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Peak Side Lobe Levels of Legendre and Rudin-Shapiro Sequences: Families of Binary Sequences

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Abstract: ***The peak side lobe level (PSL) is numerically estimated for Rudin-shapiro sequences and Legendre sequences which belong to the families of Binary sequences. Notable similarities are presented between PSL and merit factor behavior under cyclic rotations of the sequences (i.e. 1/4,1/2,3/4) rotations and we obtain a maximum merit factor of 3.5 in case of Rudin-shapiro sequence and maximum merit factor of 6 in case of Legendre sequence. In addition a detailed comparison of both Rudin-shapiro and Legendre sequence is provided.***

Index Terms: **Auto correlation for a-periodic sequences, Legendre sequences merit factor, peak side lobe level (PSL), Rudin-Shapiro sequences**

I. INTRODUCTION

Pulse compression techniques are used to reduce the length of the pulse to attain good range resolution, accuracy and target classification. Digital pulse compression techniques are widely adopted pulse compression techniques. There are many types of digital codes Barkers code is one of them. Barker code is a binary phase coded sequence of 0, pi values that gives equal side lobe values when passed through a matched filter. Though barker code gives rise to low autocorrelation side lobes, they are short codes with maximum length of 13.

Auto correlation is a method which is frequently used for extraction of fundamental frequency. If a phase shifted signal is obtained, the distance between the correlation peaks is taken as the fundamental period of the signal.

Auto correlation is one parameter which determines the goodness of a sequence. It should have very large values at zero shifts and very low values at non zero shifts. Other factors like discrimination, merit factor and energy efficiency is also similar parameters used to determine goodness of a sequence.

A length n binary sequence, given as A= (a_0, a_1……a_{n-1}) where a_i= 1 or -1 for each i=0, 1….n-1. The auto correlation function of an aperiodic sequence at a shift u is defined using the term

$$C_a(u) = \sum_{i=0}^{n-u-1} a_i a_i + u$$

Based on autocorrelation various goodness parameters such as

A. Discrimination (d): it is defined as the ratio between main peaks in auto correlation to absolute amplitude of side lobes.

B. Merit factor (F (A)): which is defined as the ratio between energy in the main lobe of autocorrelation to total energy in the side lobes. This merit factor introduced by Golay [1].

$$F(A) = \frac{n^2}{2\sum_{u=1}^{n-1}[C_A(u)]^2} \quad for\, n > 1$$

C. The third measure is the PSL given by

$$M(A) := \max_{1 \le u \le n-1} |C_A(u)|$$

A_n is denoted as set of the binary sequence of length n, we evaluate the behavior as n->?

$$M_n := \min_{A \in A_n} M(A)$$

We compare its asymptotic behavior with that of $1/F_n$ where F_n = max F (A). In order to compute M_n, it is required to evaluate 2^n sequences; an effective algorithm reduces the exponential value from 2^n to 1.4^n. So far the value of M_n has been computed till n=70, we will use the function "o" to indicate the desired PSL growth rate.

Practical boundaries for M_n values

1. $M_n \le 2$ for $n \le 21$ [2], where M_n=1, evaluated for barker sequence of length n= 2,3,4,5,7,11 and 13.
2. $M_n \le 3$ for $n \le 48$ [3] for $n \le 40$ cohen, Fox and Baden ,1990 for $n \le 48$)
3. $M_n \le 4$ for $n \le 70$ [4] for $49 \le n \le 61$ [5] for $61 \le n \le 70$)
4. Levanon and Mozeson [6, table (6.3)] listed a sequence for values of $n \le 69$.

Theoretical bounds on M_n values

In the early 1968 some theorems were put forward which gives the theoretical bounds of M_n.

Theorem1.1(Moon and Moser [7]): If K(n) is a function of n such that K(n)=o($\sqrt{n}$) then the proportion of sequences A∈An for which M(A)> K(n) approaches 1 as n→∞.

Theorem 1.2 (Moon and Mooser [7]): For any fixed $\varepsilon > 0$ the proportion of sequences $A \in An$, such that Mn≤ $(\sqrt{2}+\varepsilon)\sqrt{n\, lnn}$ approaches 1 as n→ ∞.

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The constant term in theorem 1.2 has been improved giving rise to

Theorem 1.3 (Mercer [8]): For any fixed ε >0 Mn≤ $((\sqrt{2}+\varepsilon)\sqrt{n}\ln n)$ where n is sufficiently large.

II. BINARY SEQUENCES

Binary sequences are widely employed in digital pulse compression techniques. Binary sequences have an important property that every counting number can be expressed as sum of one or more of its terms. Binary sequence is a Boolean valued function it is a sequence of 0's and 1's. Binary sequences are generally employed for controlling the synchronization between the transmitter and the receiver. Here we discuss mainly two types of binary sequences i.e., the Legendre sequence and the Rudin-shapiro sequence, and compare their efficiencies.

