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Application of Phase Plots for Dynamic Stability Analysis of Compliant Offshore Structures

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ABSTRACT - Compliant offshore structures, by yielding to wave and current actions, avoid unacceptably high hydro-dynamic loads leading to economic designs and thus have always been favored for deep sea water operations. As these structures have large displacements with inherent non-linearities, so prediction of behavior of these structures in oceanic environment along with earthquake loads is difficult and is always met with many challenges. Oceanic waves are better modeled as stochastic process, so there is a need to investigate the stochastic stability of flexible offshore structures as well. The presence of strong geometric non-linearity and non-linearity arising due to fluid structure interaction leads to the possibility of dynamic instability of the systems. Efforts have been made to use simplified realistic mathematical models to determine dynamic stability and chaotic behavior of compliant offshore structures. A comprehensive review of literature available on the subject matter was made and relevant portions have been presented in the paper along with a case study of Dynamic Stability analysis of Single Hinged Articulated Tower [SHAT] using two dimensional Phase Plots.

Index Terms: **Bifurcation, Compliant Structures, Limit Cycle, Phase Plot, Single Hinged Articulated Tower**

I. INTRODUCTION

Structural stability is a fundamental property of a dynamical system which means that the qualitative behavior of the trajectories is unaffected by small perturbations. Structural stability deals with perturbations of the system itself. Most of the researchers in past have carried out dynamic analysis of the structure under Wave, Earthquake or Wind loads or a combination of these loads and studied its behavior(Banik, A.K.and Datta T.K., 2003[1], Chakrabarti, S. And Cotter, D., 1979[6], Chandrasekaran and Gaurav, S., 2008[7], Chua, L.O. and Ushida, A., 1981[8], Hasan, S. D., Islam, N. and Moin, K., 2011[11], Lina, H., Youngang, T. and Cong, YI., 2006[13]). But very few researchers have carried out Dynamic Stability analysis of the structure under these loads (Banik, A.K., 2004[3],Banik, A.K. and Datta, T.K., 2009[2], Chakrabarti S.K., 2001[5], Friedmann, P.,Hammond, C.E., and Woo, Tze-hsin,1977[9]). In order to explore the various theories available for Dynamic Stability analysis, a literature review was carried out to search for suitable and practical method for determination of Dynamical Stability behavior of Compliant offshore Structures. Accordingly, several case studies were carried out using Single Hinged Articulated Tower (SHAT). One of the case study has been presented in the paper to demonstrate use of Phase Plots for dynamic stability analysis of SHAT.

II. LITERATURE REVIEW FOR DYNAMIC STABILITY PROBLEM SOLUTION

Literature was reviewed from the aspect of Dynamic Stability determination of compliant offshore structures and following methods were explored in general to find out solution to stability problems:

- Floquet Theory by applying Incremental Harmonic Balance(IHB) Method[3], [10]
- Duffing oscillator Method[4],[14],[15]
- Vander Pol oscillator Method[14]
- Use of Mathieu type instability concept[12],[14]
- Stability solutions Using Minimum Potential Energy Concept[16]
- Limit Cycle method for Stability determination[14]
- Bifurcations as a measure of instability in two dimensional flows[14]

In the instant study, Stability solutions were obtained by using Limit Cycle method and identification of Bifurcations occurring in motion during various phases of Wave / Earthquake loading on SHAT. For the benefit of readers, the background concept given by Mallik, A.K. and Bhattacharjee, J.K.,,2005[14] and associated analysis technique in present study is being explained in the following paras:

A. Limit Cycle Method for Stability Determination

In two dimensional non-linear flows there is a possibility of the existence of isolated, closed phase trajectories called limit cycles. Like equilibrium points, limit cycles also represents a kind of steady-state which can be approached. Once the system reaches a limit cycle, then it remains on the limit cycle with x and y varying periodically. Such limit cycles can be stable, unstable or semi-stable. For a stable cycle, with any disturbance away from it, the phase trajectories tend to return to the limit cycle. For an unstable limit cycle, with slightest disturbance away from it, the phase trajectories continually move away from the limit cycle. In case of semi-stable limit cycle, with a disturbance towards one direction, say inside the limit cycle, the phase trajectories tend to return whereas any disturbance towards the opposite direction, say outside the limit cycle, the phase trajectories continually move away from the limit cycle. An example of a two-dimensional flow exhibiting a stable limit cycle is given by

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$\dot{x}$ = y and $\dot{y}$ = - φ (x^2- ν) y – x, φ > 0, ν > 0 (2.1)

B. *Bifurcation as a Measure of Instability in Two Dimensional Flows*

The characteristics of a two dimensional flow can change qualitatively when a parameter passed through a critical value. Consider the flow given by equation (2.1) i.e.

