single-frequency GPS/Galileo PPP solution through rigorous modelling of the stochastic characteristics of Galileo signal. In addition, this research studies the effect of the satellite elevation angle on the stochastic characteristics. A receiver system noise test is performed to determine the stochastic characteristics of Galileo E1 and E5a and GPS L1 signals, respectively. To verify the obtained results, the newly developed stochastic model is implemented to assess the effect of the stochastic characteristics on the combined GPS/Galileo PPP solution precision and convergence time. The results of the combined GPS/Galileo solution show an improvement of up to 30% in the solution convergence time and a positioning accuracy at the sub-decimeter level.

II. GPS AND GALILEO OBSERVATION MODELS

GNSS observations are affected by errors and biased, which must be considered to obtain accurate positioning. The accuracy of precise point positioning depends on the ability to mitigate all kinds of errors. These errors can be categorized as satellite-related errors, signal propagation-related errors and receiver/antenna-related errors [3]. GNSS errors attributed to the satellites include satellite clock errors, orbital errors, satellite hardware delay, satellite antenna phase centre variation, and satellite initial phase bias. Errors attributed to signal propagation include the delays of the GNSS signal as it passes through the ionospheric and tropospheric layers. Errors attributed to receiver/antenna configuration include, among others, the receiver clock errors, multipath error, receiver noise, receiver hardware delay, receiver initial phase bias, and receiver antenna phase center variations. In addition to the above errors and biases, combining GPS and Galileo observation in a PPP model introduces additional errors such as GPS to Galileo time offset (GGTO) due to the fact that each system uses a different time frame. GPS system uses the GPS time system, which is referenced to coordinated universal time (UTC) as maintained by the US Naval Observatory (USNO). On the other hand, Galileo satellite system the Galileo system time (GST), which is a continuous atomic time scale with a nominal constant offset with respect to the international atomic time (TAI) [5]. Moreover, GPS and Galileo use different reference frames, which should be considered in the combined PPP solution. The mathematical models of GPS and Galileo observables, code and carrier phase, can be written respectively as:

$$\begin{aligned} P_G &= \rho_G(t_G,(t-\tau)_G) + c[dt_r(t_G) - dt^s(t-\tau)_G] + T_G \\ &\quad + I_G + c[d_r(t_G) + d^s(t-\tau)_G] + d_{mp} + e_{PG} \end{aligned} \tag{1}$$

$$\begin{aligned} P_E &= \rho_E(t_E,(t-\tau)_E) + c[dt_r(t_E) - dt^s(t-\tau)_E] + T_E \\ &\quad + I_E + c[d_r(t_E) + d^s(t-\tau)_E] + d_{mp} + e_{PE} \end{aligned} \tag{2}$$

$$\begin{aligned} \Phi_G &= \rho_G(t_G,(t-\tau)_G) + c[dt_r(t_G) - dt^s(t-\tau)_G] + T_G - I_G \\ &\quad + c[\delta_r(t_G) + \delta^s(t-\tau)_G] + \lambda[N_G + \varphi_r(t_0) - \varphi^s(t_0)] + \delta_{mp} + \varepsilon_{\varphi G} \end{aligned} \tag{3}$$

$$\begin{aligned} \Phi_E &= \rho_E(t_E,(t-\tau)_E) + c[dt_r(t_E) - dt^s(t-\tau)_E] + T_E - I_E \\ &\quad + c[\delta_r(t_E) + \delta^s(t-\tau)_E] + \lambda[N_E + \varphi_r(t_0) - \varphi^s(t_0)] + \delta_{mp} + \varepsilon_{\varphi E} \end{aligned} \tag{4}$$