A. TYPES OF BINARY SEQUENCES

1. Legendre Sequences

Legendre sequences, belong to the family of binary sequences of primal lengths, there are number of useful properties of Legendre sequences, which used together with quadratic reciprocity, can be used to compute its efficiency. For obtaining a better merit factor the Legendre sequence is rotated by a factor.

2. Rudin-Shapiro Sequence

Rudin-Shapiro sequence belongs to the family of binary sequences of length 2m, where m= {0, 1, 2…}. It has two complimentary pair denoted as X and Y. Rudin-shapiro sequence is an example of a binary sequence which has no periodic property. It has been proved that the merit factor has no change even when the Rudin-shapiro sequence undergoes rotations by factor r where r= (1/4, 1/2, 3/4).

2.1 Rudin-Shapiro Sequence: Rudin - Shapiro sequence is a type of binary sequence, which consists of a pair of complimentary sequences that is A, A^1.

$$A=(a_o, a1\ldots\ldots a_{n-1}) \text{ of length } n$$

$$A`=(a_o{}^1, a_1{}^1\ldots\ldots\ldots a_{n`-1}{}^1) \text{ of length } n`$$

The sequences A, A^1 gives rise to another sequence B= (b_o, b1……b_{n+n^1-1}) of length $n+n^1$

$$bi=\begin{cases} ai & for\ 0<i<n \\ a`i-n & for\ n<i<n+n` \end{cases}$$

The merit factor of the sequence A is equal to merit factor of A` which is also equal to the merit factor of the sequence B. Complimentary pair Xm, Y^m of the Rudin – Shapiro sequence have the length in order of 2^m (i.e. 1,2,4,8…..) and are defined such that $X^0=Y^0=1$.

$$\left.\begin{array}{l} X^{(m)}:=X^{(m-1)};Y^{(m-1)} \\ Y^{(m)}:=X^{(m-1)};-Y^{(m-1)} \end{array}\right\} for\ m>0$$

For the generated sequence merit factor is calculated using the auto correlation function C_a (u).In 1968, little wood determined the exact merit factor of a Rudin-Shapiro sequence of any length 2^m

Theorem 2.3 (Little wood[9,p.28]).the merit factor of a both sequences X^m and Y^m of a Rudin- Shapiro sequence the merit factor can also be calculated using the formula:

$$\frac{3}{1-(-1/2)^m}$$

The sequences are rotated by a factor r (r=1/2,1/4,3/4) and a new sequence is formed.

$$b_i := a(i+\lfloor rn \rfloor)\ \ mod\ n$$

For each new sequence the merit factor is examined.

Note that for a Rudin Shapiro sequence, the merit factors of all the new sequences generated by rotating the old sequence are equal.

B. Legendre Sequence: The legendary sequence belongs to the family of binary sequences of prime length n (i.e. length= 3 or 5 or7…). The legendary sequence is also referred as quadratic residue sequence. Consider a Legendre sequence X={x_0, x1……} of length n, it is defined as follows:

$$x_i := \begin{cases} 1, if\ i\ a\ quadratic\ residue\ mod\ n \\ -1, \quad otherwise \end{cases}$$

By always assuming x_0 =1 always, we can use the quadratic residue mod method that means
If there is an integer such that 0<x<p such that $^{X}{}_{n}$=q (mod p)

If the congruence has a solution then q is said to have a quadratic residue i.e. (q mod p) [10]. The trivial case q=0 is excluded from list of quadratic residues, so that the number of quadratic residues (mod n) is taken as one less than the number of squares of (mod n). The other source includes 0 as solution if congruence has no solution. Then q is said to be quadratic non residue (mod p).

Thus the general legendary sequence is formed in terms of 1 and -1.The auto correlation values are generated for the given legendary sequence that is Ca (u) values.Based on auto correlation values the merit factor is calculated using the formula F (A).For obtaining a better merit factor sequence A= (a_o, a1…..a_{n-1}) of length n is rotated through a rotational factor r a new sequence B is

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generated which is of same length n and given as B= (b_o, b1…..b_{n-1}), such that

$$b_i := a(i + \lfloor rn \rfloor)\ mod\ n$$

In 1988 Høholdt and Jensen [11], building on earlier work of Turyn (reported in and Golay [12], established By calculating the B sequence for different r values (i.e. r= 1/4, 1/2, 3/4) different merit factors are evaluated (i.e. F (A)).

III. COMPARISON BETWEEN LEGENDRE AND RUDIN SHAPIRO SEQUENCE

Legendre sequences are prime length sequences which has a highest merit factor (i.e. maximum of 6) when compared to Rudin-Shapiro sequence of length in order of 2^m, whose merit factor is approximately commutated as 3.5.

1. The family of Legendre sequence and their rotations has the most desirable PSL growth which is of order $O(\sqrt{n}\ln n)$, this is not likely to occur in Rudin-Shapiro sequences.

