

Thermo elastic Analysis of Thick Hollow Cylinder with Radiation Conditions

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Abstract. The main aim of this paper is to study the thermo elastic response of a thick hollow cylinder in which sources are generated according to the linear function of the temperature, with boundary conditions of the radiation type. The solutions are based on theory of integral transformations with boundary conditions of the radiation type on the surfaces, with independent radiation constants. The results are obtained in series form in terms of Bessel functions. Some numerical calculations for temperature change, the displacement and the stress distributions are carried out and depicted graphically.

Keywords: Transient problem, thick hollow cylinder, temperature distribution, thermal stress, integral transform

I. INTRODUCTION

Nowacki [6] has determined steady-state thermal stresses in a thick circular plate subjected to an axisymmetric temperature distribution on the upper face with zero temperature on the lower face and circular edge. Wankhede [9] has determined the quasi-static thermal stresses in circular plate subjected to arbitrary initial temperature on the upper face with lower face at zero temperature. However, there aren't many investigations on transient state. Roy Choudhari [8] has succeeded in determining the quasi-static thermal stresses in a circular plate subjected to transient temperature along the circumference of circular upper face with lower face at zero temperature and the fixed circular edge thermally insulated. In a recent work, some problems have been solved by Noda et al. [7] and Deshmukhet al. [3]. In all aforementioned investigations an axisymmetrically heated plate has been considered. Recently, Nasser [1,2] proposed the concept of heat sources in generalized thermo elasticity and applied to a thick plate problem. They have not however considered any Thermo elasticity problem with boundary conditions of radiation type, in which sources are generated according to the linear function of the temperatures, which satisfies the time-dependent heat conduction equation.

This paper is concerned with the transient thermo elastic problem in a hollow cylinder in which sources are generated according to the linear function of temperature, occupying the space

$$D = \{(x, y, z) \in R^3 : a \leq (x^2 + y^2)^{1/2} \leq b, -h \leq z \leq h\},$$

where $r = (x^2 + y^2)^{1/2}$ with radiation type boundary conditions.

II. STATEMENT OF THE PROBLEM

Consider a hollow cylinder in which sources are generated according to the linear function of temperature.

The material of the hollow cylinder is isotropic, homogenous and all properties are assumed to be constant. Heat conduction with internal heat source and the prescribed boundary conditions of the radiation type, where the stresses are required to be determined. The equation for heat conduction in cylindrical coordinates [7] is:

$$\kappa \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \theta}{\partial r} \right) + \frac{\partial^2 \theta}{\partial z^2} \right] + \Theta(r, z, t, \theta) = \frac{\partial \theta}{\partial t} \quad (1)$$

Where $\Theta(r, z, t, \theta)$ is the internal source function, and $\kappa = \lambda / \rho C$, λ being the thermal conductivity of the material, ρ is the density and C is the calorific capacity, assumed to be constant. For convenience, we consider the under given functions as the superposition of the simpler function [4]:

$$\Theta(r, z, t, \theta) = \Phi(r, z, t) + \psi(t) \theta(r, z, t) \quad (2)$$

$$T(r, z, t) = \theta(r, z, t) \exp \left[- \int_0^t \psi(\zeta) d\zeta \right] \quad (3)$$

$$\chi(r, z, t) = \Phi(r, z, t) \exp \left[- \int_0^t \psi(\zeta) d\zeta \right] \quad (4)$$

or for the sake of brevity, we consider

$$\chi(r, z, t) = \frac{\delta(r - r_0) \delta(z - z_0)}{2\pi r_0} \exp(-\omega t), \quad a \leq r_0 \leq b,$$

$$-h \leq z_0 \leq h, \quad \omega > 0 \quad (5)$$

Substituting equations (2) and (3) to the heat conduction equation (1), one obtains

$$\kappa \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial z^2} \right] + \chi(r, z, t) = \frac{\partial T}{\partial t} \quad (6)$$

Where κ is the thermal diffusivity of the material of the hollow cylinder (which is assumed to be constant), subject to the initial and boundary conditions

$$M_r(T, 1, 0, 0) = T_0 \text{ for all } a \leq r \leq b, -h \leq z \leq h \quad (7)$$

$$M_r(T, 1, k_1, a) = F_1(z, t), \text{ for all } -h \leq z \leq h, \quad t > 0 \quad (8)$$

$$M_r(T, 1, k_2, b) = F_2(z, t) \text{ for all } -h \leq z \leq h, \quad t > 0 \quad (9)$$

$$M_z(T, 1, k_3, h) = \exp(-\omega t) \delta(r - r_0) \quad (10)$$

$$M_z(T, 1, k_4, -h) = 0 \text{ for all } 0 \leq r \leq b, \quad t > 0 \quad (11)$$

The most general expression for these conditions can be given by

$$M_g(f, \bar{k}, \bar{k}, \hat{f}) = (\bar{k} f + \bar{k} \hat{f})_{g=s}$$

where the prime (^) denotes differentiation with respect to \mathcal{G} ; $\delta(r-r_0)$ is the Dirac Delta function having $a \leq r_0 \leq b$; $\omega > 0$ is a constants; $\exp(-\omega t)\delta(r-r_0)$ is the additional sectional heat available on its surface at $z = h$; T_0 is the reference temperature; \bar{k} and \bar{k} are radiation coefficients respectively.

