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Inverse Thermo elastic Problem of Semi infinite Rectangular Beam due to Heat Generation

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Abstract-This paper is concerned with the determination of temperature distribution, unknown temperature gradient and thermal deflection of the plate with the stated boundary conditions. The transient heat conduction equation is solved by using the finite Fourier cosine and Marchi-Fasulo transforms techniques. The results are obtained in terms of Bessel's function in the form of infinite series.

Index Terms **– Transient thermoelastic problem, temperature distribution, thermal stresses, semi-infinite rectangular beam.**

I. INTRODUCTION

Adams and Bert [1], Tanigawa and Komatsubara [4] and Vihak et.al [5] have studied the direct problem of the thermo elasticity in a rectangular plate under thermal shock. Khobragade et.al. [2, 6, 7] has studied the inverse steady state thermoelastic problem to determine the temperature, displacement function and thermal stresses at the boundary of a thin rectangular plate. They have used the finite Fourier sine transform techniques. Khobragade and Lamba [3] have studied three dimensional coupled thermoelastic response of infinitely long hallow circular cylinder due to axisymmetric heating, considered under the thermo-mechanical coupling effect. This approach is based upon expressions for both temperature and the stress distribution and are determined from field equation of motion. Numerical calculations are carried out and results are depicted graphically.

In the present paper, an attempt has been made to determine the temperature distribution, displacement function and thermal stresses of a thin rectangular beam occupying the region D : $-a \le x \le a$; $-b \le y \le b$, $0 \le z \le \infty$ with known boundary conditions. Here Marchi-Fasulo transforms and Fourier cosine transform techniques have been used to find the solution.

II. STATEMENT OF THE PROBLEM

Consider a thin rectangular plate occupying the space D : $-a \le x \le a$; $-b \le y \le b$, $0 \le z \le \infty$ The displacement components u_x and u_y u_z in the x and y and z directions respectively as Tanigawa et.al [4] are

$$u_x = \int_{-a}^{a}\left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial y^2}+\frac{\partial^2 U}{\partial z^2}-v\frac{\partial^2 U}{\partial x^2}\right)+\lambda T\right]dx \quad (1)$$

$$u_y = \int_{-b}^{b}\left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial z^2}+\frac{\partial^2 U}{\partial x^2}-v\frac{\partial^2 U}{\partial y^2}\right)+\lambda T\right]dy \quad (2)$$

$$u_z = \int_{0}^{\infty}\left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}-v\frac{\partial^2 U}{\partial z^2}\right)+\lambda T\right]dz \quad (3)$$

Where E, ν, and λ are the young's modulus, Poisson's ratio and the linear coefficient of the thermal expansion of the material of the beam respectively and U (x,y,z,t) is the Airy's stress functions which satisfy the differential equation [4]:

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)^2 U(x,y,z,t) = -\lambda E\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right) \times T(x,y,z,t) \quad (4)$$

where T(x,y,z,t) denotes the temperature of a rectangular beam satisfy the following differential equation [4] :

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{g(x,y,z,t)}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \quad (5)$$

Where K is the thermal conductivity and α is the thermal diffusivity of the material,

subject to initial condition

$$T(x,y,z,0) = f(x,y,z) \quad (6)$$

The boundary conditions are

$$\left[T(x,y,z,t)+k_1\frac{\partial T(x,y,z,t)}{\partial x}\right]_{x=a} = f_1(y,z,t) \quad (7)$$

$$\left[T(x,y,z,t)+k_2\frac{\partial T(x,y,z,t)}{\partial x}\right]_{x=-a} = f_2(y,z,t) \quad (8)$$

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$$[T(x,y,z,t)]_{y=0} = f_3(x,z,t) \qquad (9)$$

$$[T(x,y,z,t)]_{y=\xi} = f_4(x,z,t) \qquad (10)$$

(10)

$$[T(x,y,z,t)]_{y=b} = G(x,z,t) \qquad (11)$$

$$\left.\frac{\partial T(x,y,z,t)}{\partial z}\right|_{z=0} = 0 \qquad (12)$$

$$\left.\frac{\partial T(x,y,z,t)}{\partial z}\right|_{z=\infty} = 0 \qquad (13)$$

(13)

The stress components in terms of U(x, y, z , t) are given by

$$\sigma_{xx} = \left[\frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2}\right] \qquad (14)$$

$$\sigma_{yy} = \left[\frac{\partial^2 U}{\partial z^2} + \frac{\partial^2 U}{\partial x^2}\right] \qquad (15)$$

(15)

$$\sigma_{zz} = \left[\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2}\right] \qquad (16)$$

The equations (1) to (16) constitute the mathematical formulation of the problem under consideration.