$\dot{x}$ = y and $\dot{y}$ = - φ (x^2- ν) y – x, φ > 0, ν > 0
with - ∞ < ν < ∞

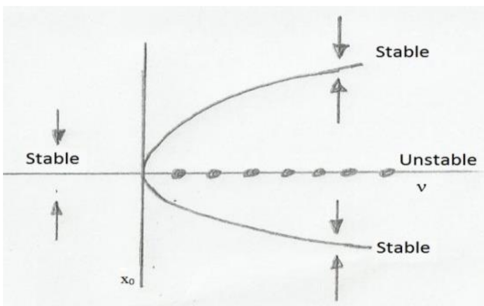
Using the methods outlined in [14], it is easy to show that the fixed point (x_0, y_0) is a stable focus for $\nu < 0$. Flow staring from any point in the phase plane ultimately spirals into this equilibrium state. However, the same fixed point transforms into an unstable focus for ν > 0, and phase trajectories from the neighbourhood of this equilibrium state spirals out. Such phenomenon of qualitative changes in the flow characteristics as one parameter passes through a critical value, with other parameters, if any, remaining unchanged, is called bifurcation. 'Bifurcation diagram' represents characteristics of various types of bifurcations graphically. In this diagram a physical variable is plotted against the parameter and at the critical value of the parameter a change in the physical variable is marked. Bifurcation point is the point on the diagram corresponding to the critical value of the parameter. Here we shall discuss two important bifurcations:

B(i) Pitchfork Bifurcation

Consider the flow

$\dot{x}$ = f(x, ν) = ν x - x^3, - ∞ < ν < ∞ (2.2)

When x_0 = 0, $\pm\sqrt{\nu}$ are given as fixed points, then by linear stability analysis it can be verified that for $\nu < 0$, x_0 = 0 is stable and for $\nu > 0$, it is unstable. The other two fixed points $+\sqrt{\nu}$ and $-\sqrt{\nu}$ exist only for $\nu > 0$ and can be easily verified to be stable. Fig.1 shows, bifurcation diagram for two different values of ν.



(Fig.1 – Pitchfork Bifurcation)

B(ii) Hopf Bifurcation

If one replaces the variable x and the parameter ν by complex number (z = x + iy) in equation (2.2), Hopf bifurcation is obtained.

Let us consider $\dot{z}$ = (ν + i℘) z - $|z^2|$ z
(2.3)

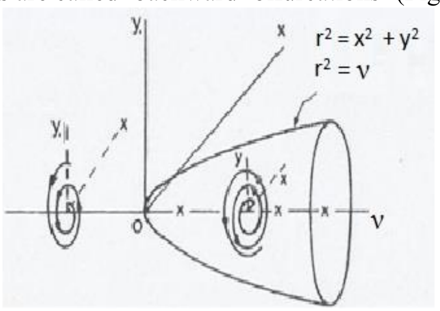
Equating the real and imaginary parts of the two sides of eqn. (2.3), two dimensional flow is obtained as

$\dot{x}$ = x[ν - (x^2 + y^2)] - ℘y
$\dot{y}$ = y[ν - (x^2 + y^2)] - ℘x
(2.4)

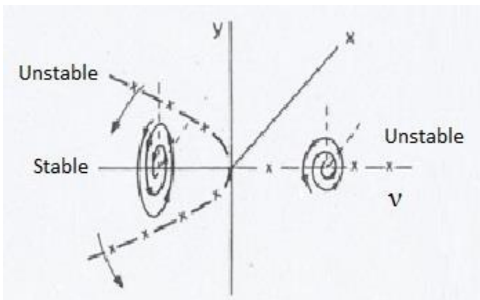
We consider ℘ > 0 and a constant with ν as a parameter. Point x_0 = 0, y_0 = 0, is a stable focus for ν < 0 and is an unstable focus for ν > 0. By substituting x = rcos Θ and y= rsin Θ, eqn. (2.4) in polar coordinates, can be written as

$\dot{r}$ = r (ν - r^2) and $\dot{\Theta}$ = ℘
(2.5)

It can be summarised that, a bifurcation takes place at the origin when the fixed points loses its stability and a stable limit cycle of time period 2π / ℘ is born. Fig.2 shows the bifurcation diagram where for two different values of ν, typical phase trajectories are also indicated. Here a new frequency is generated at the origin i.e. beyond the bifurcation point. The bifurcations can be classified as 'forward' bifurcations' and 'backward' bifurcations'. In 'forward' bifurcations', growth of the unstable solutions is ultimately arrested by the nonlinearity and finally a stable equilibrium is reached. If the sign of the nonlinear term in eqn. (2.4) is taken as positive, then the corresponding bifurcations are called 'backward' bifurcations' (Fig.3).