The Navier's equations without the body forces for axisymmetric two-dimensional thermo elastic problem can be expressed as [7]

$$\nabla^2 u_r - \frac{u_r}{r} + \frac{1}{1-2\nu} \frac{\partial e}{\partial r} - \frac{2(1+\nu)}{1-2\nu} \alpha_t \frac{\partial \theta}{\partial r} = 0 \quad (12)$$

$$\nabla^2 u_z - \frac{1}{1-2\nu} \frac{\partial e}{\partial z} - \frac{2(1+\nu)}{1-2\nu} \alpha_t \frac{\partial \theta}{\partial z} = 0 \quad (13)$$

where u_r and u_z are the displacement components in the radial and axial directions, respectively and the dilatation e as

$$e = \frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z} \quad (14)$$

The displacement function in the cylindrical coordinate system are represented by the Goodier's thermo elastic displacement potential $\phi(r, z, t)$ and Love's function L as [7]

$$u_r = \frac{\partial \phi}{\partial r} - \frac{\partial^2 L}{\partial r \partial z},$$

$$(15) u_z = \frac{\partial \phi}{\partial z} + 2(1-\nu) \nabla^2 L - \frac{\partial^2 L}{\partial z^2} \quad (16)$$

in which Goodier's thermo elastic potential must satisfy the equation [7]

$$\nabla^2 \phi = \left(\frac{1+\nu}{1-\nu} \right) \alpha_t \theta \quad (17)$$

and the Love's function L as [10] must satisfy the equation

$$\nabla^2 (\nabla^2 L) = 0 \quad (18)$$

where

$$\nabla^2 = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) + \frac{\partial^2}{\partial z^2}$$

The component of the stresses are represented by the use of the potential ϕ and Love's function L as

$$\sigma_{rr} = 2G \left\{ \left(\frac{\partial^2 \phi}{\partial r^2} - \nabla^2 \phi \right) + \frac{\partial}{\partial z} \left(\nu \nabla^2 L - \frac{\partial^2 L}{\partial r^2} \right) \right\}$$

$$(19) \sigma_{\theta\theta} = 2G \left\{ \left(\frac{1}{r} \frac{\partial \phi}{\partial r} - \nabla^2 \phi \right) + \frac{\partial}{\partial z} \left(\nu \nabla^2 L - \frac{1}{r} \frac{\partial L}{\partial r} \right) \right\} \quad (20)$$

$$\sigma_{zz} = 2G \left\{ \left(\frac{\partial^2 \phi}{\partial z^2} - \nabla^2 \phi \right) + \frac{\partial}{\partial z} \left((2-\nu) \nabla^2 L - \frac{\partial^2 L}{\partial z^2} \right) \right\} \quad (21)$$

$$\sigma_{rz} = 2G \left\{ \frac{\partial^2 \phi}{\partial r \partial z} + \frac{\partial}{\partial r} \left((1-\nu) \nabla^2 L - \frac{\partial^2 L}{\partial z^2} \right) \right\} \quad (22)$$

where G and ν are the shear modulus and Poisson's ratio respectively.

The boundary condition on the traction free surface stress functions are

$$\sigma_{zz}|_{z=\pm h} = \sigma_{rz}|_{z=\pm h} = 0 \quad (23)$$

The equations (1) to (23) constitute the mathematical formulation of the problem under consideration.

III.SOLUTION OF THE PROBLEM

Transient Heat Conduction Analysis:

In order to solve fundamental differential equation (6) under the boundary conditions (8) and (9), we first introduce the integral transform [5] of order n over the variable r .

Let n be the parameter of the transform, then the integral transform and its inversion is given by

$$\bar{f}(n) = \int_a^b r f(r) S_p(k_1, k_2, \mu_n r) dr, \quad (24)$$

$$f(r) = \sum_{n=1}^{\infty} (\bar{f}_p(n) / C_n) S_p(k_1, k_2, \mu_n r) \quad (25)$$

$$C_n = \frac{b^2}{2} \left\{ \frac{S_p^2(k_1, k_2, \mu_n b) - S_{p-1}(k_1, k_2, \mu_n b)}{S_{p+1}(k_1, k_2, \mu_n b)} \right\} - \frac{a^2}{2} \left\{ \frac{S_p^2(k_1, k_2, \mu_n a) - S_{p-1}(k_1, k_2, \mu_n a)}{S_{p+1}(k_1, k_2, \mu_n a)} \right\} \quad (26)$$

Operational property:

$$\int_a^b x \left\{ \frac{\partial^2 f}{\partial x^2} + \frac{1}{x} \frac{\partial f}{\partial x} - \frac{p^2}{x^2} f \right\} S_p(k_1, k_2, \mu_n x) dx$$

$$= \frac{b}{k_2} S_p(k_1, k_2, \mu_n b) \left\{ f + k_2 \frac{\partial f}{\partial x} \right\}_{x=b} - \frac{a}{k_1} S_p(k_1, k_2, \mu_n a) \left\{ f + k_1 \frac{\partial f}{\partial x} \right\}_{x=a} - \mu_n^2 \bar{f}(n). \quad (27)$$

Applying the transform defined in equation (24) to the equations (3) to (5) and (7), and taking into account equations (8) and (9), one obtains

$$\kappa \left[-\mu_n^2 \bar{T}(n, z, t) + \frac{\partial^2 \bar{T}(n, z, t)}{\partial z^2} \right] + \bar{\chi}(n, z, t) = \frac{\partial \bar{T}(n, z, t)}{\partial t} \quad (28)$$

$$M_r(\bar{T}, 1, 0, 0) = \bar{T}_0 \quad (29)$$

$$M_z(\bar{T}, 1, k_3, h) = \exp(-\omega t) r_0 S_0(k_1, k_2, \mu_n r_0), \quad (30)$$

$$M_z(\bar{T}, 1, k_4, -h) = 0 \quad (31)$$

Where \bar{T} is the transformed function of T and n is the transform parameter, and μ_n are the positive roots of the characteristic equation.

$$J_0(k_1, \mu a) Y_0(k_2, \mu b) - J_0(k_2, \mu b) Y_0(k_1, \mu a) = 0$$

and F_1, F_2 are assumed to be zero.