2. The merit factor is different for different rotations of a Legendre sequence whereas the merit factor remains same for different rotations of Rudin – Shapiro sequence.

3. Legendre sequence maintains periodic property where as Rudin-Shapiro sequence does not maintain aperiodic property.

4. In Rudin-Shapiro sequence a pair of complimentary sequence are considered, whose merit factors if calculated are equal, for a Legendre sequence only single sequence is considered.

IV. EXPERIMENT RESULTS

A. Estimation of PSL in Legendre Sequence

The desired growth of PSL in Legendre sequence is compared with factors $\sqrt{n}$ and $\sqrt{n}\ln n$. Consider a set R= {0, 1/n….n-1/n}, considering a Legendre sequence X={X_0, X1…… we calculate the function M (x_r) for all r$\in R$, for different values of n.The graph is evaluated based on the table-1values.

Table 1: merit factors for different rotations to different lengths of Legendre Sequences

Length	merit factor	for 1/4th	for 1/2th	for 3/4th
59	1.5582	5.6327	1.788	*6.1940*
127	1.5335	*6.0048*	1.4827	5.0435
131	1.5309	5.8570	1.6549	*6.3231*
179	1.5129	5.8533	1.5754	*6.0205*
229	1.4903	*6.0222*	1.3562	*6.0222*
251	1.5152	*6.0613*	1.5791	*6.0380*
419	1.5085	*6.0152*	1.5458	*6.0401*
467	1.5021	5.8485	1.5197	*6.0416*
491	1.5055	5.8263	1.5324	*6.1812*
563	1.5037	5.9560	1.5240	*6.0093*
659	1.5048	5.9672	1.5272	*6.0557*
971	1.5046	5.9937	1.5238	*6.0649*
1019	1.5025	6.0056	1.5155	5.9937
1091	1.5049	5.9989	1.5246	6.08
1213	1.4981	6.0069	1.4677	6.0069
1259	1.5023	5.9937	1.5137	6.0146
1279	1.5074	6.0128	1.4982	5.8773
1283	1.5004	5.9487	1.5062	6.003
1307	1.5004	5.9501	1.506	6.0014
1423	1.4996	6.0203	1.4984	5.881
1427	1.5016	5.9879	1.5104	6.0067
1471	1.5054	5.8952	1.4985	6.0088
1531	1.5001	5.9456	1.5041	6.0054
1571	1.5019	6.0245	1.5113	5.9851
1619	1.5011	5.9777	1.5079	6.0076
1667	1.5004	6.0026	1.5052	5.9638
1811	1.5033	6.0373	1.5164	6.0229
1931	1.5022	5.9949	1.5117	6.0329
1949	1.4988	6.0172	1.4882	6.0172
1979	1.5027	6.0047	1.5136	6.0395
2099	1.5013	6.0007	1.5078	6.0015
2179	1.4993	6.0177	1.5001	5.9261
2213	1.499	6.0097	1.4847	6.0097
2309	1.499	6.0039	1.489	6.0039
2339	1.5009	6.0053	1.5062	5.9902
2371	1.4999	5.9579	1.5023	6.0076
2381	1.4991	6.0106	1.4848	6.0106
2411	1.5016	6.0081	1.5089	6.0108
2459	1.5008	5.9867	1.5056	6.0064
2579	1.501	6.0248	1.5063	5.9768
2659	1.4998	5.9622	1.5017	6.0046
2939	1.502	6.0201	1.5098	6.0151
3011	1.5006	5.9807	1.5045	6.0136
3251	1.5018	6.0183	1.5092	6.0161
3299	1.5012	6.0058	1.5066	6.0085
3331	1.4999	5.9708	1.5015	6.0036
3449	1.4993	6.0088	1.5023	6.0088
3461	1.4994	6.0098	1.4926	6.0098
3467	1.5002	5.9777	1.5026	6.0068
3491	1.5006	6.0073	1.5041	5.989
3539	1.5006	5.9908	1.5039	6.005
3571	1.4999	5.9663	1.5013	6.0088