(Fig 2 – Forward Bifurcations)



(Fig 3 – Backward Bifurcations)

III. STABILITY SOLUTION USING TWO DIMENSIONAL PHASE PLOT, LIMIT CYCLE & BIFURCATION TECHNIQUE

A phase portrait is a collection of trajectories that represent the solution of these equations in the phase space. To obtain phase plots, velocities are plotted on abscissa and displacement / rotation are plotted on x-axis. In general, a

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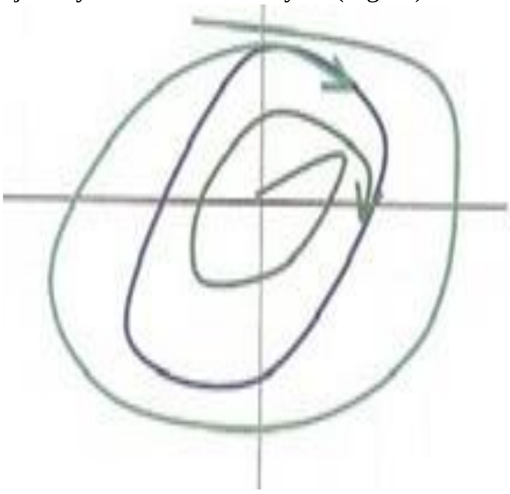
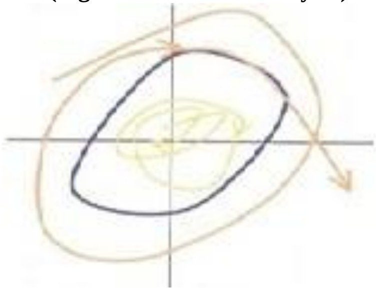
phase portrait contains information about both the transient and the asymptotic behaviours of the solutions of a system. The phase plots are extensively used to identify transitions of solutions from stable to unstable zone. When a system approaches periodic behaviour and a closed curve in phase plane is appeared, the closed path is called a limit cycle. A close trajectory of a dynamic system which has nearby open trajectories spiralling towards it both from inside and outside is called stable limit cycle (Fig-4a). If nearby open trajectories spiral away from closed path on both sides, the close trajectory is unstable limit cycle (Fig-4b).

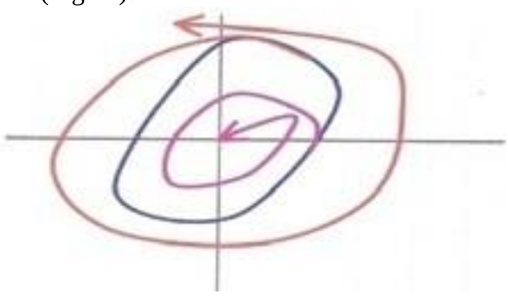


(Fig. 4a – Stable Limit Cycle)



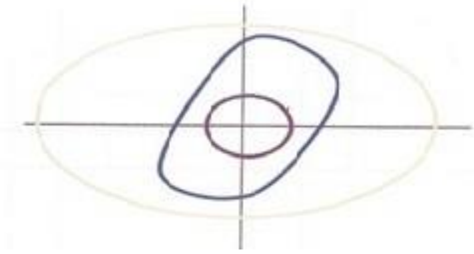
(Fig. 4b – Unstable Limit Cycle)

The thick line closed trajectory is limit cycle and other paths are neighbouring open trajectories. When the neighbouring trajectories spiral towards the limit cycle from one side and spiral out from the other side, it is semi-stable limit cycle (Fig-4c).



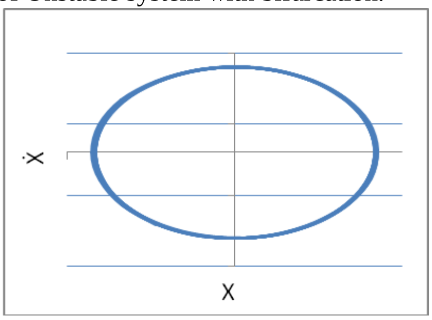
(Fig. 4c – Semi-stable Limit Cycle)

If nearby trajectories neither approach nor recede from closed trajectory, it is neutrally-stable limit cycle (Fig-4d) (Islam Saiful, A.B.M, 2013[12]). A Stable limit cycle attracts all neighbouring trajectories. In reality, the stable limit cycle indicate self-sustained oscillations. It is agreed that the trajectories for various initial states of this structural system converge to the limit cycle as described by Van der Pol Oscillator. Sub-harmonic oscillations occur when the time period of subsequent cycle lessens by 1/n times than the previous time period. When the time period of subsequent cycle increases n times of previous time period, the oscillation is super-harmonic.

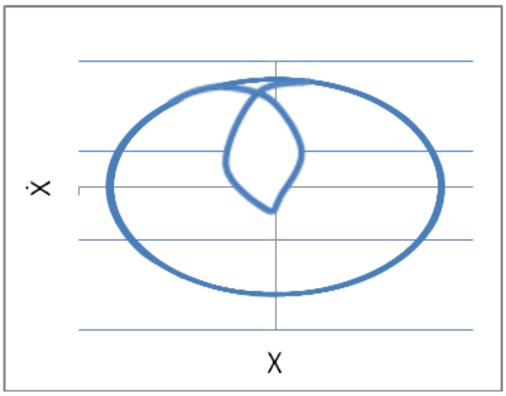


(Fig. 4d – Neutrally-stable Limit Cycle)

In another consideration, instability phenomenon in Phase plots is shown in the form of symmetry breaking bifurcations caused by nT sub-harmonic / super-harmonic oscillations and a periodic responses. As discussed in para II-B, the bifurcation concept is a mathematical study of changes in qualitative or topological behavior of structure. Without occurrence of bifurcation, the system seems to be quiet stable. In reality bifurcations may occur in both continuous and discrete systems. In a dynamical system, a bifurcation occurs when a small smooth change made to the bifurcation parameter causes a sudden qualitative or topological alteration in structural behavior. When the symmetry of a phase plot is disturbed, bifurcation is termed as symmetry breaking bifurcation. Fig.5a shows the Phase Plot of Stable system with no bifurcation and Fig.5b shows Phase Plot of Unstable system with bifurcation.



