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Analysis of Combined rectangular footing by Winkler's Model and Finite Element Method

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***Abstract*— as we know the foundation is the most important member of the building and a precise analysis of footing will result in more safe and economic design. In the present study the comparison of analysis of combined rectangular footing using Rigid (conventional) Method and Finite Element Analysis of combine footing using Winkler Model is carried out.**

***Index Terms*—combined rectangular footing, Finite Element Analysis, Winkler's Model.**

I. INTRODUCTION

Foundation is a structural member, made of brick work, masonry or concrete a structure which carries the entire load from superstructure and it will distribute the load on soil bellow it. The strength, stability & support of structure are fully dependent on foundation. If some parts of superstructure fail, then repairs, modifications, additions & alterations are possible to save the structure, but in case of foundation failure it is much difficult and very costly.

II. TYPES OF FOUNDATION

"Foundation" is the main part of a building which is broadly classified into two main categories:

- Shallow foundation
- Deep foundation

Depending upon the nature of soil below foundation, site condition, type and amount of loading, type of super-structure we have to choose appropriate foundation.

III. COMBINED FOOTING

Combined footing is one in which a footing supports a line of two or more columns, it is called a combined footing. A combined footing may have either rectangular or trapezoidal shape or be a series of pads connected by narrow rigid beams called a strap footing.

IV. RECTANGULAR COMBINED FOOTING

It may not be possible to place columns at the center of a spread footing if they are near the property line, near mechanical equipment locations, or irregularly spaced. Columns located off- center will usually result in a non-uniform soil pressure. To avoid the non-uniform soil pressure, an alternative is to enlarge the footing and place one or more of the adjacent columns in the same line on it the footing geometry is made such that the resultant of the columns is in the center of the footing area. The footing can be rectangular if the column that is eccentric with respect to a spread footing carries a smaller load than the interior columns. Bridge piers are also founded on very rigid combined rectangular footings.

V. ANALYSIS OF FOOTING

To get safe and economic sizes of footing we have to analyzes the footing accurately. Structural analysis comprises the set of physical laws and mathematics required to study and predicts the behavior of structures. To perform an analysis a structural engineer must determine such information as structural loads, geometry, support conditions, and materials properties. The results of such an analysis typically include support reactions, Bending and shear stresses and displacements. This information is then compared to criteria that indicate the conditions of failure. There are four types of analysis in use of combined footing:

- ❖ Rigid Analysis
- ❖ Elastic Analysis
- ❖ Simplified Elastic Approach
- ❖ Elastoplast Approach

In the present study Finite Element Analysis of combine footing using Winkler Model will be carried out. The objective of this study is to obtain displacement, base pressure shear force and bending moment. Further a Comparison will be carried out with Rigid Analysis (Conventional Analysis). Winkler (1871) Presented nearly 140 years ago a book "Constructing a Permanent way of Railroad" he suggested to analyze the rails as an infinite beam with the stiffness E (Young's Modulus), J (M.I. of cross-section of beam) supported by the foundation being which is consist of continuous closely spaced, independent linear springs.