The kernel function $S_0(k_1, k_2, \mu_n r)$ can be defined as

$$S_0(k_1, k_2, \mu_n r) = J_0(\mu_n r) [Y_0(k_1, \mu_n a) + Y_0(k_2, \mu_n b)] - Y_0(\mu_n r) [J_0(k_1, \mu_n a) + J_0(k_2, \mu_n b)]$$

with

$$\left. \begin{aligned} J_0(k_i, \mu r) &= J_0(\mu r) + k_i \mu J_0'(\mu r) \\ Y_0(k_i, \mu r) &= Y_0(\mu r) + k_i \mu Y_0'(\mu r) \end{aligned} \right\} \text{for } i=1,2$$

In which $J_0(\mu r)$ and $Y_0(\mu r)$ are Bessel functions of first and second kind of order $p=0$ respectively.

We introduce another integral transform [4] that responds to the boundary conditions of type (10) and (11):

$$\bar{f}(m, t) = \int_{-h}^h f(z, t) P_m(z) dz, \quad (32)$$

$$f(z, t) = \sum_{m=1}^{\infty} \frac{\bar{f}(m, t)}{\lambda_m} P_m(z) \quad (33)$$

where

$$P_m(z) = Q_m \cos(a_m z) - W_m \sin(a_m z)$$

$$Q_m = a_m (k_3 + k_4) \cos(a_m h),$$

$$\lambda_m = \int_{-h}^h P_m^2(z) dz = h [Q_m^2 + W_m^2] + \frac{\sin(2a_m h)}{2a_m} [Q_m^2 - W_m^2]$$

The eigen values a_m are the positive roots of the characteristic equation

$$\begin{aligned} & [k_3 a \cos(ah) + \sin(ah)] [\cos(ah) + k_4 a \sin(ah)] \\ &= [k_4 a \cos(ah) - \sin(ah)] [\cos(ah) - k_3 a \sin(ah)] \end{aligned}$$

Further applying the transform defined in equation (32) to the equations (28), (29) and using equations (30) and (31), one obtains

$$\frac{d\bar{T}^*}{dt} + \kappa(\Lambda_{n,m}) \bar{T}^* = H(\mu_n, a_m) \quad (34)$$

where

$$\Lambda_{n,m} = \mu_n^2 + a_m^2$$

and

$$\begin{aligned} H(\mu_n, a_m) &= \left\{ \frac{P_m(h)\kappa}{k_3} - \frac{P_m(-h)\kappa}{k_4} + \frac{P_m(z_0)}{2\pi r_0} \right\} \\ &\times r_0 S_0(k_1, k_2, \mu_n r_0) \exp(-\omega t) \end{aligned}$$

Where \bar{T}^* denotes Marchi-Fasulo integral transform of \bar{T} and m is the transform parameter.

The general solution of equation (34) is given by

$$\begin{aligned} \bar{T}^*(n, m, t) &= \frac{F(n, m)}{\kappa(\Lambda_{n,m}) - \omega} \exp(-\omega t) \\ &+ \left[\bar{T}_0^* - \frac{F(n, m)}{\kappa(\Lambda_{n,m}) - \omega} \right] \exp(-\kappa(\Lambda_{n,m}) t) \end{aligned}$$

where

$$\begin{aligned} F(n, m) &= \left\{ \frac{P_m(h)\kappa}{k_3} - \frac{P_m(-h)\kappa}{k_4} + \frac{P_m(z_0)}{2\pi r_0} \right\} \\ &\times r_0 S_0(k_1, k_2, \mu_n r_0) \end{aligned}$$

Applying inversion theorems of transformation rules defined in equations (33) and equation (28) to the equation (35), one obtains

$$\begin{aligned} T(r, z, t) &= \sum_{n=1}^{\infty} \frac{1}{C_n} \left\{ \sum_{m=1}^{\infty} \frac{1}{\lambda_m} [\wp_{n,m} \exp(-\omega t) \right. \\ &\quad \left. + (\bar{T}_0^* - \wp_{n,m}) \exp(-\kappa(\Lambda_{n,m}) t)] \right\} \\ &\times P_m(z) S_0(k_1, k_2, \mu_n r) \quad (36) \end{aligned}$$

where

$$\wp_{n,m} = \frac{F(n, m)}{\kappa(\Lambda_{n,m}) - \omega}$$

Taking into account the first equation of equation (3), the temperature distribution is finally represented by

$$\begin{aligned} \theta(r, z, t) &= \sum_{n=1}^{\infty} \frac{1}{C_n} \left\{ \sum_{m=1}^{\infty} \frac{1}{\lambda_m} [\wp_{n,m} \exp(-\omega t) \right. \\ &\quad \left. + (\bar{T}_0^* - \wp_{n,m}) \exp(-\kappa(\Lambda_{n,m}) t)] \right\} \\ &\times P_m(z) S_0(k_1, k_2, \mu_n r) \exp\left[\int_0^t \psi(\zeta) d\zeta \right] \quad (37) \end{aligned}$$

The function given in equation (37) represents the temperature at every instant and at all points of hollow cylinder when there are conditions of radiation type.