(Fig.5a -No Bifurcation and Stable System)



(Fig.5b- Bifurcation and Unstable System)

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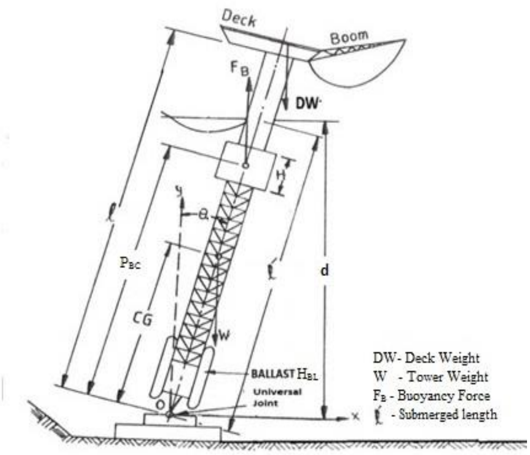
IV. RESULT AND DISCUSSION

CASE STUDY - SHAT SUBJECTED TO NORTHRIDGE EARTHQUAKE AT ZERO SECOND

For the present study, a Single Hinged Articulated Tower (SHAT) as shown in Fig.6, with following major structural parameters was subjected to Northridge Earthquake (NWH360) with time history duration of 39.98s. The Earthquake start time was taken as zero second. As it is very difficult to imagine sea without sea wave, almost negligible size regular sea wave was also generated in the sea. In order to get complete picture of different phases of loading and stabilization, the behavior of structure was examined for 2000 s.



(Fig.6 – SHAT Model)

SHAT Parameters

1. Height of Tower: 400m
2. Height of Ballast Chamber: 120 m
3. Sea Water Depth: 350 m
4. Height of Buoyancy chamber: 70 m
5. Position of Centre of Buoyancy chamber: 310 m
6. Structure Angular Velocity: 0.213 rad/s
7. Time Period: 29.47s

Various structural responses of the Tower viz. Hinge angle rotation, Tip Displacement, Shear Force, Axial Force, Bending Moment, Base Shear, Stabilizing / Destabilizing moments etc., obtained after Earthquake loading of SHAT, are given in Table I, II & III at Appendix-I. During Earthquake, the maximum hinge angle rotation was obtained as 8.41 degrees which was well outside the serviceability limits of the tower for any sort of usage / operation. In the present case, from Table III, it is seen that the Tower was subjected to net positive stabilizing moments, hence the structure was safe and motion got stabilized after the Earthquake excitation was removed. Except for 300s of Earthquake excitation and stabilization period, the Tower could be used. Detailed transient responses have been presented in form of figures pertaining to Time History plots and two dimensional Phase Plots which are given at Appendix-II. Kindly refer these figures (Fig.6 to Fig.11) for understanding the behavior of tower under Earthquake impact and its dynamic stability analysis. Fig.6 provides Time history plot for Hinge rotation for complete duration of 2000s of wave loading and Fig. 7 provides an enlarged view of Time history of Hinge rotation from 0 – 500s. Fig.7 shows, how the magnitude of responses, generated after Earthquake excitation, are getting reduced due to hydrodynamic damping. To examine the Stability phenomenon under various phases of loading, Phase plots were obtained for complete duration of loading as well as for different Time intervals. These are given at Fig.8 to Fig.11.The Phase plot for complete 2000s of loading (Fig.8) shows clustering of points on left side as well as right side. Cluster of points on left side has been scaled in Fig.9 where the phase plot has been plotted only for Earthquake excitation period from 0 – 40s. From here it is seen that there is a complete chaos during Earthquake and motion is Unstable. Bifurcations are clearly visible during this Phase. The maximum heel angle is 8.14 degrees which is the highest among all the cases analyzed in earlier studies. During this period, the Tower cannot be put to any use. Comparisons with other studies show that when Earthquake is applied in calm sea, the responses are much higher as compared to cases where regular / random waves exist in the sea. The stabilization phase is shown in Fig.10, from 40-300s duration, after the impact of Earthquake is over. Due to hydrodynamic dampening, the magnitude of responses reduces with time. Phase plot shows that the trajectory is trying to close showing the stable limiting behaviour. The responses are gradually becoming harmonic, symmetric and periodic. Finally stabilized behaviour of SHAT is shown in Fig.11 where Phase plot from 1000-2000s has been plotted. This shows that due to longer duration of hydrodynamic damping effects, the motion becomes periodic, harmonic and symmetric. The sub-harmonic and super-harmonic characteristics of motion are no more existent.