where the subscript G refers to the GPS satellite system and the subscript E refers to the Galileo satellite system; P_G and P_E are pseudoranges for the GPS and Galileo systems, respectively; Φ_G and Φ_E are the carrier phase measurements of the GPS and Galileo systems, respectively; $dt_r(t)$, $dt^s(t-\tau)$ are the clock error for receiver at reception time t and satellite at transmitting time t-τ, respectively; $d_r(t)$, $d^s(t-\tau)$ are frequency dependent code hardware delay for receiver at reception time t and satellite at transmitting time t-τ, respectively; $\delta_r(t)$, $\delta^s(t-\tau)$ are frequency-dependent carrier phase hardware delay for receiver at reception time t and satellite at transmitting time t-τ, respectively; T is the tropospheric delay; I is ionospheric delay; dmp is code multipath effect; δmp is the carrier phase multipath effect; λ is the wavelengths of carrier frequencies, respectively; $\Phi_r(t_0)$, $\Phi^s(t_0)$ are frequency-dependent initial fractional phases in the receiver and satellite channels; N is the integer number of cycles for the carrier phase measurements, respectively; c is the speed of light in vacuum; and ρ is the true geometric range from receiver at reception time to satellite at transmission time; e_P, ε_Φ are the relevant noise and un-modeled errors. The IGS satellite precise orbital and clock corrections contain the satellite hardware delay of the ionosphere free linear combination of GPS L1 and L2 signals [6]. On the other hand, CONGO satellite precise orbital and clock corrections include the satellite hardware delay of the ionosphere free linear combination of Galileo E1 and E5a signals [7]. In our model, the receiver and the GPS satellite hardware delays are lumped to the receiver clock error. This, in turn introduces a new term in the Galileo observation equations, which represent the difference between the satellite hardware delays of GPS and Galileo signals. A new unknown is considered in our model to account for the system time offset and biases, as well as the new satellite hardware difference term. The receiver and satellite hardware delays can be lumped to the receiver clock error and to the GGTO as all of these errors are timing errors. Equations 5 to 8 show the final combined GPS and Galileo PPP model.

$$P_G = \rho_G + c[dt_r - dt^s_{IGS}] + T_G + I_G + e_{PG} \quad (5)$$

$$\Phi_G = \rho_G + c[dt_r - dt^s_{IGS}] + T_G - I_G + \lambda\tilde{N}_G + \varepsilon_{\varphi G} \quad (6)$$

$$P_E = \rho_E + c[dt_r - dt^s_{CON}] + ISB + T_E + I_E + e_{PE} \quad (7)$$

$$\Phi_E = \rho_E + c[dt_r - dt^s_{CON}] + ISB + T_E - I_E + \lambda\tilde{N}_E + \varepsilon_{\varphi E} \quad (8)$$

where $\tilde{N}$ is the ambiguity parameter including frequency-dependent initial fractional phases in the receiver and satellite channels; ISB is the newly introduced unknown parameter. To combine GPS and Galileo observations in a PPP solution, it is essential that the stochastic characteristics of the noise terms in the above equations are described using the proper model.

III. SYSTEM NOISE TEST

The receiver measurement noise results from the limitations of the receiver's electronics and can be determined through receiver calibration or test. Two tests are usually carried out to determine the system noise level, namely the zero and short baselines tests. The zero baseline test employs one antenna followed by a signal splitter that feeds two or more GPS receivers. Using the zero baseline test, several receiver problems can be investigated, such as inter-channel biases and cycle slips. The single antenna cancels out the real world systematic problems such as multipath in addition to the preamplifier's noise. The short baseline test, on the other hand, uses two receivers a few meters apart and is usually carried out over two consecutive days. In this case, the double difference residuals of one day would contain the system noise and the multipath effect. As the multipath effect repeats almost every day for GPS system, differencing the double difference residuals of the two consecutive days cancels out the multipath effect and leaves the scaled system noise. However, multipath effect is not repeatable for the Galileo satellite system as the satellites take about 14 hours 4 minutes 41 seconds to orbit the Earth [5]. In this research, a short baseline test is used to determine stochastic characteristics of the E1 and E5a signals, assuming that multipath does not exist. Usually, this test is performed using the same type of receivers. Unfortunately, in this research, two different receivers were available (Septentrio and Trimple) for the test, which can observe the Galileo measurements. This, however, were considered when processing the data as shown in the sequel. The pseudorange and carrier phase equations can be re-written as, assuming no multipath and dropping the time argument:

$$P_i = \rho + c[dt_r - dt^s]_i + c[d_r + d^s]_i + T_i + I_i + e_{Pi} \quad (9)$$

$$\Phi_i = \rho + c[dt_r - dt^s]_i + c[\delta_r + \delta^s]_i + T_i - I_i + \lambda\tilde{N} + e_{\Phi i} \quad (10)$$

Differencing the pseudorange and carrier phase equations of each receiver cancels out the geometric term, satellite and receiver clock error, and tropospheric delays, as shown in (11) and (12). The remaining terms include the satellite and receiver hardware delays, ionosphere delay, the ambiguity parameter and the system noise.

$$\Delta R_1 = P_{R1} - \Phi_{R1} = c[d_r - d^s]_1 + c[\delta_r - \delta^s]_1 + \Delta\lambda\tilde{N}_1 + 2I + e_P \quad (11)$$

$$\Delta R_2 = P_{R2} - \Phi_{R2} = c[d_r - d^s]_2 + c[\delta_r - \delta^s]_2 + \Delta\lambda\tilde{N}_2 + 2I + e_p \quad (12)$$

It should be pointed out that the noise parameters in Equations 11 and 12 are essentially those of the pseudorange measurements. The phase measurement noise has been neglected due to its small size compared to that of the pseudorange measurements [4]. The receiver hardware delay is assumed to be stable over the observation period (four hours in this research), while the ambiguity parameter and initial phase bias are constants for a continuous session of measurements [5]. As such, they can be removed from the model by differencing with respect to the first value of the series. Using this approach, only the differenced system noise remains in the model.