III. SOLUTION OF THE PROBLEM

Applying finite Marchi-Fasulo transform (twice) defined in [] and Fourier cosine transform to the equations, we have

$$\frac{d\overline{\overline{T}}_c^*}{dt} + \alpha q^2 \overline{\overline{T}}_c^* = \frac{\alpha \overline{\overline{g}}_c^*}{k} + \Psi$$

This is a linear equation whose solution is given by

$$\overline{\overline{T}}_c^*(m,n,\eta,t) = \int_0^t \left[\frac{\alpha \overline{\overline{g}}_c^*}{k} + \Psi\right] e^{-\alpha q^2 (t-t')} dt' \qquad (17)$$

Where, $\phi = \frac{Q_n(a)}{k_1} f_1 - \frac{Q_n(a)}{k_2} f_2$

, $\Phi = \frac{P_m(b)}{k_3} f_3 - \frac{P_m(-b)}{k_4} f_4$, $\phi + \Phi = \Psi$

Now, applying inversion of Fourier Cosine transform and finite Marchi-Fasulo transform [7] to the equation (16),one obtains the expression for temperature distribution as

$$T(x,y,z,t) = \frac{2\eta}{\pi} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left(\frac{Q_n(x)}{\lambda_n}\right) (\sin y) \Lambda(z) \qquad (18)$$

$$G(x,z,t) = \frac{2\eta}{\pi} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left(\frac{Q_n(x)}{\lambda_n}\right) (\sin b) \Lambda(z) \qquad (19)$$

Where,

$$\Lambda(z) = \int_0^{\infty} B(t) \cos(\eta z) dz \ ,$$

$$B(t) = \int_0^t \left[\frac{\alpha \overline{\overline{g}}_c^*}{k} + \Psi\right] e^{-\alpha q^2 (t-t')} dt' \qquad (20)$$

which are the required solutions.

IV. AIRY'S STRESS FUNCTIONS

Substituting the value of temperature distribution T(x,y,z,t) from (16) in equation (4)

one obtains

$$U(x,y,z,t) = -\frac{2\eta\pi E}{\pi} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left(\frac{Q_n(x)}{\lambda_n}\right) (\sin y) \Lambda(z)$$

(20)

V. DISPLACEMENT COMPONENTS

Substituting the values of Airy's stress function from equation (18) in the equation (1) to (3), one obtains

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$$u_x = -\frac{2\eta\lambda}{\pi}\Lambda(z) \times \int_{-a}^{a}\left\{\begin{array}{l}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{Q_n(x)}{\lambda_n}\right)\left(-m^2\sin my\right)-\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}-\eta^2\left(\frac{Q_n(x)}{\lambda_n}\right)(\sin my)\\ -\upsilon\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{Q_n''(x)}{\lambda_n^2}\right)(\sin my)+\lambda\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{Q_n(x)}{\lambda_n}\right)(\sin my)\end{array}\right\}dx \quad (21)$$

$$u_y = -\frac{2\eta\lambda}{\pi}\Lambda(z) \times \int_{0}^{\xi}\left\{\begin{array}{l}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\eta^2\left(\frac{Q_n(x)}{\lambda_n}\right)(\sin my)-\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{Q_n''(x)}{\lambda_n^2}\right)(\sin my)\\ +\upsilon\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{Q_n(x)}{\lambda_n}\right)\left(-m^2\sin my\right)+\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{Q_n(x)}{\lambda_n}\right)(\sin my)\end{array}\right\}dy \quad (22)$$

$$u_z = -\frac{2\eta\lambda}{\pi}\Lambda(z) \times \int_{0}^{\infty}\left\{\begin{array}{l}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{Q_n''(x)}{\lambda_n^2}\right)(\sin my)+\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{Q_n(x)}{\lambda_n}\right)\left(-m^2\sin my\right)\\ +\upsilon\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\eta^2\left(\frac{Q_n(x)}{\lambda_n}\right)(\sin my)-\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{Q_n(x)}{\lambda_n}\right)(\sin my)\end{array}\right\}dz \quad (23)$$

VI. DETERMINATION OF STRESS FUNCTION

Substituting the value of Airy's stress function U(x,y,z,t) from equation (18) in the equation (13) to (15) one obtain the stress functions as,

$$\sigma_{xx} = -\frac{2\eta\lambda E}{\pi}\Lambda(z) \times\sum_{n=1}^{\infty}\left(\frac{Q_n(x)}{\lambda_n}\right)\left[\sum_{m=1}^{\infty}\left(-m^2\sin my\right)-\eta^2\sum_{m=1}^{\infty}(\sin my)\right] \quad (24)$$

$$\sigma_{yy} = -\frac{2\eta\lambda E}{\pi}\Lambda(z)\sum_{m=1}^{\infty}(\sin my)\left[\sum_{n=1}^{\infty}\left(\frac{Q_n''(x)}{\lambda_n^2}\right)-\eta^2\sum_{n=1}^{\infty}\left(\frac{Q_n(x)}{\lambda_n}\right)\right] \quad (25)$$

$$\sigma_{zz} = -\frac{2\eta\lambda E}{\pi}\Lambda(z) \times\left[\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{Q_n''(x)}{\lambda_n^2}\right)(\sin my)+\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{Q_n(x)}{\lambda_n}\right)\left(-m^2\sin my\right)\right] \quad (26)$$

VII. MATERIAL PROPERTIES

The numerical calculations has been carried out for an Aluminum (pure) rectangular beam with the material properties as,

Density ρ =169 lb/ft ^{3}

Specific heat = 0.208 Btu/lb OF

Thermal conductivity K = 117Btu/(hr. ft OF)

Thermal diffusivity α = 3.33 ft^2/hr.