V. CONCLUSION

Compliant Offshore structures are inherited with strong geometric non-linearity and non-linearity arising due to fluid structure interaction. As these structures are also subjected to stochastic regular / random waves along with earthquake, so determination of their structural responses and dynamic Stability behaviour is being seen as a challenge by the researchers. Many researchers in the past have worked on determination of responses of Compliant Offshore structures, but very few have worked on their dynamic stability analysis. Present study provides a practical solution to the dynamic stability analysis problems of non-linear structures subjected to transient loads. Two dimensional Phase Plots provide a powerful tool for carrying out dynamic Stability Analysis of Compliant Offshore structures such as Articulated Towers, Spar Platforms[12] etc. This method of two dimensional Plots, provides insight into the response behaviour of these structures and helps in exploring the possibility of their dynamic instability and chaotic motion during various phases of loading. The two dimensional representation of phase trajectories moving towards or away from limit cycle determine the stability of motion. The dynamic behaviour of structure is further evaluated by the presence or absence of bifurcations in the Phase Plots.

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APPENDIX-I

TABLE-I

	Heel Angle (deg/rad.)	Deck Displ. (m)	Time (sec.)	Shear Force (N)	Axial Force (N)	Bending Moment (Nm)	Base Shear Force (N)	Base Axial Force (N)
Max^{m}	8.14 deg./ 0.1429E+0 0rad	0.5696E+2	55.38	0.6922E+8	0.5110E+8	0.1417E+13	0.6922E+8	0.0
Min^{m}	-0.27deg./ -0.4744E-02ra	-0.1898E+1	0.20	-0.8477E+8	-0.2767E+9	-0.1745E+13	-0.8477E+8	-.2566E+9

TABLE- II

	Mean	RMS	SD

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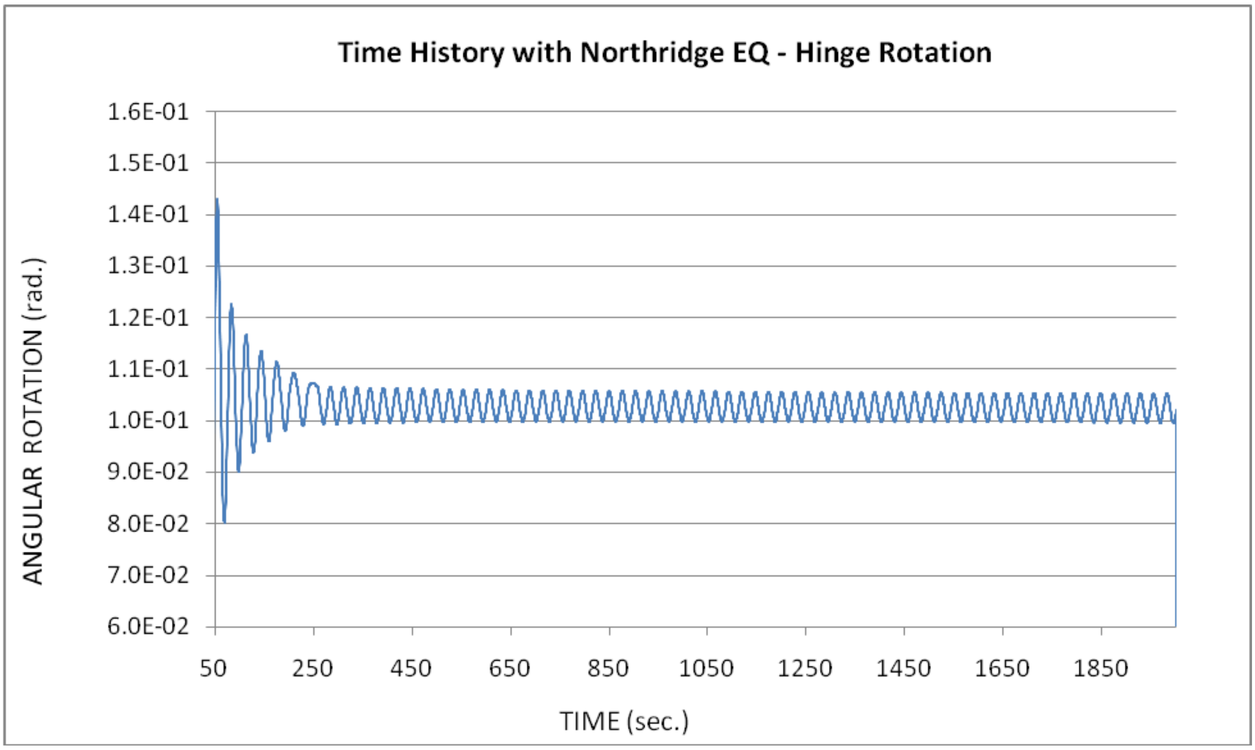
Heel Angle(rad.)	0.5990E-01	0.7859E-01	0.5086E-01
Tip Displacement(m)	0.2390E+02	0.3138E+02	0.2031E+02
Base Shear Force(N)	0.1930E+07	0.4297E+07	0.3842E+07
Base Axial Force(N)	0.1730E+09	0.1750E+09	0.2644E+08

