

Solution of Linear Programming Problem by KKL Method

N. W. Khobragade, N. K. Lamba and P. G. Khot*
 Department of Mathematics, MJP Educational Campus,
 * Department of Statistics, MJP Educational Campus
 RTM Nagpur University, Nagpur 440 033, India.

Abstract - In this paper, a new approach to the solution of linear programming problems is suggested, which is based on the iterative procedure. The proposed KKL method is computationally more efficient and easier as compared to the traditional simplex method. The method suggested for the solution of LPP namely “KKL method” where KKL is formed from the first letter of author’s surname.

Key words and phrases: KKL Simplex Method, Optimum Solution, LPP.

I. INTRODUCTION

Linear programming is an optimization technique for finding an optimal (maximum or minimum) value of a function called objective function of several independent variables, the variables being subject to various restrictions (or constraint) expressed as equation or inequalities. The term ‘linear’ indicates that the function to be maximized or minimized is linear in the nature and that the corresponding constraint represented by a system of linear inequality or linear equation involving variables. The linear programming has its own importance in obtaining the solution of a problem where two or more activities complete for limited resources.

Mathematically:

Maximize the objective function CX

Subject to $AX = B, X \geq 0$

Where $X = n \times 1$ column matrix,

$A = m \times n$ coefficient matrix,

$B = m \times 1$ Column vector,

$C = 1 \times n$ row vector.

There are four methods to obtain the solution of the above problem; these methods can be classified as:

- (i). the graphical method,
- (ii). the systematic trial and error method,
- (iii). the vector method,
- (iv). the simplex method.

In this paper, we shall introduce and explain the Computational Algorithm of the KKL method. Computational procedure of the KKL method requires the construction of “New KKL tableau” which can be done in different ways, all of which, however, led to the same optimal solution. The initial KKL tableau is formed by writing the coefficient of constraints of a linear programming problem in a systematic tabular form. The

rule used for the construction of initial KKL table is same in both the maximization and minimization problems.

II. COMPUTATIONAL ALGORITHM OF THE KKL METHOD

Step 1: Formulate the linear programming model of the real world problem that is obtained as mathematical representation of the problem, objective function and constraints as stated below:

Maximize $M = c_1x_1 + c_2x_2 + \dots + c_nx_n$

Subject to constraints:

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \leq b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \leq b_2$$

.....

$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n \leq b_m$$

$$x_1, x_2, x_3, \dots, x_n \geq 0$$

If the objective function is minimized, then convert it into maximization problem of by using the rule Minimum $M = -(\text{Maximum } (-M))$.

All b_i 's, $i = 1, 2, \dots, m$ must be non negative. If any one of b_i is negative, multiply corresponding inequality by (-1), so as to get all b_i 's, $i = 1, 2, \dots, m$ non-negative.

Step 2: Convert all in equations of the constraints into equations by introducing slack variables and assign a zero coefficient to the corresponding variable in the objective function. Thus we can reformulate the problem in terms of equations as follows:

Maximize

$$M = c_1x_1 + c_2x_2 + \dots + c_nx_n + 0p_1 + 0p_2 + \dots + 0p_m$$

Subject to constraints:

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n + p_1 = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n + p_2 = b_2$$

.....

$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n + p_m = b_m$$

where $x_1, x_2, x_3, \dots, x_n \geq 0$ and

$$p_1, p_2, p_3, \dots, p_m \geq 0$$

Step 3: An initial basic feasible solution is obtained by setting

$$x_1 = x_2 = x_3 = \dots = x_n = 0$$

Thus we get $p_1 = b_1, p_2 = b_2, \dots, p_m = b_m$.

Step 4: For computational efficiency and simplicity, the initial basic feasible solution, the constraint of the standard linear programming problem as well as the function can be displayed in a tabular form, called the new KKL tableau as given below:

NEW KKL TABLE:

| Row(R) | Basic | Z | y_1 | ... | y_n | P_1 | ... | P_m | R.H.S |
|-------------------------------|-------|-----|----------|-----|----------|-------|-----|-------|-------|
| Initial row (R*) | Z | 1 | C_1 | ... | C_n | 0 | ... | 0 | 0 |
| First row(R ₁) | P_1 | 0 | a_{11} | ... | a_{1n} | 1 | ... | 0 | b_1 |
| Second row(R ₂) | P_2 | 0 | a_{21} | ... | a_{2n} | 0 | ... | 0 | b_2 |
| ... | ... | ... | ... | ... | ... | ... | ... | ... | ... |
| ... | ... | ... | ... | ... | ... | ... | ... | ... | ... |
| m^{th} Row(R _m) | P_m | 0 | a_{m1} | ... | a_{mn} | 0 | ... | 1 | b_m |

The interpretation of data of the above tableau is given as under:

(i). The Initial row (R*), called the objective row of the new KKL table indicates the coefficients of (m+n) variables in the objective function.