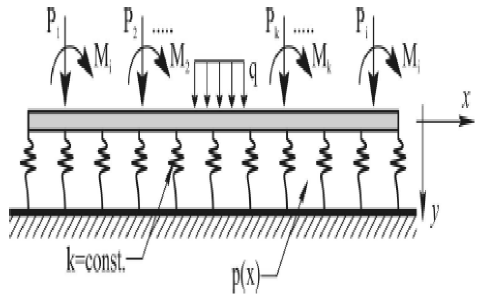

Fig 1 Winkler's Model

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V. FORMULATION OF FINITE ELEMENT METHOD FOR COMBINED FOOTING

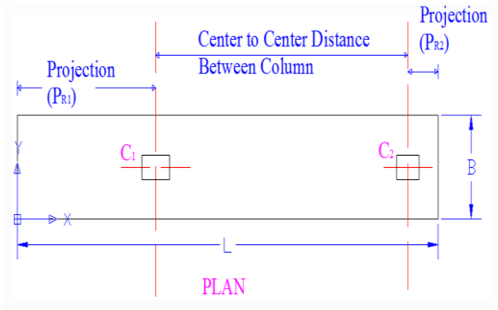

Fig 2 : Plan of combined footing

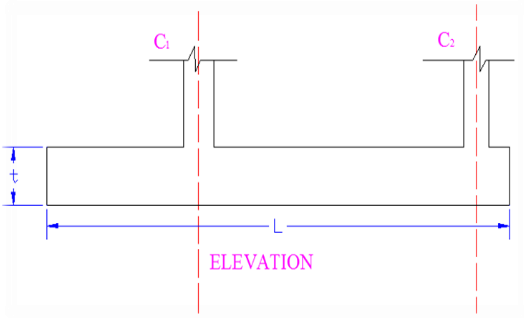

Fig 3 : Elevation of combined footing

To analyze the footing by finite element method, firstly discretize the given problem. In the figure bellow (fig. 3) it is shown that the given footing is divided into three number of elements and four Nodal points, now according to Winkler's Model the soil is made-up of springs, therefore the springs are attached at each nodal points, which is having stiffness "K"

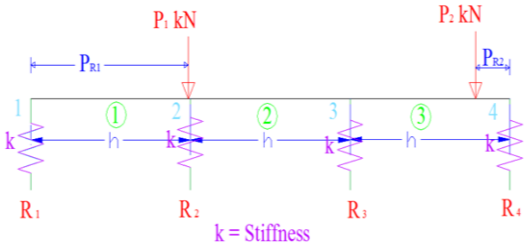

Fig 4 : Discretizing the Problem

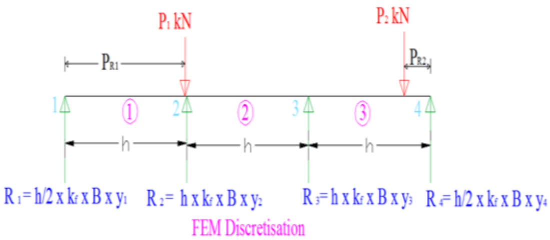

Fig 5 : Spring Force

Now, the spring will give some reaction to the beam which is equal to product of effective area, Modulus of Sub grade reaction and Deflection at that point as given in fig. No. 4 Now, Find out the Stiffness matrix for each element that is local stiffness matrix then combine it and find out global stiffness matrix.

$$k=\frac{E\times I}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L & 0 & 0 & 0 & 0 \\ 6L & 4L^2 & -6L & 2L^2 & 0 & 0 & 0 & 0 \\ -12 & -6L & 24 & 0 & -12 & 6L & 0 & 0 \\ 6L & 2L^2 & 0 & 8L^2 & -6L & 2L^2 & 0 & 0 \\ 0 & 0 & -12 & -6L & 24 & 0 & -12 & 6L \\ 0 & 0 & 6L & 2L^2 & 0 & 8L^2 & -6L & 2L^2 \\ 0 & 0 & 0 & 0 & -12 & -6L & 12 & -6L \\ 0 & 0 & 0 & 0 & 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

Fig 6 : Global stiffness matrix

Scene, the base is made-up of springs then add the stiffness contribution of that springs.

$$[K]=\frac{E\times I}{L^3}\begin{bmatrix} 12+\left(k_1\times\frac{L^3}{E\times I}\right) & 6L & -12 & 6L & 0 & 0 & 0 & 0 \\ 6L & 4L^2 & -6L & 2L^2 & 0 & 0 & 0 & 0 \\ -12 & -6L & 24+\left(k_2\times\frac{L^3}{E\times I}\right) & 0 & -12 & 6L & 0 & 0 \\ 6L & 2L^2 & 0 & 8L^2 & -6L & 2L^2 & 0 & 0 \\ 0 & 0 & -12 & -6L & 24+\left(k_3\times\frac{L^3}{E\times I}\right) & 0 & -12 & 6L \\ 0 & 0 & 6L & 2L^2 & 0 & 8L^2 & -6L & 2L^2 \\ 0 & 0 & 0 & 0 & -12 & -6L & 12+\left(k_4\times\frac{L^3}{E\times I}\right) & -6L \\ 0 & 0 & 0 & 0 & 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

Fig 7 : Final stiffness matrix

It will gives us the final stiffness matrix [K].

Finally, we know, $F = K \times \delta$

$\delta = F \times K^{-1}$

We can get the deformation at various nodal points

Once the deformations are calculated, then base pressure and reaction offered by the springs can be calculate by following formula.

Base Pressure = (modulus of sub grade reaction) x (Deflection at a point)

Reaction by spring = (Stiffness of Spring) x (Deflection at that point)

It is recommended to write a computer code for calculation in the present study, computer code is written in MATLab. MATLab is powerful software known for matrix calculation.