IV. THERMOELASTIC SOLUTION

Referring to the fundamental equation (1) and its solution (37) for the heat conduction problem, the solution for the displacement function are represented by the Goodier's thermo elastic displacement potential ϕ governed by equation (17) are represented by

$$\begin{aligned} \phi(r, z, t) &= \left(\frac{1+\nu}{1-\nu} \right) \alpha_t \\ &\times \sum_{n=1}^{\infty} \frac{1}{C_n} \left\{ \sum_{m=1}^{\infty} \frac{-1}{\lambda_m(\Lambda_{n,m})} [\wp_{n,m} \exp(-\omega t) \right. \\ &\quad \left. + (\bar{T}_0^* - \wp_{n,m}) \exp(-\kappa(\Lambda_{n,m}) t)] P_m(z) \right\} \\ &\times S_0(k_1, k_2, \mu_n r) \exp\left[\int_0^t \psi(\zeta) d\zeta \right] \quad (38) \end{aligned}$$

Similarly, the solution for Love's function L are assumed so as to satisfy the governed condition of equation (18) as

$$L(r, z, t) = \left(\frac{1+\nu}{1-\nu}\right) \alpha_t \times \sum_{n=1}^{\infty} \frac{1}{C_n} \left\{ \sum_{m=1}^{\infty} \frac{-1}{\lambda_m(\Lambda_{n,m})} [\wp_{n,m} \exp(-\omega t) + (\bar{T}_0^* - \wp_{n,m}) \exp(-\kappa(\Lambda_{n,m})t)] \right\} \times [B_{nm} \sinh(\mu_n z) + C_{nm} z \cosh(\mu_n z)] \times S_0(k_1, k_2, \mu_n r) \exp\left[\int_0^t \psi(\zeta) d\zeta\right] \quad (39)$$

Using equations (38) and (39) in equations (14) and (15), one obtains

$$u_r = \left(\frac{1+\nu}{1-\nu}\right) \alpha_t \sum_{n=1}^{\infty} \frac{\mu_n}{C_n} \left\{ \sum_{m=1}^{\infty} \frac{-1}{\lambda_m(\Lambda_{n,m})} [\wp_{n,m} \exp(-\omega t) + (\bar{T}_0^* - \wp_{n,m}) \exp(-\kappa(\Lambda_{n,m})t)] \right\} \times [P_m(z) - (B_{nm}\mu_n + C_{nm}) \cosh(\mu_n z) + C_{nm} z \sinh(\mu_n z)] \times S_0'(k_1, k_2, \mu_n r) \exp\left[\int_0^t \psi(\zeta) d\zeta\right] \quad (40)$$

$$u_z = \left(\frac{1+\nu}{1-\nu}\right) \alpha_t \sum_{n=1}^{\infty} \frac{1}{C_n} \left\{ \sum_{m=1}^{\infty} \frac{-1}{\lambda_m(\Lambda_{n,m})} [\wp_{n,m} \exp(-\omega t) + (\bar{T}_0^* - \wp_{n,m}) \exp(-\kappa(\Lambda_{n,m})t)] \right\} \times \{ [-a_m(Q_m \sin(a_m z) + W_m \cos(a_m z)) - \mu_n^2(-1+2\nu)(B_{nm} \sinh(\mu_n z) + C_{nm} z \cosh(\mu_n z))] - 2(-1+2\nu)C_{nm} \sinh(\mu_n z) \mu_n] S_0(k_1, k_2, \mu_n r) + \mu_n(2(1-\nu)) [B_{nm} \sinh(\mu_n z) + C_{nm} z \cosh(\mu_n z)] \times [\mu_n S_0''(k_1, k_2, \mu_n r) + \frac{S_0'(k_1, k_2, \mu_n r)}{r}] \} \exp\left[\int_0^t \psi(\zeta) d\zeta\right] \quad (41)$$

Thus, making use of the two displacement components, the dilatation can be obtained. Then, the stress components can be evaluated by substituting the values of thermo elastic displacement potential ϕ [7] from equation (38) and Love's function L [10] from equation (39) in equations (19) to (22), one obtains

$$\sigma_{rr} = 2G \left(\frac{1+\nu}{1-\nu}\right) \alpha_t \sum_{n=1}^{\infty} \frac{1}{C_n} \times \left\{ \sum_{m=1}^{\infty} \frac{-1}{\lambda_m(\Lambda_{n,m})} \left[\wp_{n,m} \exp(-\omega t) + (\bar{T}_0^* - \wp_{n,m}) \exp(-\kappa(\Lambda_{n,m})t) \right] \right\} \times \left\{ -P_m(z) \left[\frac{S_0'(k_1, k_2, \mu_n r)}{r} - a_m^2 S_0(k_1, k_2, \mu_n r) \right] + \mu_n^2(\nu-1) S_0''(k_1, k_2, \mu_n r) \right. \\ \left. + [\mu_n(B_{nm} \cosh(\mu_n z) + C_{nm}(z \sinh(\mu_n z) + \cosh(\mu_n z)) \right]$$

$$+ \mu_n \nu [B_{nm} \cosh(\mu_n z) \mu_n + C_{nm}(z \sinh(\mu_n z) + \cosh(\mu_n z))] \times \left[\frac{S_0'(k_1, k_2, \mu_n r)}{r} + \mu_n S_0(k_1, k_2, \mu_n r) \right] + 2\nu C_{nm} \mu_n^2 \cosh(\mu_n z) S_0(k_1, k_2, \mu_n r) \} \exp\left[\int_0^t \psi(\zeta) d\zeta\right] \quad (42)$$