(ii). From 1st to mth row are called constraint rows of the table, represents the coefficient of the constraints.

(iii). The first column labeled (R) denotes the m rows, also known as objective column, the second column labelled 'Basic variable', in the initial KKL tableau. These variables are the slack variables. The third column indicates the coefficient of 'z' whose value for the first row is 1 and for all other rows, it is zero.

(iv). The body matrix (under non-basic variables) in the initial KKL tableau consists of the decision variable in the constraint set.

(v). The Identity matrix in the initial KKL tableau represents the coefficients of the slack variables that have been added to the original inequality to make them equations.

Step 5: Test the Solution for optimality: -Examine the initial row (R*) of the above KKL tableau.

(i). If all the elements in the initial row (R*) are positive, the current solution is optimal.

(ii). If there exists some negative number, the current solution can be improved by entering the column, which contain less negative term, let it be y_r .

Then

(a). Select the maximum positive term in the corresponding column of y_r , let it be a_{rj} $1 \leq j \leq n$, which is our new pivot element and find out the corresponding identity element, let it lies in the column p_k where $1 \leq k \leq m$, we drop this column p_k .

(b). Repeat the above process till all the coefficient of the Initial Row (R*) becomes positive. If one of the coefficients of the row R₁ is negative, then repeat the above process to find the optimum solution.

III. STATEMENT OF THE PROBLEM

Problem (1): Use KKL method to

$$\text{Maximize } z = 4x_1 + 10x_2$$

Subject to the constraints:

$$2x_1 + x_2 \leq 50$$

$$2x_1 + 5x_2 \leq 100$$

$$2x_1 + 3x_2 \leq 90$$

$$x_1, x_2 \geq 0$$

Solution: By introducing slack variables $p_1 \geq 0$, $p_2 \geq 0$ and $p_3 \geq 0$ respectively, the set of constraints of the LPP are converted into the system of equations.

The modified objective function is to

$$\text{Maximize } z = 4x_1 + 10x_2 + 0p_1 + 0p_2 + 0p_3$$

Subject to the constraints:

$$2x_1 + x_2 + p_1 = 50$$

$$2x_1 + 5x_2 + p_2 = 100$$

$$2x_1 + 3x_2 + p_3 = 90$$

$$x_1, x_2, p_1, p_2, p_3 \geq 0$$

Initial table:

| Row | Basic | z | y_1 | y_2 | P_1 | P_2 | P_3 | RHS |
|----------------|-------|---|-------|-------|-------|-------|-------|-----|
| R ₁ | z | 1 | -4 | -10 | 0 | 0 | 0 | 0 |
| R ₂ | P_1 | 0 | 2 | 1 | 1 | 0 | 0 | 50 |
| R ₃ | P_2 | 0 | 2 | 5 | 0 | 1 | 0 | 100 |
| R ₄ | P_3 | 0 | 2 | 3 | 0 | 0 | 1 | 90 |

↑ ↓

Where P_1 , P_2 and P_3 denotes the slack variables and R_1, R_2, R_3, R_4 represents the first, second, third and fourth row respectively. From the above table, it is clear that the least negative coefficient of z lies in column y_1 which is (-4) , that will enter into the basis. The pivot element is 2 (arbitrary in column y_1) and the corresponding identity element lies in the P_1 column, hence we drop column vector P_1 .

First table: Introduce y_1 and drop P_1 , we get

| Row | Basic | z | y_1 | y_2 | P_1 | P_2 | P_3 | RHS |
|----------------|-------|---|-------|-------|-------|-------|-------|-----|
| R ₁ | z | 1 | 0 | -8 | 2 | 0 | 0 | 100 |
| R ₂ | y_1 | 0 | 1 | 1/2 | 1/2 | 0 | 0 | 50 |
| R ₃ | P_2 | 0 | 0 | 4 | -1 | 1 | 0 | 50 |
| R ₄ | P_3 | 0 | 0 | 2 | -1 | 0 | 1 | 40 |

Since y_2 in the initial row has only one negative value i.e. (-8) , y_2 will enter into the basis. The pivot element is 4, and the corresponding identity element which lies in column P_2 . Hence we drop P_2 .