VI. ANALYSIS OF COMBINED RECTANGULAR FOOTING

Analyze the footing having length equal to 7.5 meters and width equal to 1.3 meters. Assuming that the footing is resting on clayey medium Dance soil having safe bearing capacity 250 kN/m2 and K_p (sub grade reaction by Plate load test) equal to 50000 kN/m^3 According to Terzaghi (1955) Applying correction to K_p

$$k_f = K_p\left[\frac{B_p(B_f+30)}{B_f(B_p+30)}\right]^2\times\left(\frac{m+0.5}{1.5\times m}\right)\times\left(1+\frac{2\times D_f}{B}\right)$$

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m = L/B = 7.5/ 1.3 = 5.77

Assuming the depth of foundation = 1.5 m

We get, modulus of sub grade reaction k_f = 3.56 x 10^4 kN/m^3

Now, Calculation of Moment of Inertia,

$$I = B \times d^3 / 12$$

Assuming the depth of footing = 440 mm

Therefore, we get the M I = (1.3 x 0.443) / 12

= 9.228 x 10^{-3} m^4

Now, calculate the Young's Modulus of Elasticity of footing

$$E_c = 5000\sqrt{20}$$

$$E_c = 2.24 \times 10^4 \text{ N/mm}^2$$

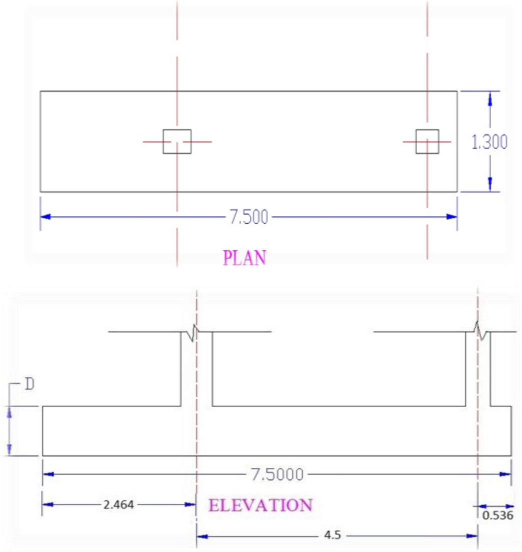


Fig 8 : Plan and Elevation of Combined footing

Now, Discretizing the given Problem into three No of elements and four nodal points taking member length (h)

(h) = 2.5 m

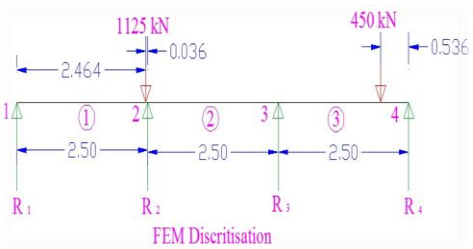


Fig 9 : Finite Element Discretizing

After discretizing the problem, calculate the Local and Global Stiffness Matrix Add the stiffness contribution of springs at Proper location, and get final stiffness matrix [k]. Find the consistent Nodal load vectors for each Element and arrange it properly to get Vector matrix {F}.

Now, we know, F= K x δ

δ = F x K^{-1}

We can get the deformation at various nodal points

Table 1 : Deflection and Rotation at Nodal points

v1		-0.00265606
θ1		-0.00221554
v2		-0.00625918
θ2	=	0.00010386
v3		-0.00391829
θ3		0.00052333
v4		-0.00421458
θ4		1.0654E-05

V_n represent the vertical deflection at Nodal point 'n'

θ_n represent the rotation at Nodal point 'n'

After getting deformation at nodal points we can calculate the Base pressure and Reaction by the spring

Base Pressure = (modulus of sub grade reaction) x (Deflection at a point)

Table 2: Base pressure at nodal points

P1	=	-94.5557503
P2	=	-222.826862
P3	=	-139.491112
P4	=	-150.039071

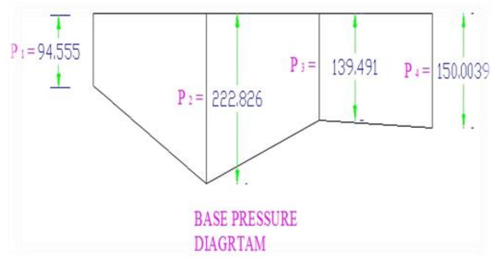
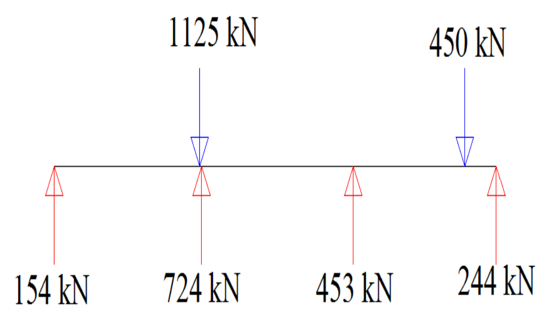