IV. STOCHASTIC MODEL DEVELOPMENT

In PPP, most of existing observation stochastic models are empirical functions such as sine, cosine, exponential and

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polynomial functions. Most of these stochastic models are functions of the satellite elevation angles [8]. Unfortunately, existing stochastic models may not be valid for all receiver types and GNSS signal frequencies. As such, it is essential that new stochastic models are developed for the Galileo signal. The data series developed in Section 3 are divided into nine bins depending on the satellite elevation angle, starting from 0° to 90° with increments of 10° (i.e., 0º to 10º, 10º to 20º, etc.). The standard deviation of the differenced system noise each bin is estimated. A least squares regression analysis is performed to obtain the best-fit model of the estimated standard deviations. Three empirical functions were tested for this purpose, namely an exponential, a polynomial and a rational model. The best-fit model is selected based on the goodness of fit test, i.e., the one with the largest R2 (R-squared) statistic [2]. Figures 1 through 3 show the 95% confidence levels for the best-fit models. Table 1 summarizes the results of all three tested functions. As shown, the exponential function was found to be the best-fitting model in the least-squares sense.

Table 1 Summary results of regression fitting functions with 95% confidence level

	Exponential function			Polynomial function			Rational function		
	$STD = a \times e^{(-b \times ELE)} + c$			$STD = -a \times ELE^3 + b \times ELE^2 - c \times ELE + d$			$STD = \frac{(a \times ELE^2 - b \times ELE + c)}{(ELE + d)}$		
	E1	E5a	L1	E1	E5a	L1	E1	E5a	L1
a	0.6383	0.3692	0.6830	1.835e-6	5.892e-6	1.473e-6	3.5e-3	4.315e-3	6.087e-3
b	0.0763	0.0753	0.0730	3.688e-4	1.445e-4	3.195e-4	0.2703	0.5155	0.6533
c	0.2150	0.0974	0.1751	0.02443	0.01556	0.0228	22.93	28.36	36.57
d	-	-	-	0.7557	0.4014	0.7156	28.35	69.29	49.69
R^2	0.9995	0.9993	0.9994	0.9988	0.9977	0.9990	0.9990	0.9977	0.9984

Where ELE is the satellite elevation angle in degrees; STD is the observation standard deviation.

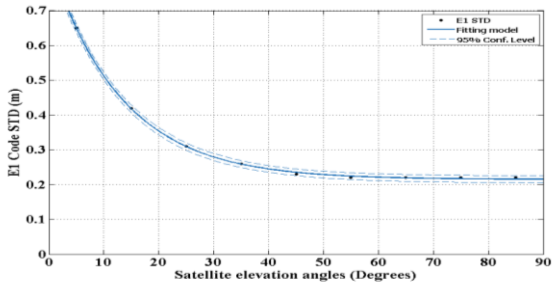

Fig 1 the standard deviation of Galileo E1 signal using exponential fit model

Figure 1 shows the variation of the stochastic characteristics of the Galileo E1 signal with respect to the satellite elevation angles. In addition, Figure 1 shows the results of the regression analysis using the exponential model with the 95% confidence level limits. The stochastic characteristic of the Galileo E1 signal is almost constant above the elevation angle of 45°. Equation 13 shows the computed exponential model.

$$STD = 0.6383 \times e^{(-0.07634*ELE)} + 0.2150 \quad (13)$$

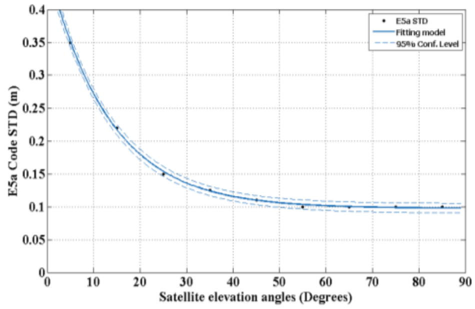

Fig 2 the standard deviation of Galileo E5a signal using exponential fit model

Figure 2 shows the variation of the standard deviation of the Galileo E5a signal with respect to the satellite elevation angles. In addition, Figure 2 shows the results of the regression analysis using the exponential model with the 95% confidence level limits. Equation 14 shows the computed exponential model.