$$\sigma_{\theta\theta} = 2G \left(\frac{1+\nu}{1-\nu}\right) \alpha_t \times \sum_{n=1}^{\infty} \frac{1}{C_n} \left\{ \sum_{m=1}^{\infty} \frac{-1}{\lambda_m(\Lambda_{n,m})} [\wp_{n,m} \exp(-\omega t) + (\bar{T}_0^* - \wp_{n,m}) \exp(-\kappa(\Lambda_{n,m})t)] \right\} \times \{ -P_m(z) [\mu_m^2 S_0''(k_1, k_2, \mu_n r) - a_m^2 S_0(k_1, k_2, \mu_n r)] + \frac{\mu_n(\nu-1)}{r} S_0^1(k_1, k_2, \mu_n r) \times [(\mu_n B_{nm} + C_{nm}) \cosh(\mu_n z) + \mu_n C_{nm} z \sinh(\mu_n z)] + \mu_n^2 \nu [(\mu_n B_{nm} + C_{nm}) \cosh(\mu_n z) + \mu_n C_{nm} z \sinh(\mu_n z)] \times [S_0''(k_1, k_2, \mu_n r) + S_0(k_1, k_2, \mu_n r)] + 2\nu C_{nm} \mu_n^2 \cosh(\mu_n z) S_0(k_1, k_2, \mu_n r) \} \exp\left[\int_0^t \psi(\zeta) d\zeta\right]$$

$$\sigma_{zz} = 2G \left(\frac{1+\nu}{1-\nu}\right) \alpha_t \times \sum_{n=1}^{\infty} \frac{1}{C_n} \left\{ \sum_{m=1}^{\infty} \frac{-1}{\lambda_m(\Lambda_{n,m})} [\wp_{n,m} \exp(-\omega t) + (\bar{T}_0^* - \wp_{n,m}) \exp(-\kappa(\Lambda_{n,m})t)] \right\} \times \{ -\mu_n P_m(z) [\mu_n S_0''(k_1, k_2, \mu_n r) + \frac{S_0(k_1, k_2, \mu_n r)}{r}] + \mu_n^2 [B_{nm} \cosh(\mu_n z) + C_{nm} z \sinh(\mu_n z)] (2-\nu) \times [\mu_n S_0''(k_1, k_2, \mu_n r) + r^{-1} S_0(k_1, k_2, \mu_n r)] + (1-\nu) \mu_n S_0(k_1, k_2, \mu_n r) + \mu_n C_{nm} \cosh(\mu_n z) [(2-\nu) [\mu_n S_0''(k_1, k_2, \mu_n r) + \frac{S_0(k_1, k_2, \mu_n r)}{r}]] + (1-\nu) \mu_n^2 S_0(k_1, k_2, \mu_n r) \} \exp\left[\int_0^t \psi(\zeta) d\zeta\right] \quad (44)$$

$$\sigma_{rz} = 2G \left(\frac{1+\nu}{1-\nu} \right) \alpha_t \times \sum_{n=1}^{\infty} \frac{1}{C_n} \left\{ \sum_{m=1}^{\infty} \frac{-1}{\lambda_m (\Lambda_{nm})} [\wp_{n,m} \exp(-\omega t) + (\bar{T}_0^* - \wp_{n,m}) \exp(-\kappa (\Lambda_{n,m}) t)] \right\} \times \{ [-\mu_n a_m (Q_m \sin(a_m z) + W_m \cos(a_m z))] \times S_0'(k_1, k_2, \mu_n r) + [(B_{nm} \sinh(\mu_n z) + C_{nm} z \cosh(\mu_n z)) \mu_n^2 [\nu \mu_n + \frac{(1-\nu)}{r}] - 2\nu \mu_n^2 C_{nm} \sinh(\mu_n z)] S_0'(k_1, k_2, \mu_n r) + (1-\nu) [B_{nm} \sinh(\mu_n z) + C_{nm} z \cosh(\mu_n z)] \times [\mu_n^3 S_0'''(k_1, k_2, \mu_n r) - \frac{S_0(k_1, k_2, \mu_n r)}{r^2}] \} \times \exp \left[\int_0^t \psi(\zeta) d\zeta \right] \quad (45)$$

V. DETERMINATION OF UNKNOWN ARBITRARY FUNCTION B_{nm} AND C_{nm}

Applying boundary conditions (7 - 11) to the equations (36) and (37) one obtains

$$P_m(h) \bar{X} [h \cosh(\mu_n h) \bar{Y} - \bar{Z}] - a_m \bar{f} \mu_n S_0'(k_1, k_2, \mu_n r) B_{nm} = \frac{\bar{g} [(2-\nu) \bar{X} + (1-\nu) \mu_n S_0(k_1, k_2, \mu_n r)]}{[(2-\nu) \bar{X} + (1-\nu) \mu_n S_0(k_1, k_2, \mu_n r)]} \{ (h \mu_n) \cosh(\mu_n h) [h \cosh(\mu_n h) \bar{Y} - \bar{Z}] - \bar{g} \sinh(\mu_n h) \bar{Y} \} \quad (46)$$

and

$$P_m(h) \bar{X} [\sinh(\mu_n h) \bar{Y}] - a_m \bar{f} \mu_n S_0'(k_1, k_2, \mu_n r) C_{nm} = \frac{\cosh(\mu_n h) [(2-\nu) \bar{X} + (1-\nu) \mu_n S_0(k_1, k_2, \mu_n r)]}{[(2-\nu) \bar{X} + (1-\nu) \mu_n S_0(k_1, k_2, \mu_n r)]} \{ (h \mu_n) \cosh(\mu_n h) [h \cosh(\mu_n h) \bar{Y} - \bar{Z}] - \bar{g} \sinh(\mu_n h) \bar{Y} \} \quad (47)$$