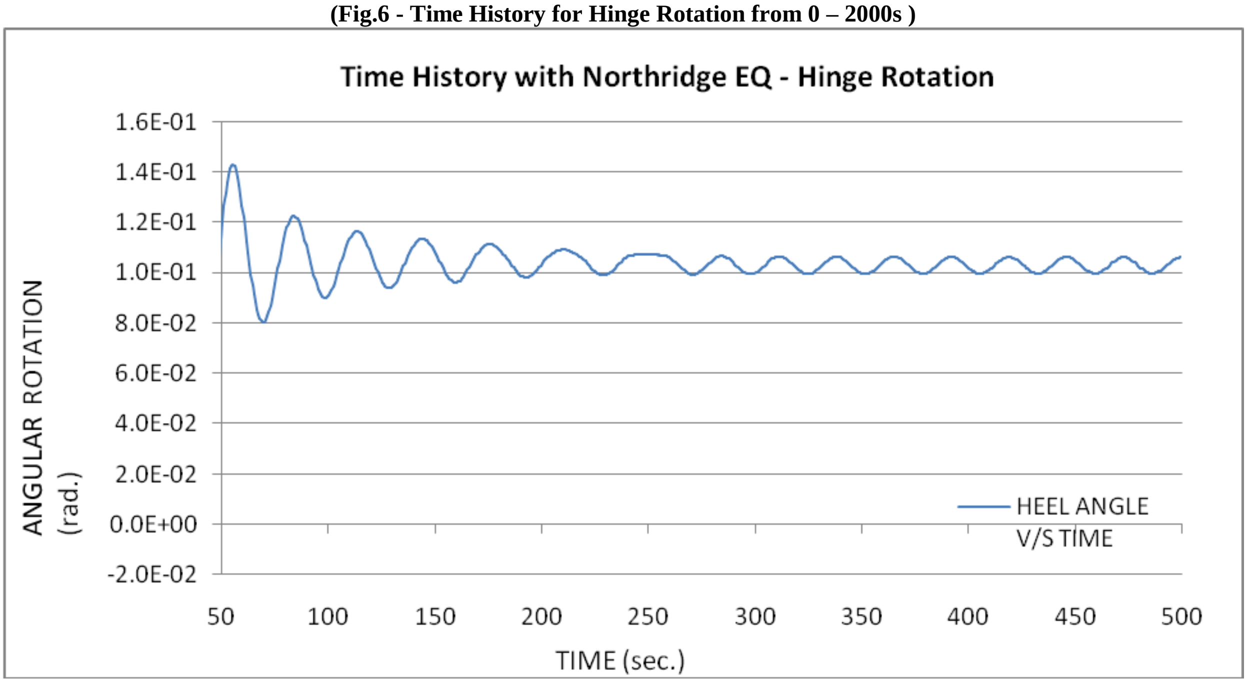
TABLE- III

Stabilizing Moment(Nm)	Maxm. Moment due to Drag and Inertia(Nm)	Moment of Tower Weight and Deck(Nm)	Total Destabilizing Moment(Nm)	Net Stabilizing Moment(Nm)
8.65E+10	1.15E+10	2.87E+10	4.01E+10	4.64E+10

APPENDIX-II



(Fig.6 - Time History for Hinge Rotation from 0 – 2000s)



(Fig.7 - Enlarged view of Time History for Hinge Rotation from 0 – 500s)

