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Alternative Approach to Linear Fractional Programming

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Abstract- In this paper, an alternative approach to the Linear Fractional Programming Problem (LFP) is proposed in which the objective function, and the constraints are in the form of linear inequality. We review some recent development in Linear Fractional Programming. It will be shown that this method mainly used for solving algebraic Problems and depend upon the concept of selecting pivot vector on the basis of new rules of method described below. We provide the necessary context by including some major existing results. A simple example is given to illustrate the proposed method.

Key words and phrases: **Linear Fractional Programming, Optimum solution.**

I. INTRODUCTION

In various applications of nonlinear programming a ratio of two functions is said to be maximized or minimized. In other applications the objective function involves more than one such ratio. Ratio Optimization is commonly called fractional programming. A linear fractional Programming Problem is defined as follows:

Maximize (or Minimize)

$$M(x)\ or\big(m(x)\big)=\frac{p^T\ x+r}{q^T\ x+s}$$

Subject to the constraints:

$$Ux=v\ ;\ x\geq 0$$

Here p and q are vectors in R^n, represent the Objective function coefficients.

and x, p and q are $n\times 1$ vectors, U is an $m\times n$ matrix, $U=\left(u_{ij}\right)$ where $1\leq i\leq m$ and $1\leq j\leq n$, $v\,\varepsilon\,R^m$ is $m\times 1$ vectors and r, s are scalars.

The above defined Linear Fraction maximum Problems (i.e. the ratio objective that have numerator and denominator) arise in various circumstances in management sciences, research and interest; since they are useful in management Production Planning, Financial and corporate Planning, maximization of return on investment, Health care and Hospital Planning, Maximization of cost/time give rise to a Fractional Programming. Linear Fractional Programming Problems is studied by many authors charnes et al.(1962) reformulate Linear Fractional Programming Problem into a Linear Programming Problem. Bialas and Karwan (1982, 1984) studied and developed Bi-Level Programming Problem. Bitran and Novaes (1973) derived dated objective function to solve Linear Fractional Program by solving a sequence of Linear Programs only re-computing the local gradient of the objective function. Also Birtan and Magnant I (1976) was discussed some aspect concerning duality and sensitivity analysis in Linear fraction programs and Singh C.(1981) discussed a useful study about the Optimality condition in Fractional Programming.

The Linear Fractional Programming Problems has its own importance in obtaining the solution of a Problem where two or more activities complete for limited resources. In this paper an alternative approach for solving linear Fractional Programming (LPF) is Proposed which depends mainly on the linear Fractional Function and the constraint Functions in the form of linear inequality. This paper proposed Powerful technique for solution of algebraic Problems based upon the new concept of selecting pivot vector which helps to reduced number of iteration as compared to other methods. An example is given to clarify the developed theory and the proposed method.

The New Fractional Programming Algorithm for solving Linear Fractional Programming Problem is stated below:

Step 1: Consider the Fractional Programming Problem defined as:-

Maximize

$$M(x)=\frac{p^T\ x+r}{q^T\ x+s}$$

Subject to the constraints:

$$Ux=v\ ;\ x\geq 0$$

where $x\in R^n$, A is an $m\times n$ matrix, we point out that the non-negative conditions are included in the set of constraints, p and q are n-vectors, b is $m\times 1$ vector and r, s are scalars.

Let x_ω be the initial basic feasible solution such that

$$\omega x_\omega = v \ \text{or}\ x_\omega=\omega^{-1}v,\ x_\omega\geq 0$$

Where $\omega=\left(\omega_1,\omega_2,\ldots\ldots\ldots\ \ldots.,\omega_m\right)$

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Next assume that

$M' = p_\omega^T x_\omega + r$ and $M'' = q_\omega^T x_\omega + s$

Where p_ω^T and q_ω^T denotes the vectors associated with basic variables in the numerator and the denominator of the objective function respectively.
In addition we assume that for this basic feasible solution

$y_j = \omega^{-1} u_j$, $M' = p_\omega^T y_j$ and $M'' = q_\omega^T u_j$

Step 2: There is a possibility of finding another basic feasible solution with the modified value of $M = M'/M''$. We wish to confine our attention to those basic feasible solution in which only one column of ω is changed. Now if the basic feasible solution is denoted by x'_ω , then $x'_\omega = (\omega')^{-1} v$ where $\omega' = (\omega'_1, \omega'_2, \ldots\ldots\ldots \ldots, \omega'_m)$.
The column of the new matrix ω' are given by $v'_i = v_i (i \neq r)$ and $v'_r = u_j$.
Next, we obtain the value of the new basic variables in terms of the original ones and the y_{ij} i.e. $x'_{\omega i} = x_{\omega i} - x_\omega (y_{ij} / y_{rj})$

$x'_{\omega i} = x_{\omega r} / y_{rj} = \eta (say)$

where

$u_j = \sum_{i=1}^{m} y_{ij} v_i$.

Step3: We are interest in finding a new basic feasible solution with an improved value of the objective function.
Let the new objective function be $M_1 = M'_1 / M''_1$
we have

$$M'_1 = M' + \frac{\eta (M'_j - p_j)}{\sum_{i=1}^{m} \chi}$$

And

$$M''_1 = M'' + \frac{\eta (M''_j - q_j)}{\sum_{i=1}^{m} \chi},$$

where $\sum_{i=1}^{m} \chi$ is the sum of the corresponding column.