Fig 10 : Base Pressure Diagram

Reaction by spring = (Stiffness of Spring) x (Deflection at that point)

R1	=	-1.54E+02
R2	=	-7.24E+02
R3	=	-4.53E+02
R4	=	-2.44E+02

Table 3: Spring Reaction at Nodal points



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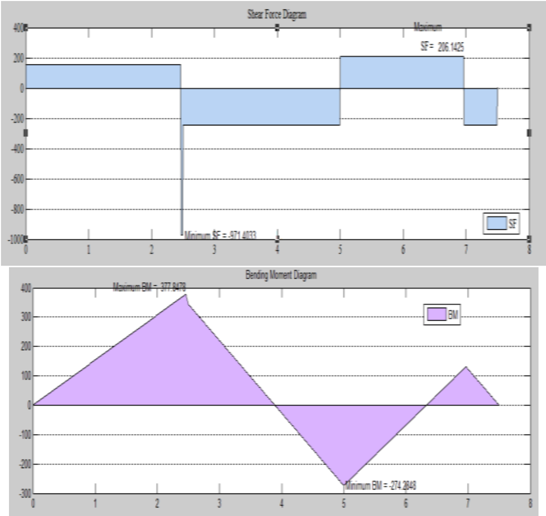


Fig 11 : Spring reaction & loading, SFD, BMD by FEM (Program output)

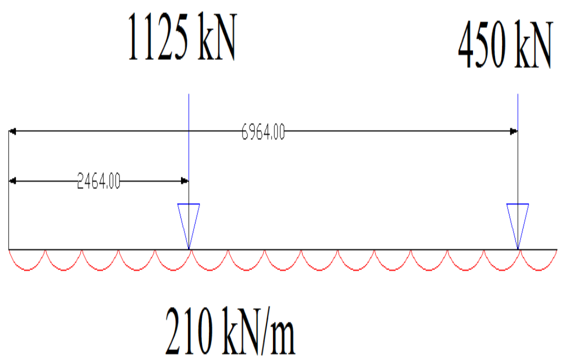
VII. CONVENTIONAL ANALYSIS OF RECTANGULAR COMBINED FOOTING

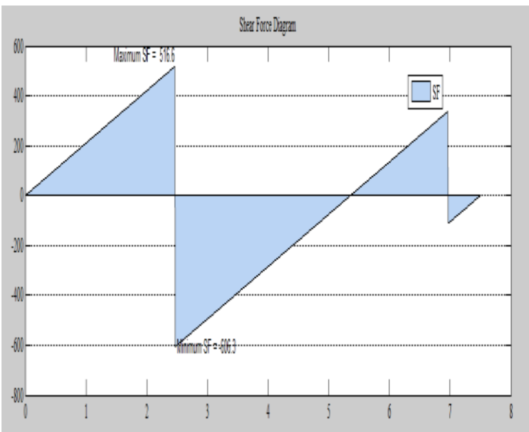
Calculation of Shear force and Bending Moment as per rigid (conventional) method Calculating Upward Soil Pressure per meter length as Uniformly Distributed Load,

$$w = \text{Total Load} / \text{length of footing}$$

$$w = (P_1 + P_2) / (L)$$

We get , $w = 210$ kN/m





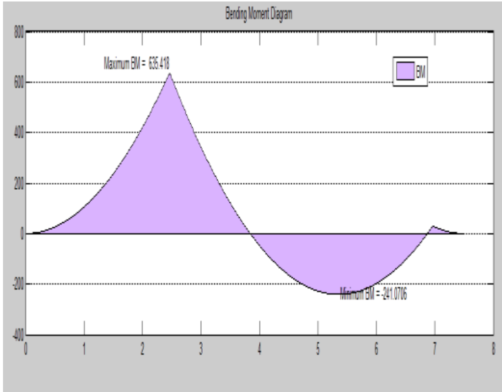


Fig 12 SFD and BMD by Convention method (Program output)

VIII. PARAMETRIC STUDY OF RECTANGULAR COMBINED FOOTING

Further the analysis is carried out for various Number of elements and effect of modulus of sub grade reaction is also studied.