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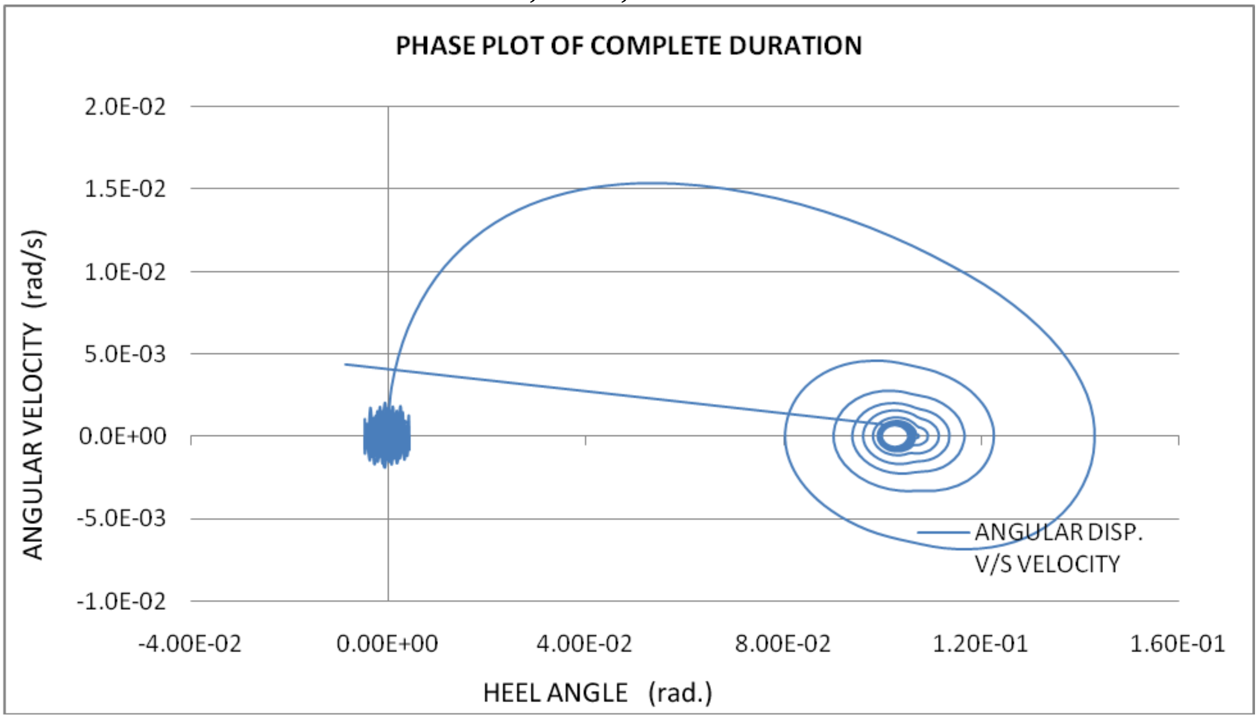
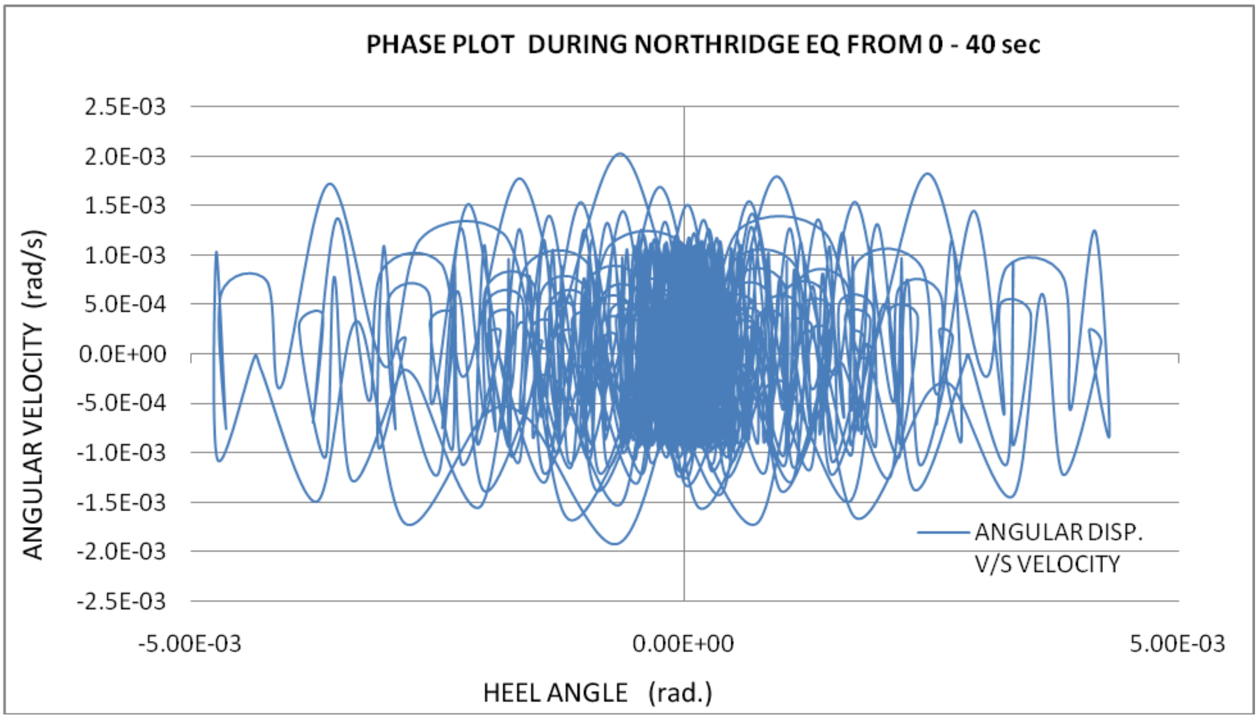
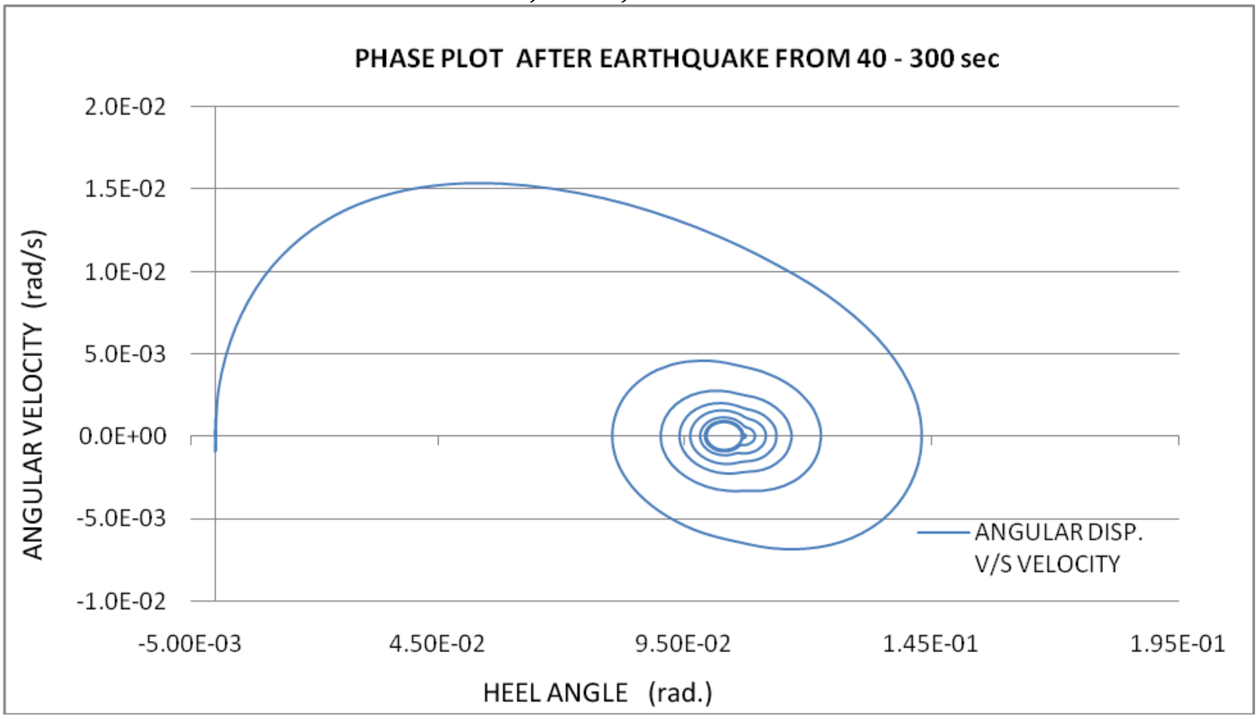