where

$$\bar{X} = S_0''(k_1, k_2, \mu_n r) + \frac{S_0(k_1, k_2, \mu_n r)}{r}$$

$$\bar{Y} = \mu_n^2 (\nu \mu_n + (1-\nu)) \frac{S_0'(k_1, k_2, \mu_n r)}{r} + (1-\nu) \left(\mu_n^3 S_0'''(k_1, k_2, \mu_n r) - \frac{S_0(k_1, k_2, \mu_n r)}{r^2} \right)$$

$$\bar{f} = W_m \cos(a_m h) + Q_m \sin(a_m h)$$

$$\bar{g} = \cosh(\mu_n h) + (\mu_n h) \sinh(\mu_n h)$$

$$\bar{z} = 2\nu \mu_n^2 \sinh(\mu_n h) S_0'(k_1, k_2, \mu_n r)$$

VI. SPECIAL CASE

Set

$$\psi(\zeta) = \delta(t - t_0), \quad 0 < t_0 < t \quad (48)$$

$$T_0 = 0 \quad (49)$$

$$T_0^* = 0 \quad (50)$$

Substituting the values of equation (48) and (50) into equations (37) and (42) to (45) one obtains the expressions for the temperature and stresses respectively as follows:

$$\theta(r, z, t) = e \sum_{n=1}^{\infty} \frac{1}{C_n} \left\{ \sum_{m=1}^{\infty} \frac{\wp_{n,m}}{\lambda_m} [\exp(-\omega t) - \exp(-\kappa (\Lambda_{n,m}) t)] \right\} \times P_m(z) S_0(k_1, k_2, \mu_n r) \quad (51)$$

$$\sigma_{rr} = \sigma_0 \sum_{n=1}^{\infty} \frac{1}{C_n} \left\{ \sum_{m=1}^{\infty} \frac{-\wp_{n,m}}{\lambda_m (\Lambda_{nm})} [\exp(-\omega t) - \exp(-\kappa (\Lambda_{n,m}) t)] \right\} \times \{ -P_m(z) \left[\frac{\mu_n S_0'(k_1, k_2, \mu_n r)}{r} - a_m^2 S_0(k_1, k_2, \mu_n r) \right] + \mu_n^2 (\nu - 1) S_0''(k_1, k_2, \mu_n r) \times [\mu_n (B_{nm} \cosh(\mu_n z) + C_{nm} (z \sinh(\mu_n z) + \cosh(\mu_n z))) + \mu_n \nu [B_{nm} \cosh(\mu_n z) + C_{nm} (z \sinh(\mu_n z) + \cosh(\mu_n z))] \times [r^{-1} S_0'(k_1, k_2, \mu_n r) + \mu_n S_0(k_1, k_2, \mu_n r)] + 2\nu C_{nm} \mu_n^2 \cosh(\mu_n z) S_0(k_1, k_2, \mu_n r)] \} \quad (52)$$

$$\sigma_{\theta\theta} = \sigma_0 \sum_{n=1}^{\infty} \frac{1}{C_n} \left\{ \sum_{m=1}^{\infty} \frac{-\wp_{n,m}}{\lambda_m (\Lambda_{nm})} [\exp(-\omega t) - \exp(-\kappa (\Lambda_{n,m}) t)] \right\} \times \{ -P_m(z) [\mu_m^2 S_0''(k_1, k_2, \mu_n r) - a_m^2 S_0(k_1, k_2, \mu_n r)] + \frac{\mu_n (\nu - 1)}{r} S_0^1(k_1, k_2, \mu_n r) \times [(\mu_n B_{nm} + C_{nm}) \cosh(\mu_n z) + \mu_n C_{nm} z \sinh(\mu_n z)] + \mu_n^2 \nu [(\mu_n B_{nm} + C_{nm}) \cosh(\mu_n z) + \mu_n C_{nm} z \sinh(\mu_n z)] \times [S_0''(k_1, k_2, \mu_n r) + S_0(k_1, k_2, \mu_n r)] + 2\nu C_{nm} \mu_n^2 \cosh(\mu_n z) S_0(k_1, k_2, \mu_n r)] \} \quad (53)$$

$$\sigma_{zz} = \sigma_0 \sum_{n=1}^{\infty} \frac{1}{C_n} \left\{ \sum_{m=1}^{\infty} \frac{-\wp_{n,m}}{\lambda_m (\Lambda_{nm})} [\exp(-\omega t) - \exp(-\kappa (\Lambda_{n,m}) t)] \right\} \times \{ -\mu_n P_m(z) \left[\mu_n S_0''(k_1, k_2, \mu_n r) + \frac{S_0(k_1, k_2, \mu_n r)}{r} \right] + \mu_n^2 [B_{nm} \cosh(\mu_n z) + C_{nm} z \sinh(\mu_n z)] \times \left[(2-\nu) [\mu_n S_0''(k_1, k_2, \mu_n r) + r^{-1} S_0(k_1, k_2, \mu_n r)] + (1-\nu) \mu_n S_0(k_1, k_2, \mu_n r) \right] \}$$