M'_j and M''_j refer to the original basic feasible solution.

Step 4: The value of the objective function will improve if $M_1 > M$, or

$$M''(M'_j - p_j) - M'(M''_j - q_j) > 0$$

Let

$$\xi_j = -M''(M'_j - p_j) + M'(M''_j - q_j)$$

Now ξ_j is less than zero if

$(M''_j - q_j) > 0$, $(M''_j - q_j) < 0$ or

$(M''_j - q_j) = 0$

We conclude that a given a basic feasible solution $x'_\omega = \omega^{-1} v$, if for any column u_j in U but not in ω , $\xi_j < 0$ holds, and if at least one $y_{ij} > 0$, then it is possible to obtain a new basic feasible solution by replacing one of the column in ω by u_j and the new value of the objective function satisfies $M_1 > M$.

Step 5: For any $u_j \in U$ not in ω at least one $y_{ij} < 0$, $1 \leq i \leq m.$
We have basic feasible solution

$$\sum_{i=1}^{m} x\omega_i v_{j=v} ,$$

add and subtracts $\eta' u_j$ (where η' is any scalar), one obtains

q_ω	p_ω	x_ω	y_1	y_2	y_3	y_4	$x_{\omega i} / y_{ij}$
0	0	$S_1 = 15$	3	5	1	0	5
0	0	$S_2 = 10$	5	2	0	1	2
M''	M'	$M = 0$					
		p_j :	5	3	0	0	
		q_j :	5	2	0	0	
		$M'_j - p_\omega$	-5/8	-3/7	0	0	
		$M'_j - p_\omega$	-5/8	-2/7	0	0	
		ξ_j	-5/8	-3/7	0	0	

$$\sum_{i=1}^{m} x\omega_i v_j - \eta' u_j + \eta' u_j = v$$

But

$$-\eta' \sum_{i=1}^{m} y_{ij} v_i = -\eta' u_j$$

$$\therefore \sum_{i=1}^{m} (x\omega_i - \eta' y_{ij}) v_i + \eta' u_j = v$$

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when $\eta' > 0$, we have $(x\omega_i - \eta' y_{ij}) \geq 0$.

Step 6: In the algorithm if we start with a basic feasible solution and if there is a vector u_j not in the basis having $\xi_j < 0$, then there exists another basic feasible solution such that $M_1 > M$. Thus changing one vector at a time so long as there is some u_j not in the basis with the condition of $\xi_j < 0$ and at each step, M is increased.

This procedure continues up to a finite number of steps because of finite basis.

This process will terminate only when all $\xi_j \geq 0$, for every column u_j in U.

II. STATEMENT OF THE PROBLEM

Use alternative approach to solve the following Linear Fractional Programming Problem:

Maximize $(5x_1 + 3x_2)/(5x_1 + 2x_2 + 1)$

Subject to the constraints:

$3x_1 + 5x_2 \leq 15,\ 5x_1 + 2x_2 \leq 10,\ x_1, x_2 \geq 0$

III. SOLUTION OF THE PROBLEM

Maximize $M = (5x_1 + 3x_2)/(5x_1 + 2x_2 + 1)$

Convert the inequality constraints into equations by introducing slack variable

Subject to the constraints:

$3x_1 + 5x_2 + S_1 = 15,$

$5x_1 + 2x_2 + S_2 = 10,$

where $x_1, x_2 \geq 0$ and slack variable $S_1, S_2 \geq 0$

The initial basic feasible solution is given in the following tables:-

Initial iteration:

First Iteration:-Introduce y_1 and drop y_4.

		p_j :	5	3	0	0	
		q_j :	5	2	0	0	
q_ω	p_ω	x_ω	y_1	y_2	y_3	y_4	$x_{\omega i}/y_{ij}$
0	0	$S_1 = 9$	0	19/5	1	-3/5	45/19
5	5	$x = 2$	1	2/5	0	1/5	5
M''	M'	$M = 10/11$					
		$M'_j - p_\omega$	0	-5/21	0	-5/2	
		$M'_j - p_\omega$	0	0	0	-5/2	
		ξ_j	0	-22/21	0	-5/2	

Second Iteration:-Introduce y_2 and drop y_3.

		p_j :	5	3	0	0	
		q_j :	5	2	0	0	
q_ω	p_ω	x_ω	y_1	y_2	y_3	y_4	$x_{\omega i}/y_{ij}$
2	3	$x_2 = 45/19$	0	1	1	-3/5	45/19
5	5	$x_1 = 20/19$	1	0	0	1/5	5
M''	M'	$M = 0$					
		$M'_j - p_\omega$	0	0	25/7	8	
		$M'_j - p_\omega$	0	0	0	19/2	
		ξ_j	0	0	+ve	+ve	

Since $\xi_j \geq 0$, and hence the optimum solution exist and its value is given by

$x_1 = 20/19$ and $x_2 = 45/19$ thus we reached maximum M=235/209.

IV. CONCLUSION

We observed that the optimum solution obtained in less or at the most equal number of iteration by our modified method than traditional method and our technique gives better result as compared to other methods. Hence the number of iterations required is reduced by our methodology. Also we require less time to simplify the numerical problems.

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