$$STD = 0.3692 \times e^{(-0.0753*ELE)} + 0.0974 \quad (14)$$

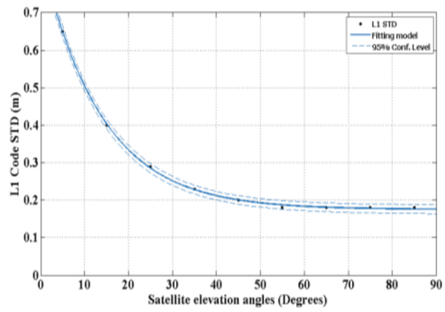

Fig 3 the standard deviation of GPS L1 signal using exponential fit model

Figure 3 shows the variation of the standard deviation of the GPS L1 signal with respect to the satellite elevation angles. In addition, Figure 3 shows the results of the regression analysis using the exponential model with the 95% confidence level limits. Equation 15 shows the computed exponential model.

$$STD = 0.683 \times e^{(-0.0730*ELE)} + 0.1751 \quad (15)$$

V. GPS/GALILEO PPP USING DEVELOPED STOCHASTIC MODELS

To verify the determined stochastic models of the Galileo E1 and E5a signals, Natural Resources Canada (NRCan)

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GPSPace PPP software was modified to handle the Galileo observations in addition to the newly developed stochastic models. In order to assess the newly developed stochastic models, the combined GPS/Galileo PPP solution is implemented by using both the empirical sine function, which has traditionally been used to describe the stochastic characteristics of the measurement noise, and the newly developed stochastic models. Four stations were used to verify our model: two stations in North America (UNB and USN) and two in Europe (Delft and GOP). The NOAA ionospheric correction model was used to correct for the ionospheric delay [9]. In addition, the NOAA model is used along with the Vienna mapping function to correct the tropospheric delay [1]. IGS precise orbital and clock corrections are used for GPS satellites, while the CONGO precise satellite orbital and clock corrections are used for Galileo satellites. Unfortunately, precise orbit and clock data from a single GPS/Galileo network solution were not available, which might create a small bias in the PPP results. Only the results of the DLFT station is presented in this paper as other stations have the same results.

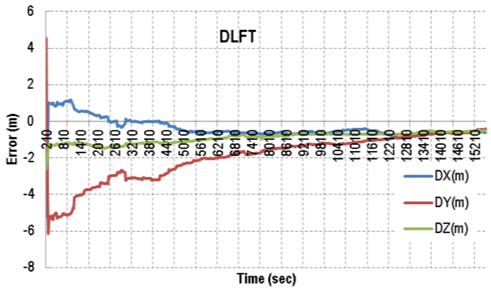

Fig 4 Combined Galileo (E1)/GPS (L1) PPP results using traditional sine function

Figure 4 shows the results of the combined single frequency Galileo/GPS of E1 and L1 signals using the empirical sine function. The results of the combined GPS/Galileo PPP solution using the sine function show a decimeter level of accuracy however the solution takes a long convergence time, almost three hours, to achieve this level of accuracy.

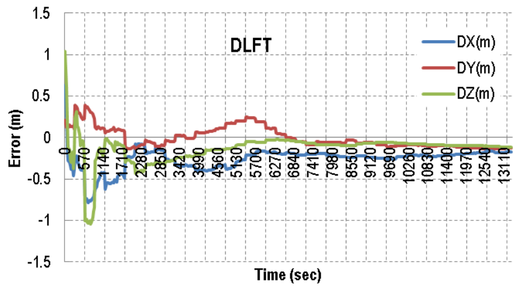

Fig 5 Combined Galileo (E1)/GPS (L1) PPP results using the new stochastic model

Figure 5 shows the results of the combined single frequency Galileo E1 and GPS L1 signals using the newly developed stochastic model. The results show that the new stochastic model improves the positioning accuracy and the convergence time, as well.

VI. CONCLUSION

New stochastic models of Galileo E1 and E5a signals have been developed in this research. Three empirical functions are considered, namely exponential, polynomial and rational functions. It has been found that the exponential function gives the best fit, based on regression analysis. The newly developed stochastic models are used through combining GPS L1 with Galileo E1 signals, respectively, to implement a single-frequency PPP solution. The results of the combined GPS/Galileo PPP solution of both L1/E1 signals showed that a sub-decimetre positioning accuracy is attainable by using the observations proper stochastic models. In addition, the convergence time improves up to 30% of the combined GPS/Galileo L1/E1 PPP results by using the newly developed stochastic model.

VII. ACKNOWLEDGMENT

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