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3659	1.5011	6.0004	1.5062	6.0146
3691	1.4998	5.9701	1.5007	6.0018
3701	1.4994	6.0062	1.4927	6.0062
3779	1.5013	6.0172	1.5067	6.0029
3851	1.5006	6.0085	1.504	5.9906
3923	1.5004	5.9888	1.5031	6.0043
4091	1.5013	6.0107	1.5065	6.0104
4099	1.4998	5.9724	1.5009	6.0042
4211	1.5003	6.0002	1.5027	5.9914
4259	1.5013	6.0058	1.5068	6.0188
4283	1.5002	6.0053	1.5021	5.9819
4451	1.5007	6.0022	1.5041	6.0021
4651	1.4999	5.9729	1.501	6.0079
4691	1.5001	5.9844	1.5017	6.0023
4787	1.5003	6.0022	1.5025	5.9913
4931	1.5009	6.0005	1.505	6.0139
5051	1.5005	6.0007	1.5031	5.9993
5099	1.5012	6.0076	1.5059	6.0148
5147	1.5	5.9812	1.501	6.0027
5171	1.5008	6.0077	1.5045	6.0041
5651	1.5004	5.9939	1.5029	6.0067
5779	1.4998	5.973	1.5002	6.0066
5851	1.5	6.0013	1.501	5.9853
5939	1.5006	6.0056	1.5034	6.0002
5981	1.4996	6.0022	1.4962	6.0022
6011	1.5002	5.9915	1.5018	6.002
6131	1.5003	6.0036	1.5024	5.995
6299	1.5009	6.0128	1.5047	6.0046
6301	1.4996	6.0016	1.4949	6.0016
6451	1.4998	6.0017	1.5004	5.982
6491	1.5003	5.992	1.5021	6.0055
6659	1.5	5.9882	1.501	6.0005
6691	1.4999	6.0031	1.5007	5.984
6779	1.5006	6.0072	1.5032	6
6899	1.5004	6.0023	1.5025	5.9991
7019	1.5007	6.0059	1.5037	6.006
7211	1.5003	5.9977	1.5022	6.0027
7451	1.5003	5.9961	1.5021	6.0037
7499	1.5002	5.996	1.5018	6.0015
7523	1.5003	5.9995	1.502	6
7691	1.5006	6.0065	1.5031	6.0018
7907	1.4999	5.9877	1.5005	6.0002
8147	1.5003	6.0013	1.502	5.9988
8219	1.5002	6.0007	1.5017	5.997

8291	1.5006	6.013	1.5032	5.9974
8363	1.5002	5.9965	1.5016	6.0014
8741	1.4997	6.0018	1.4968	6.0018
8819	1.5006	6.0076	1.5031	6.0027
8867	1.5	5.9901	1.5007	6.0017
8923	1.4999	6.0004	1.5002	5.9874
9059	1.5003	5.9955	1.5018	6.0047
9349	1.4998	6.0037	1.4966	6.0037
9371	1.5005	6.0061	1.5027	6.0022
9419	1.5001	5.9949	1.5013	6.0018
9467	1.5003	6.0006	1.5018	6.0004
9491	1.5004	5.9997	1.5022	6.0045
9539	1.5007	6.0051	1.5034	6.0085
9811	1.4999	6.0006	1.5003	5.9887
9851	1.5003	5.9946	1.502	6.0089
10091	1.5002	6.0019	1.5014	5.9967
10099	1.4999	5.9903	1.5004	6.0007
10139	1.5006	6.0095	1.503	6.0017
10211	1.5003	6.0039	1.5017	5.9973
10331	1.5006	6.0047	1.5028	6.0058
10499	1.5002	6.0002	1.5014	5.9993
10691	1.5003	5.9974	1.5017	6.0044
10709	1.4998	6.0067	1.4979	6.0067
10739	1.5001	6.002	1.5011	5.9948
10859	1.5003	5.9988	1.5016	6.0027
10979	1.5001	5.9952	1.5009	6.0005
11059	1.4999	5.9905	1.5003	6.0009
11119	1.501	6.0002	1.4998	5.9869
11131	1.4999	5.9899	1.5003	6.0016
11171	1.5006	6.0043	1.503	6.0085
11491	1.5	6.003	1.5004	5.9892
11579	1.5003	5.9967	1.5016	6.0052
11681	1.4998	6.0026	1.5001	6.0026
11699	1.5003	6.0021	1.5017	6.0007
11779	1.5	5.9917	1.5004	6.0006
11987	1.5001	5.9967	1.5011	6.0014
12011	1.5006	6.0078	1.503	6.0056
12227	1.5001	6.0003	1.501	5.9977
12251	1.5002	6.0031	1.5014	5.9979
12379	1.4999	5.9907	1.5003	6.0012
12421	1.4998	6.0004	1.4981	6.0004
12491	1.5004	6.0042	1.5019	6.0011
12539	1.5001	5.9949	1.5009	6.002
12589	1.4998	6.0017	1.4974	6.0017