Second table: Introduce y_2 and drop P_2

| Row | Basic | z | y_1 | y_2 | P_1 | P_2 | P_3 | RHS |
|----------------|-------|---|-------|-------|-------|-------|-------|------|
| R ₁ | z | 1 | 0 | 0 | 0 | 2 | 0 | 200 |
| R ₂ | P_1 | 0 | 1 | 0 | 5/8 | -1/8 | 0 | 75/4 |
| R ₃ | P_2 | 0 | 0 | 1 | -1/4 | 1/4 | 0 | 50/4 |
| R ₄ | P_3 | 0 | 0 | 0 | -1/2 | -1/2 | 1 | 15 |

Since all the elements in the initial row are positive, the current solution is optimal. Hence the optimum basic feasible solution is

$$x_1 = 75/4, x_2 = 50/4 \text{ and } \text{Max. } z = 200.$$

Problem (2): Use KKL method to

$$\text{Minimize } Z = x_2 - 3x_3 + 2x_5.$$

Subject to the constraints:

$$3x_2 - x_3 + 2x_5 \leq 7$$

$$-2x_2 + 4x_3 \leq 12$$

$$-4x_2 + 3x_3 + 8x_5 \leq 10$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0 \text{ and } x_5 \geq 0$$

Solution: By introducing slack variables $p_1 \geq 0$, $p_2 \geq 0$ and $p_3 \geq 0$ respectively, the set of constraints of the LPP are converted into the system of equations, and converting the objective function into that of maximization; the modified linear programming problem

is as follows:

Maximize

$$z^* = -x_2 + 3x_3 - 2x_5 + 0p_1 + 0p_2 + 0p_3$$

Subject to the constraints:

$$3x_2 - x_3 + 2x_5 + p_1 = 7$$

$$-2x_2 + 4x_3 + p_2 = 12$$

$$-4x_2 + 3x_3 + 8x_5 + p_3 = 10$$

$$x_2, x_3, x_5, p_1, p_2, p_3 \geq 0.$$

Initial table:

| Row | Basic | z | y_1 | y_2 | y_3 | y_4 | y_5 | P_1 | P_2 | P_3 | RHS |
|----------------|-------|---|-------|-------|-------|-------|-------|-------|-------|-------|-----|
| R ₁ | z | 1 | 0 | 1 | -3 | 0 | 2 | 0 | 0 | 0 | 0 |
| R ₂ | P_1 | 0 | 0 | 3 | -1 | 0 | 2 | 1 | 0 | 0 | 7 |
| R ₃ | P_2 | 0 | 0 | -2 | 4 | 0 | 0 | 0 | 1 | 0 | 12 |
| R ₄ | P_3 | 0 | 0 | -4 | 3 | 0 | 8 | 0 | 0 | 1 | 10 |

where P_1, P_2 and P_3 denotes the slack variables and R_1, R_2, R_3, R_4 represents the first, second, third and fourth row respectively. From the above table, it is clear that the negative coefficient of Initial Row R_1 lies in column y_3 i.e. (-3) , that will enter into the basis. The pivot element in the same column is 4, and the corresponding identity element lies in the P_2 column, hence we drop column vector P_2 .

First Iteration: Introduce y_3 and drop P_2

| Row | Basic | z | y_1 | y_2 | y_3 | y_4 | y_5 | P_1 | P_2 | P_3 | rhs |
|----------------|-------|---|-------|-------|-------|-------|-------|-------|-------|-------|-----|
| R ₁ | z | 1 | 0 | -1/2 | 0 | 0 | 2 | 0 | 3/4 | 0 | 0 |
| R ₂ | P_1 | 0 | 0 | 5/2 | 0 | 0 | 2 | 1 | 1/4 | 0 | 10 |
| R ₃ | y_3 | 0 | 0 | -1/2 | 1 | 0 | 0 | 0 | 1/4 | 0 | 3 |
| R ₄ | P_3 | 0 | 0 | -5/2 | 0 | 0 | 8 | 0 | -3/4 | 1 | 1 |

Since y_2 in the initial row has only one negative i.e. $(-1/2)$, that will enter into the basis. The pivot element is $(5/2)$, and the corresponding identity element which lies in column P_1 . Hence we drop P_1 .