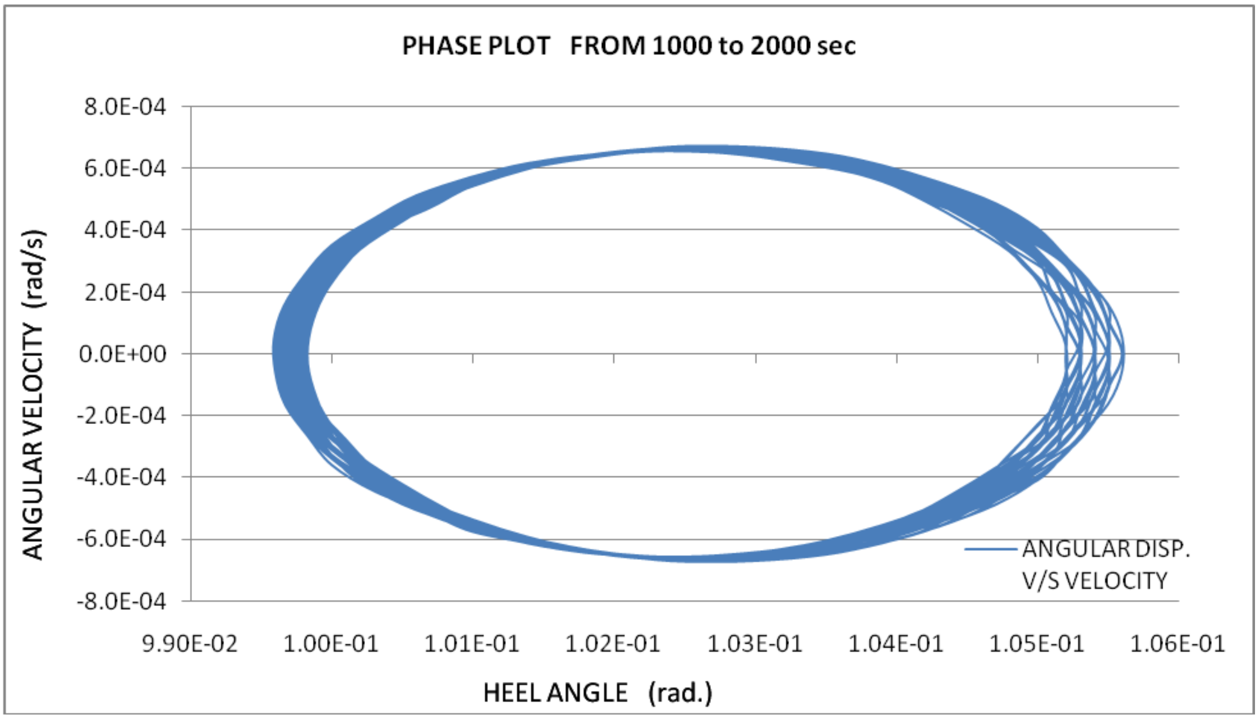
(Fig.8 – Phase Plot for complete duration of 0-2000s)



(Fig 9- Phase Plot during Earthquake Excitation from 0 – 40s)



(Fig. 10- Phase Plot during Stabilization phase from 40 – 300s after Earthquake)



(Fig. 11 – Phase Plot from 1000 – 2000s showing Stable Phase)