$$\begin{aligned}
 & + \mu_n C_{nm} \cosh(\mu_n z) \\
 & \times \left\{ [(2-\nu)[\mu_n S'_0(k_1, k_2, \mu_n r) + \frac{S_0(k_1, k_2, \mu_n r)}{r}] \right. \\
 & \quad \left. + (1-\nu) \mu_n^2 S_0(k_1, k_2, \mu_n r)] \right\} \quad (54) \\
 \sigma_{rz} = & \sigma_0 \sum_{n=1}^{\infty} \frac{1}{C_n} \left\{ \sum_{m=1}^{\infty} \frac{-\delta_{n,m}}{\lambda_m(\Lambda_{nm})} \left[\frac{\exp(-\omega t)}{-\exp(-\kappa(\Lambda_{n,m})t)} \right] \right\} \\
 & \times \{ [-\mu_n a_m (Q_m \sin(a_m z) + W_m \cos(a_m z))] \\
 & \times S'_0(k_1, k_2, \mu_n r) \\
 & + [(B_{nm} \sinh(\mu_n z) + C_{nm} z \cosh(\mu_n z)) \mu_n^2 [\nu \mu_n + \frac{(1-\nu)}{r}] \\
 & - 2\nu \mu_n^2 C_{nm} \sinh(\mu_n z)] S'_0(k_1, k_2, \mu_n r) \\
 & + (1-\nu) [B_{nm} \sinh(\mu_n z) + C_{nm} z \cosh(\mu_n z)] \\
 & \times [\mu_n^3 S''_0(k_1, k_2, \mu_n r) - \frac{S_0(k_1, k_2, \mu_n r)}{r^2}] \} \\
 & (55)
 \end{aligned}$$

where

$$\sigma_0 = 2eG \left(\frac{1+\nu}{1-\nu} \right) \alpha_t$$

VII. NUMERICAL RESULTS, DISCUSSION AND REMARKS

To interpret the numerical computations, we consider material properties of Aluminum metal, which can be commonly used in both, wrought and cast forms. The low density of aluminum results in its extensive use in the aerospace industry, and in other transportation fields. Its resistance to corrosion leads to its use in food and chemical handling (cookware, pressure vessels, etc.) and to architectural uses.

Modulus of Elasticity, E (dynes/cm ²)	6.9×10^{11}
Shear modulus, G (dynes/cm ²)	2.7×10^{11}
Poisson ratio, ν	0.281
Thermal expansion coefficient, α_t (cm/cm- ⁰ C)	25.5×10^{-6}
Thermal diffusivity, κ (cm ² /sec)	0.86
Thermal conductivity, λ (cal-cm ⁰ C/sec/cm ²)	0.48
Inner radius, a (cm)	2
Outer radius, b (cm)	10
Thickness, h (cm)	3

Table 1: Material properties and parameters used in this study

Property values are nominal.

The foregoing analysis are performed by setting the radiation coefficients constants, $k_i = 0.86$ ($i = 1, 3$) and $k_i = 1$ ($i = 2, 4$), so as to obtain considerable mathematical simplicities. The other parameters considered are $r_0 = 2.5$,

and $\omega = 1$. The derived numerical results from equation (51) to (55) has been illustrated graphically with internal heat source with available additional sectional heat on its flat surface.