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12611	1.5001	5.9973	1.501	6.0004
12619	1.5	5.9934	1.5006	6.0011
12659	1.5002	5.9963	1.5012	6.0033
12899	1.5005	6.0037	1.5026	6.0071
12907	1.4999	6.0001	1.5	5.9903
13043	1.5	5.9937	1.5004	6.0001
13109	1.4998	6.0019	1.4986	6.0019
13259	1.5002	5.9986	1.5012	6.0014
13331	1.5003	6.0011	1.5018	6.0039
13421	1.4998	6.001	1.4978	6.001
13499	1.5003	6.0041	1.5018	6.0006
13691	1.5002	5.9979	1.5011	6.0017
13709	1.4998	6.0007	1.4978	6.0007
13859	1.5005	6.0061	1.5024	6.0038
13901	1.4998	6.0023	1.4981	6.0023
13907	1.5001	6.0015	1.501	5.9972
13931	1.5	5.9921	1.5004	6.0025
14251	1.5	6.0029	1.5004	5.9916
14407	1.5002	5.99	1.4998	6
14669	1.4998	6.0014	1.4979	6.0014
14699	1.5002	6.0014	1.5014	6.0009
14731	1.5	6.0008	1.5003	5.9932
14747	1.5001	5.9949	1.5008	6.0027
14771	1.5002	6.0035	1.5011	5.9971
14867	1.5	6.0001	1.5006	5.9962
14891	1.5002	5.9992	1.5011	6.0013
15083	1.5001	6.0025	1.5009	5.9963
15107	1.4999	6.0001	1.5002	5.9931
15131	1.5002	6.0008	1.5013	6.0011
15149	1.4999	6.0005	1.4981	6.0005
15199	1.5004	5.9898	1.4999	6.0008
15319	1.5007	5.9902	1.4999	6.0004
15331	1.4999	6.0004	1.5	5.9915
15451	1.4999	6.0002	1.5001	5.9924
15683	1.5001	5.9975	1.5007	6.0004
15859	1.5	5.9937	1.5002	6.0001
16091	1.5003	6.0015	1.5014	6.0019
16141	1.4999	6.0006	1.4978	6.0006
16187	1.5	5.9967	1.5006	6
16619	1.5	5.9937	1.5003	6.0011
16691	1.5002	6.003	1.5011	5.9984
16763	1.5001	6.0009	1.5007	5.9973
16883	1.5001	5.9971	1.5006	6.0005
16931	1.5003	6.0013	1.5017	6.0046

16979	1.5002	6.0012	1.5014	6.0022
17011	1.4999	6	1.5	5.9927
17099	1.5002	6.0005	1.5012	6.0021
17291	1.5002	5.9993	1.501	6.0018
17491	1.4999	5.9939	1.5002	6.0003
17573	1.4999	6.0003	1.4984	6.0003
17579	1.5001	6.0002	1.5008	5.9989
17837	1.4999	6.0002	1.4983	6.0002
17891	1.5002	6.0035	1.5012	5.9992
17939	1.5003	5.9999	1.5016	6.0056
18059	1.5001	6.0015	1.501	5.9991
18131	1.5001	5.9978	1.5009	6.0028
18251	1.5001	6.001	1.5007	5.9978
18443	1.5002	5.9995	1.501	6.0015
18539	1.5002	5.9982	1.5011	6.004
18691	1.5	5.9945	1.5002	6.0005
18731	1.5	5.9968	1.5004	6
18899	1.5002	6.0016	1.5011	6.0003
18979	1.4999	6.0005	1.5	5.9929
19139	1.5001	5.9999	1.5008	6.0002
19211	1.5003	6.0032	1.5015	6.0024
19379	1.5004	6.0033	1.5018	6.0044
19421	1.4999	6.0012	1.4988	6.0012
19471	1.5003	5.9924	1.4999	6.0002
19531	1.4999	6.0004	1.5	5.9932
19571	1.5002	6.0025	1.5012	6.0009
19699	1.4999	6.001	1.5001	5.9931
19739	1.5001	6.0018	1.5009	5.9991
19751	1.5019	5.9919	1.4999	6.0008
19763	1.5	6.0002	1.5004	5.9963
20051	1.5001	6.0008	1.5007	5.9985
20219	1.5001	6.0007	1.5009	6.0001
20411	1.5003	6.002	1.5015	6.0039
20627	1.5001	6.0008	1.5008	5.9993
20773	1.4999	6.0005	1.4983	6.0005
20939	1.5001	5.9967	1.5005	6.0015
20963	1.5001	5.9974	1.5005	6.0008
21059	1.5002	6.0019	1.5011	6.0006
21089	1.4999	6.0005	1.4996	6.0005
21179	1.5	6.0008	1.5005	5.997
21419	1.5002	6.0021	1.501	6.0001
21467	1.5001	5.9981	1.5007	6.0013
21491	1.5	5.9968	1.5004	6.0003
21601	1.4999	5.9936	1.4986	5.9936