Second Iteration: Introduce y_2 and drop P_1

| Row | Basic | z | y ₁ | y ₂ | y ₃ | y ₄ | y ₅ | P ₁ | P ₂ | P ₃ | RHS |
|----------------|----------------|---|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|-----|
| R ₁ | z | 1 | 0 | 0 | 0 | 0 | 12/5 | 1/5 | 4/5 | 0 | 11 |
| R ₂ | y ₂ | 0 | 0 | 1 | 0 | 0 | 4/5 | 2/5 | 1/10 | 0 | 4 |
| R ₃ | y ₃ | 0 | 0 | 0 | 1 | 1 | 2/5 | 1/5 | 3/10 | 0 | 5 |
| R ₄ | p ₃ | 0 | 0 | 0 | 0 | 0 | 10 | 1 | -1/2 | 1 | 11 |

Since all the elements in the initial row are positive, the current solution is optimal. Hence the optimal basic feasible solution is

$$x_1 = 0 \quad x_2 = 4, \quad x_3 = 5 \text{ and Max. } Z = 11.$$

IV. SOLUTION OF LPP BY KKL PENALTY METHOD

In the computational procedure of the simplex method, it is more convenient to have slack variable as the starting (initial) basic variables. If the original constraint is an equation or is of the type (\geq) then in order obtain initial basic feasible solution, we put the LPP into the standard form and then a non-negative variable is added to the left side of the equation to form the identity sub matrix. The so added variable is called an artificial variable. The above methods generally employed for the solution of the linear programming problems having artificial variables.

The KKL method of penalty is an alternative method for solving linear programming problem involving artificial variables.

V. ALGORITHM OF KKL PENALTY METHOD

Step 1: Write the given LPP into its standard form and check whether there exists an initial basic feasible solution.

- (i). If there exists an initial basic feasible solution, go to step 3.
- (ii). If there does not exist an initial basic feasible solution, go to step 2.

Step 2: Add sufficient number of artificial variables to the constraints to form identity matrix. Assign a high penalty (say N) to these variables in the objective function.

Step 3: Apply KKL method to the modified LPP.

VI. STATEMENT OF THE PROBLEM

Problem (1): Use KKL penalty method to

$$\text{Maximize } z = 6x_1 + 4x_2$$

Subject to the constraints:

$$2x_1 + 3x_2 \leq 20$$

$$3x_1 + 2x_2 \leq 24$$

$$x_1 + x_2 \geq 3$$

$$x_1 \geq 0 \text{ and } x_2 \geq 0$$

Solution: By introducing slack variables $p_1 \geq 0$,

$p_2 \geq 0$ and the surplus variable $p_3 \geq 0$ respectively, the set of constraints of the LPP are converted into the system of equations, the modified linear programming problem is as follows:

$$\text{Maximize } z = 6x_1 + 4x_2$$

Subject to the constraints:

$$2x_1 + 3x_2 + p_1 = 20$$

$$3x_1 + 2x_2 + p_2 = 24$$

$$x_1 + x_2 - p_3 + A_1 = 3$$

$$x_1, x_2, p_1, p_2, p_3 \geq 0$$

We have initial KKL table:

Initial table:

| Row | Basic | z | y ₁ | y ₂ | P ₁ | P ₂ | P ₃ | A ₁ | RHS |
|----------------|----------------|---|----------------|----------------|----------------|----------------|----------------|----------------|-----|
| R ₁ | z | 1 | -6 | -4 | 0 | 0 | 0 | N | 0 |
| R ₂ | P ₁ | 0 | 2 | 3 | 1 | 0 | 0 | 0 | 30 |
| R ₃ | P ₂ | 0 | 3 | 2 | 0 | 1 | 0 | 0 | 24 |
| R ₄ | P ₃ | 0 | 1 | 1 | 0 | 0 | -1 | 1 | 3 |

↑ ↓

where P_1, P_2 denotes the slack variables, and P_3

denote the surplus variable, A_1 is an artificial variable and R_1, R_2, R_3, R_4 represents the first, second, third and fourth row respectively. From the above table, it is clear that the least negative coefficient of z lies for column

y_2 i.e. (-4), that will enter into the basis. The pivot element is 3, as it is most positive term along the column

y_2 and the corresponding identity element lies in the

P_1 column. Hence we drop P_1 .