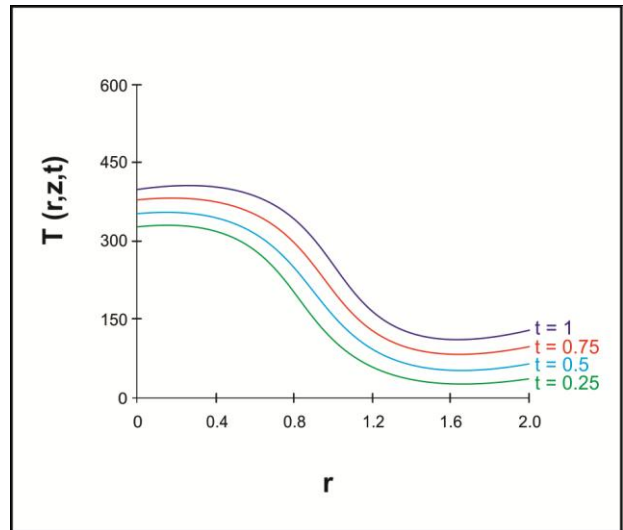


Fig 1: Temperature distribution with internal heat source

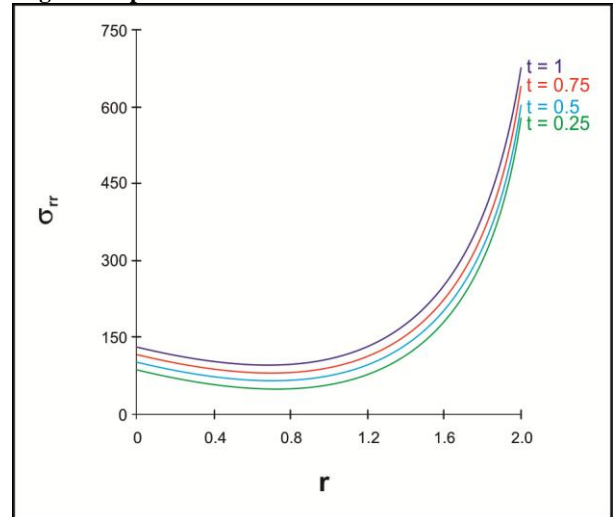


Fig 2: Radial stress distribution with internal heat source

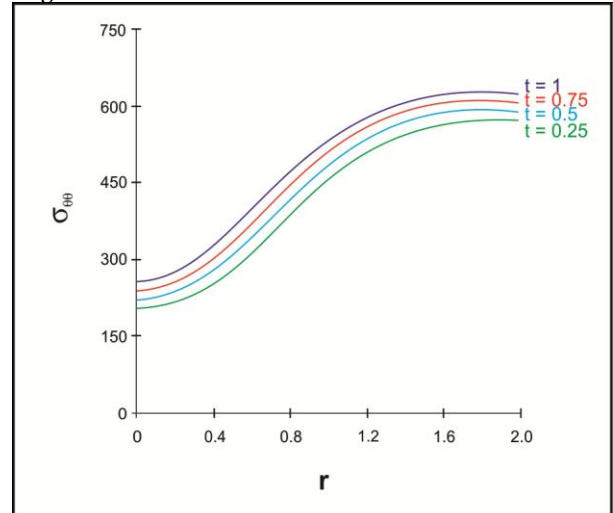


Fig 3: Tangential stress distribution with internal heat source

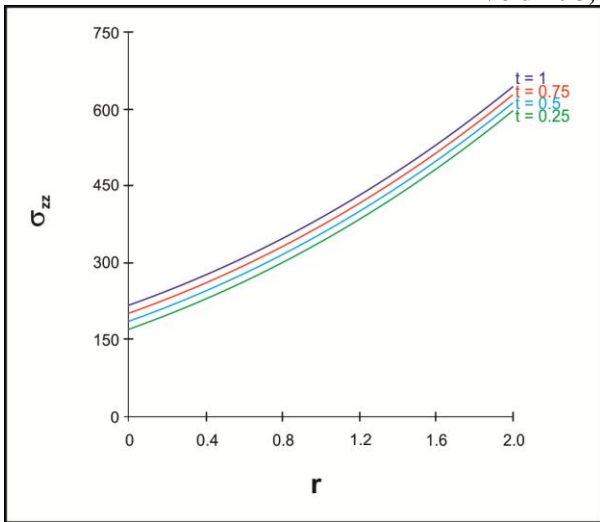


Fig 4: Axial stress distribution for varying along r-axis with internal heat source

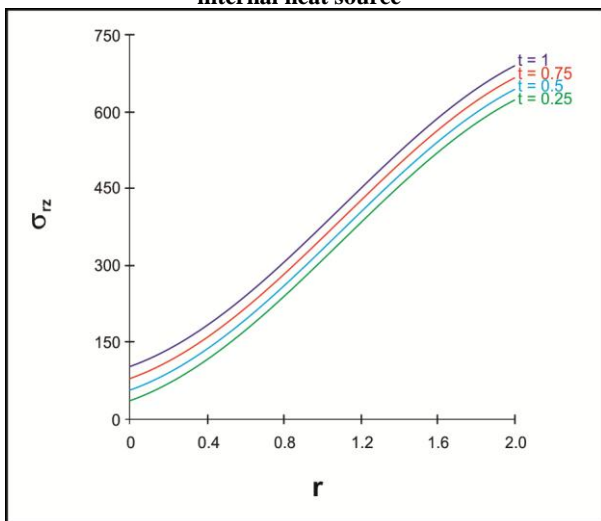


Fig 5: Shear stress distribution for varying along r-axis

In order for the solution to be meaningful the series expressed in equation (51) should converge for all $a \leq r \leq b$ and $-h \leq z \leq h$. The temperature distribution equation (51) can be expressed as

$$\theta(r, z, t) = e \sum_{n=1}^M \frac{1}{C_n} \left\{ \sum_{m=1}^{\infty} \frac{\delta_{n,m}}{\lambda_m} [\exp(-\omega t) - \exp(-\kappa \Lambda_{n,m} t)] \right\} \times P_m(z) S_0(k_1, k_2, \mu_n r) \tag{56}$$

We impose conditions so that $\theta(r, z, t)$ converge in some generalized sense to $g(r, s)$ as $t \rightarrow 0$ in the transform domain. Taking into account of the asymptotic behaviors of $P_m(z)$, μ_n , $S_0(k_1, k_2, \mu_n r)$ and C_n it is observed that the series expansion for $\theta(r, z, t)$ will be theoretically convergent due to the bounded functions. As convergence of the series for $r = b$ implies convergence for all $r \leq b$ at any value of z . An exact solution requires use of infinite number of terms in the equations. The effects of truncating of terms are brought out by the

comparison table for solutions of different functions for 5 and 10 terms. For the convergence test of the present method, we calculate the temperature values at the point $r = 4, z = 1$ with different time for 5 terms and 10 terms, as shown in the table 1.

Table 1. The temperature under different time and different number of terms at $r = 4, z = 1$

Functions	5 terms	10 terms
Time = 0.1	0.25192	0.47855
Time = 1.0	0.04453	0.06739
Time = 2.0	0.00137	0.00206

From Table 1, we can see that series solutions converge rapidly provided we take sufficient number of terms in the series.

VIII. CONCLUSION

In this study, we treated the two-dimensional thermo elastic problem of a hollow cylinder in which sources are generated according to the linear function of the temperature. We successfully established and obtained the temperature distribution, displacements and stress functions with additional sectional heat, $\exp(-\omega t) \delta(r - r_0)$ available at the edge $z = h$ of the hollow cylinder. Then, in order to examine the validity of two-dimensional thermo elastic boundary value problem, we analyze, as a particular case with mathematical model for $\psi(\zeta) = \delta(t - t_0)$ and numerical calculations are carried out. Moreover, assigning suitable values to the parameters and functions in the equations of temperature, displacements and stresses respectively, expressions of special interest can be derived for any particular case. We may conclude that the system of equations proposed in this study can be adopted to design of useful structures or machines in engineering applications in the determination of thermo elastic behavior with radiation.

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