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21611	1.5001	6.0001	1.5008	6.0006
22091	1.5002	6.0027	1.5013	6.0018
22093	1.4999	6.001	1.4984	6.001
22259	1.5002	6.0043	1.5013	6.0001
22307	1.5001	6	1.5006	5.9992
22531	1.4999	5.9937	1.5	6.0008
22571	1.5003	6.004	1.5015	6.0024
22619	1.5001	6	1.5005	5.9984
22643	1.5001	5.9982	1.5005	6.0005
22699	1.5	6.0006	1.5003	5.9965
22739	1.5002	6.0027	1.5011	6.0004
22811	1.5002	6	1.5009	6.0021
22859	1.5	5.9975	1.5005	6.0009
23459	1.5001	6.001	1.5006	5.9983
23531	1.5	6.0001	1.5005	5.9982
23747	1.5	5.9975	1.5004	6.0001
23819	1.5002	6.0022	1.5012	6.0017
23971	1.5	6.0003	1.5003	5.9971
24061	1.4999	6.0016	1.4986	6.0016
24229	1.4999	6.0006	1.4985	6.0006
24251	1.5002	5.9996	1.501	6.0032
24371	1.5001	6.0003	1.5007	5.9999
24419	1.5001	5.9993	1.5008	6.0021
24659	1.5002	6.0026	1.501	6
24683	1.5	5.9976	1.5004	6.0003
24851	1.5	5.9968	1.5004	6.0017
24923	1.5001	6.0002	1.5005	5.9989
24979	1.5	6.0001	1.5001	5.9955
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25579	1.4999	5.9945	1.5	6.0006
25763	1.5	5.9966	1.5003	6.0007
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25931	1.5002	6.0013	1.5009	6.0012
26099	1.5002	6.0026	1.5011	6.0015
26171	1.5002	6.0015	1.5012	6.0035
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26459	1.5001	5.9984	1.5005	6.0008
26539	1.5	6.001	1.5001	5.9955
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26699	1.5001	6.0009	1.5007	5.9998
26987	1.5	6.0001	1.5002	5.9966
27011	1.5003	6.002	1.5013	6.0033

27059	1.5001	5.9991	1.5006	6.0012
27179	1.5002	6.0018	1.501	6.0017
27299	1.5002	6.0027	1.5011	6.0017
27539	1.5001	6.0011	1.5006	5.9988
27611	1.5001	6.0007	1.5008	6.0012
27779	1.5	6	1.5004	5.9988
27803	1.5001	5.9998	1.5005	6.0001
27851	1.5001	6.0005	1.5007	6.0006
27947	1.5001	5.998	1.5005	6.0018
28019	1.5003	6.0023	1.5013	6.0034
28211	1.5001	6.0003	1.5006	6.0004
28229	1.4999	6.0019	1.4988	6.0019
28411	1.5	5.9948	1.5001	6.0014
28571	1.5	5.9978	1.5004	6.0011
28619	1.5002	6.0021	1.501	6.0013
28643	1.5001	6.0006	1.5006	5.9996
28859	1.5002	6.0027	1.501	6.0013
28979	1.5	5.9979	1.5003	6.0005
29123	1.5001	6.0006	1.5006	6.0002
29147	1.5	6.0001	1.5003	5.9983
29179	1.5	5.9963	1.5002	6.0006
29363	1.5	6	1.5004	5.9988
29411	1.5001	6.0005	1.5007	6.0005
29527	1.5001	5.995	1.4999	6.0001
29531	1.5002	6.0012	1.5009	6.0017
29683	1.4999	6.0001	1.5	5.9958
29819	1.5002	6.0029	1.501	6.0006
30059	1.5001	5.998	1.5004	6.0012
30203	1.5	5.9985	1.5004	6.0005
30491	1.5001	6.0004	1.5007	6.0007
30851	1.5001	5.9999	1.5006	6.0005
30971	1.5001	6.0004	1.5005	5.9998
31091	1.5	5.9969	1.5002	6.0004
31139	1.5001	6.0015	1.5007	6.0003
31259	1.5	6.0015	1.5003	5.9966
31379	1.5002	6.0023	1.5011	6.0024
31547	1.5	6.0004	1.5003	5.9982
31643	1.5	5.9985	1.5004	6.0004
31859	1.5001	6.0009	1.5005	5.9989
31891	1.5	6.0002	1.5002	5.9974
32051	1.5001	6.0011	1.5005	5.9992
32077	1.4999	6.0006	1.4988	6.0006
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32579	1.5001	6.0005	1.5006	6.0004