Poisson ratio ν = 0.35

Coefficient of linear thermal expansion α_t = 12.84 x 10^{-6}1/F

Lame constant μ = 26.67

Young's modulus of elasticity E = 70G Pa

VIII. DIMENSIONS

The constants associated with the numerical calculation are taken as

Length of rectangular beam x = 3ft

Breath of rectangular beam y = 2 ft

Height of rectangular beam z = 10^{3}ft

IX. CONCLUSION

The temperature distribution displacements and thermal stresses at any point of a thin rectangular object nave been obtained, when the boundary conditions are known with the aid of finite Marchi-Fasulo transform and semi-infinite Fourier cosine transform techniques. The results are obtained in the form of infinite series.

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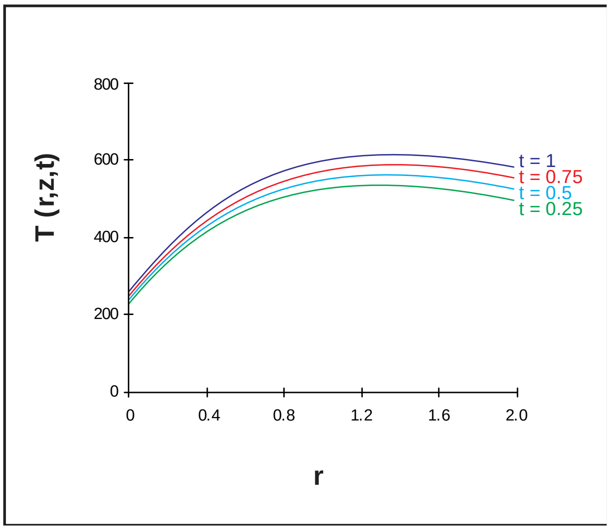

Fig. 1. Temperature distribution vs radius

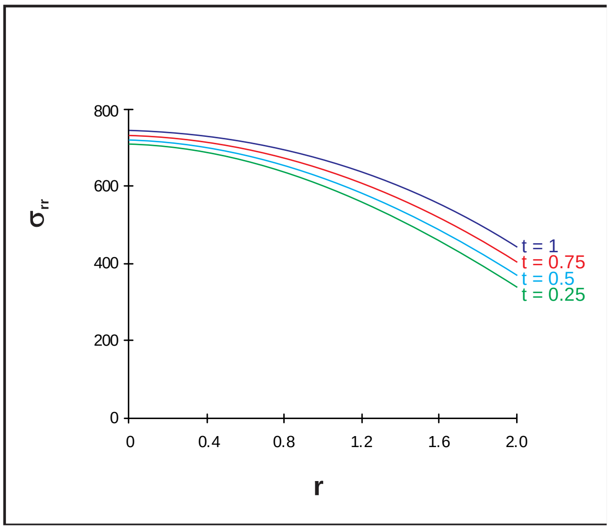

Fig. 2. Radial Stresses vs radius

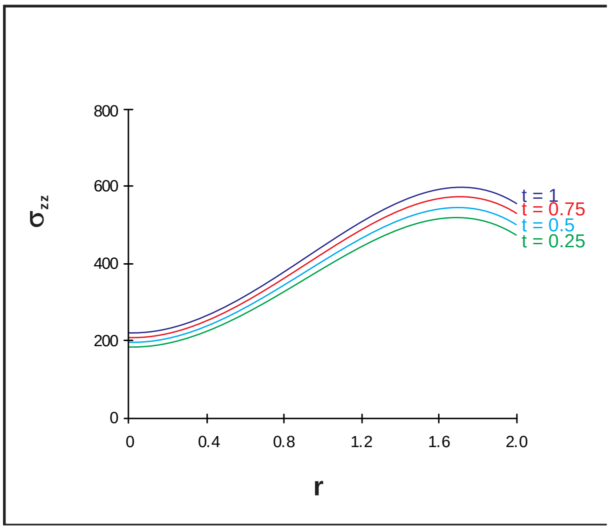

Fig. 3. Axial Stresses vs radius

AUTHOR BIOGRAPHY

Dr. N.W. Khobragade for being M.Sc in statistics and Maths he attained Ph.D. He has been teaching since 1986 for 27 years at PGTD of Maths, RTM Nagpur University, Nagpur and successfully handled different capacities.

At present he is working as Professor. Achieved excellent experiences in Research for 15 years in the area of Boundary value problems and its application. Published more than 180 research papers in reputed journals. Fourteen students awarded Ph.D Degree and four students submitted their thesis in University for award of Ph.D Degree under their guidance.