A. Number of element

The analysis is carried out for 3, 5, 10, 50, 100 and 500 elements.

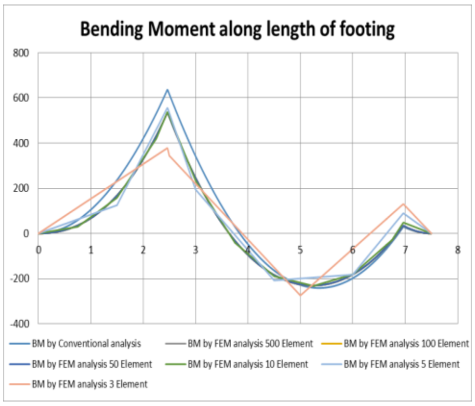


Fig 13 Bending Moment Diagram for Various Numbers of element

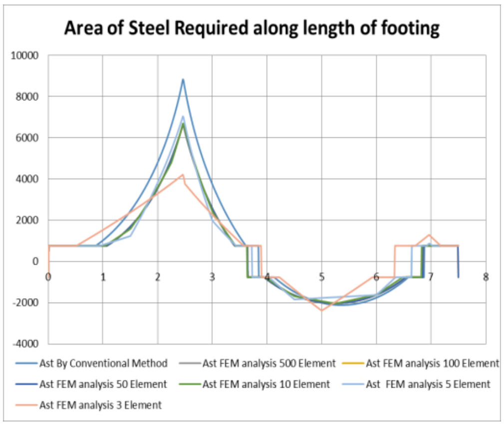


Fig 14 Area of steel required along length of footing for various Numbers of element

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B. Effect of modulus of sub grade reaction

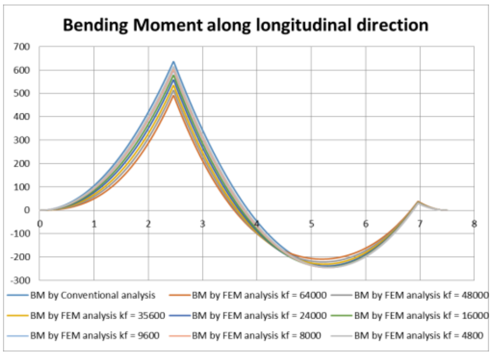

Fig 15 Bending Moment Diagram for various values of K_f

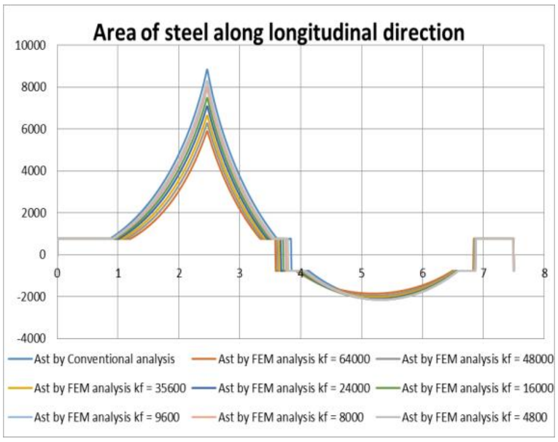

Fig 16 Area of steel required along length of footing for various values of K_f

IX. CONCLUSION

From the results obtained by analysis of combined rectangular footing following conclusion can be drawn:

1. In rigid analysis bearing pressure is low as compared to flexible analysis. Pressure variation is uniform in rigid analysis where as non- uniform or non-linear in flexible analysis.
2. The Bending moment gives higher value as compared to flexible analysis. Maximum bearing pressure obtained from flexible analysis is less than assumed allowable bearing pressure. Hence, the analysis is valid
3. For more accurate analysis it is suggested to analyze the combined footings with minimum 100 element, but 500 element analysis will gives more praise analysis
4. It is observed that increase in modulus of subgrade reaction will results in reduction of bending moment but increment in soil pressure therefore while taking value of K_f, plate load test should be carried out at each and every foundation location to determine accurate value of K_f.
5. Future scope: the study can be extended for trapezoidal footing and Strap footing, with or without Moment on one or both column.

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