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32717	1.4999	6.0006	1.499	6.0006
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33107	1.5	5.9978	1.5003	6.0004
33179	1.5001	6.0005	1.5005	5.9998
33809	1.4999	6	1.4996	6
33851	1.5001	6.0026	1.5008	6
34019	1.5001	6.0008	1.5006	6.0006
34403	1.5	6.0006	1.5004	5.9987
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35339	1.5002	6.002	1.5008	6.0007
35419	1.5	5.9977	1.5002	6.0003
35507	1.5	5.9989	1.5003	6.0003
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35747	1.5001	5.9995	1.5004	6.0005
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36083	1.5	5.9982	1.5002	6
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36563	1.5	5.9977	1.5001	6.0001
36571	1.5	5.9975	1.5001	6.0002
36779	1.5001	6.0001	1.5004	5.9998
36979	1.5	6.0004	1.5001	5.9974
37171	1.5	6.001	1.5	5.996
37379	1.5001	5.9999	1.5006	6.0011
37447	1.5	5.996	1.4999	6.0001
37571	1.5001	6.0008	1.5006	6.0009
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37691	1.5002	6.0015	1.5008	6.0014
38219	1.5001	6.0001	1.5004	6
38677	1.4999	6	1.4991	6
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38891	1.5001	6.0023	1.5007	5.9998
39079	1.5005	6.0005	1.4999	5.9959
39181	1.4999	6.0005	1.499	6.0005
39371	1.5001	6.0012	1.5006	6.0003
39419	1.5001	6.0011	1.5005	6
39563	1.5001	6.0005	1.5004	5.9993
39659	1.5	5.999	1.5003	6
39779	1.5001	6.0012	1.5008	6.0017
39971	1.5001	6.0002	1.5006	6.0017
39989	1.4999	6	1.4997	6

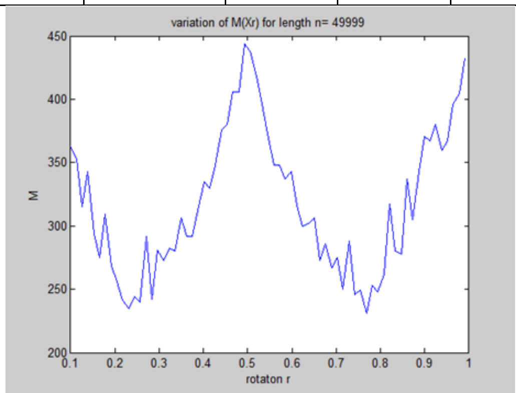

Fig a: variation of M (X_r) with the rotation factor r for n = 49999.

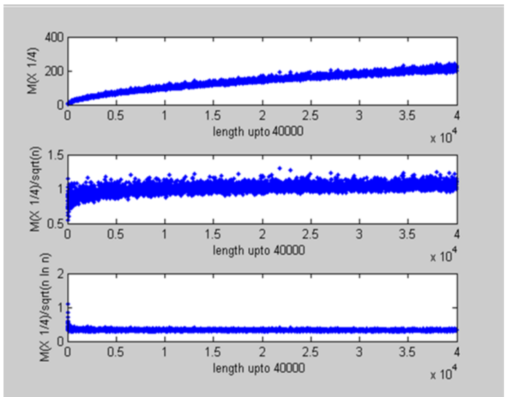

Fig b: Variation of M($X_{1/4}$), M($X_{1/4}$) / $\sqrt{n}$ and M($X_{1/4}$) / $\sqrt{n \ln n}$ for different length i.e. n = 40000.

variation of min $_{r \in R}$ ($X_{1/4}$) with length n for the first 3500 prime lengths (n $\leq$ 32609) and the difference of the sequence M($X_{1/4}$)/ $\sqrt{n \ln n}$ with n. and it is appear to approach a non zero constant.and it is the initial result of the growth of the PSL of Legendre sequences.

B. Estimation of PSL in Rudin –Shapiro Sequence

Here we pursue the same relation between the shape of the graphs of M and asymptotic 1/F as the rotational fraction r varies. We have taken the similarity lies periodic property. The property being equivalence to a difference set or partial difference set. We tested this

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assumption using the Rudin-Shapiro sequences, which have no known periodic property. The merit factor of Rudin-Shapiro sequences under cyclic rotation of the length 2^m. The graph is plotted based on values in Table-2.

Table 2: Merit factor for 2 complimentary sequences X and Y of a Rudin-Shapiro Sequence

S.No	Sequence length	M.F for X^m	M.F for Y^m	M.F for appending X^mY^m
1	M=3	2.667	2.667	2.667
2	M=4	3.200	3.200	3.200
3	M=5	2.909	2.909	2.909
4	M=6	3.047	3.047	3.047
5	M=7	3.000	3.000	3.000
6	M=8	3.011	3.011	3.011
7	M=9	2.994	2.994	2.994
8	M=10	3.002	3.002	3.002
9	M=11	2.998	2.998	2.998
10	M=12	3.007	3.007	3.007
11	M=13	2.999	2.999	2.999
12	M=14	3.000	3.000	3.000
13	M=15	2.999	2.999	2.999
14	M=16	3.000	3.000	3.000

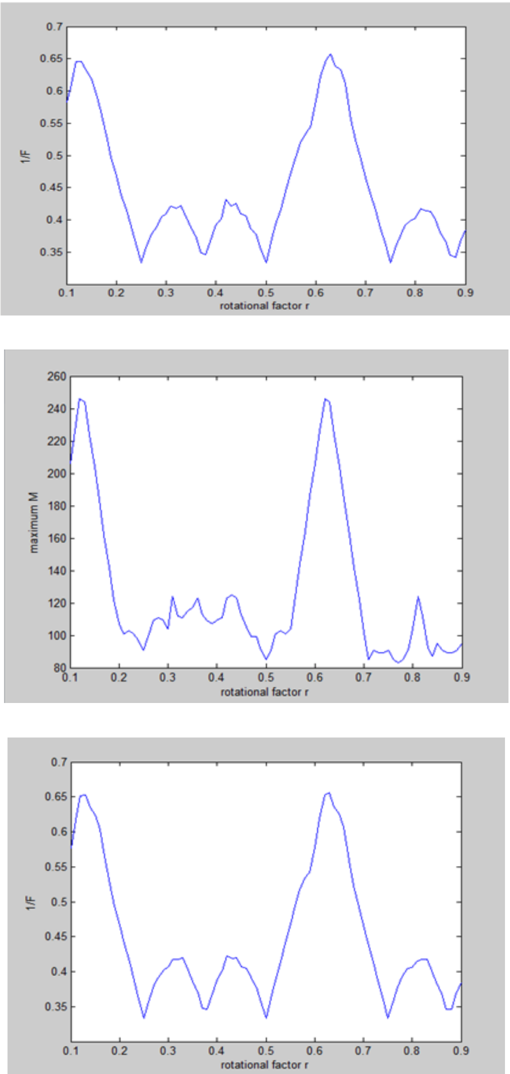

Fig c: variation of 1/F $((X^m)_r)$ with the rotational fraction r € R, for m= 10 and for m = 16. Same shapes of graph were obtained for all values of 9 ≤ m ≤ 16.

F $((X^m)_r)$ appears to lie between 3/2 and 3 for all r, when m is large.

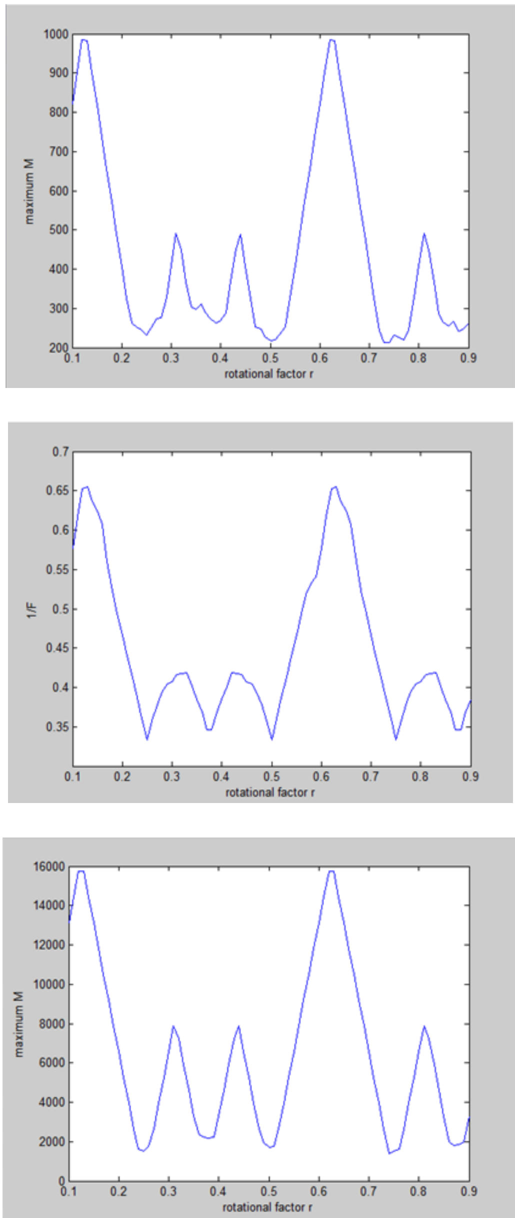

Fig d: variation of M (F $((X^m)_r)$) with r € R for m= 10, 12, and 16.

The shape of the graph becomes more perfect as m increases. apparently approaching a piecewise linear function with minima at r = 0, ¼, 3/8 , ½, ¾ and 7/8.and we observe a similarity between the graphs of M and 1/F as r varies in fig 4 and 5.this phenomenon is not restricted to sequences having an underlying periodic property.

We apply the same property and calculations for the other sequence Y^m of the Rudin –Shapiro sequence. The corresponding graphs, both for M and 1/F, appeared to be the reflection of those for X^m for r = ½.

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V. CONCLUSION

The PSL (Peak Side Lobe) level value of Legendre sequence is evaluated to have the desired growth rate of order $o\sqrt{n}\ln n$.

Legendre sequence of prime length n is so far evaluated to have the highest Merit Factor of 6.Rudin-Shapiro sequence of length2^m does not give the desired PSL growth level; it is evaluated to have the Merit factor of 3.5

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