First Iteration: Introduce y_2 and drop P_1 .

| Row | Basic | z | y ₁ | y ₂ | y ₃ | P ₁ | P ₂ | A ₁ | RHS |
|----------------|----------------|---|----------------|----------------|----------------|----------------|----------------|----------------|-----|
| R ₁ | z | 1 | -10/3 | 0 | 4/3 | 0 | 0 | N | 40 |
| R ₂ | y ₂ | 0 | 2/3 | 1 | 1/3 | 0 | 0 | 0 | 10 |
| R ₃ | P ₂ | 0 | 5/3 | 0 | -2/3 | 1 | 0 | 0 | 4 |
| R ₄ | P ₃ | 0 | 1/3 | 0 | -1/3 | 0 | -1 | 1 | -7 |

↑ ↓

Since y_1 in the initial row is only one negative i.e. (-10/3), that will enter into the basis. The pivot element is 5/3, and the corresponding identity element which lies in column p_2 . Hence we drop p_2 .

Second Iteration: Introduce y_1 and drops p_2 .

| Row | Basic | z | y ₁ | y ₂ | P ₁ | P ₂ | P ₃ | A ₁ | RHS |
|----------------|----------------|---|----------------|----------------|----------------|----------------|----------------|----------------|---------|
| R ₁ | z | 1 | 0 | 0 | 0 | 2 | 0 | N | 48 |
| R ₂ | y ₂ | 0 | 0 | 1 | 9/15 | -6/15 | 0 | 0 | 42/5 |
| R ₃ | y ₁ | 0 | 1 | 0 | -2/5 | 3/5 | 0 | 0 | 12/5 |
| R ₄ | P ₃ | 0 | 0 | 0 | -1/5 | -3/15 | -1 | 1 | -117/15 |

Since all the elements in the initial row are positive, the current solution is optimal. Therefore the optimum basic feasible solution is

$$x_1 = 12/5, x_2 = 42/5 \text{ and Max. } Z = 48.$$

Problem (2): A fertilizer company has only 1000 tonnes of nitrate, 1800 tonnes of phosphate and 1,200 tonnes of potash available per month. They can use these to make three basic fertilizers namely 5-10-5, 5-10-10 and 10-10-10, where the numbers in case represent the percentage by weight of the nitrate (N), phosphate (P), and potash (K) respectively in each of the mixture. The costs of the raw materials are given below:

| Ingredient | Cost (Rs.) per tonne |
|------------------|----------------------|
| Nitrate | 8,000 |
| Phosphate | 2,000 |
| Potash | 5,000 |
| Inert Ingredient | 250 |

The selling prices of the basic fertilizers are Rs.2,000; Rs. 2,500 and Rs.3,000 per tonnes respectively. There is a restriction that the company must produce at least 6,000 tonnes of 5-10-5 per month.

Determine how much of each of the profit of the basic fertilizer they should produce per month in order to maximize their monthly profit by using KKL penalty method.

Solution: Let F₁, F₂ and F₃ be the three products to be manufactured. Then the data of the problem can be summarized as follows:

| Fertilizer | Fertilizer Ingredient | | | |
|-------------------|-----------------------|-----------|--------|-------|
| | Nitrate | Phosphate | Potash | Inert |
| F ₁ | 5% | 10% | 5% | 80% |
| F ₂ | 5% | 10% | 5% | 80% |
| F ₃ | 5% | 10% | 5% | 80% |
| Cost per kg. (Rs) | 8,000 | 2,000 | 5,000 | 250 |

$$\begin{aligned} \text{Cost of } F_1 &= 5\% \text{ of } 8,000 + 10\% \text{ of } 2,000 + 5\% \text{ of } 5,000 + 80\% \text{ of } 250 \\ &= 400 + 200 + 250 + 200 = 1,050. \end{aligned}$$

$$\begin{aligned} \text{Cost of } F_2 &= 5\% \text{ of } 8,000 + 10\% \text{ of } 2,000 + 10\% \text{ of } 5,000 + 75\% \text{ of } 250 \\ &= 400 + 200 + 500 + 187.5 = 1,287.5 \end{aligned}$$

$$\text{Cost of } F_3 = 10\% \text{ of } 8,000 + 10\% \text{ of } 2,000$$

$$\begin{aligned} &+ 10\% \text{ of } 5,000 + 70\% \text{ of } 250 \\ &= 800 + 200 + 500 + 175 = 1,675. \end{aligned}$$

Let x₁, x₂ and x₃ be the quantity (in tonnes) of F₁, F₂, and F₃ respectively to be manufactured. Then the appropriate mathematical formulation of the given problem as LP model is:

$$\begin{aligned} \text{Maximize (total profit) } Z &= (\text{Selling price} - \text{Cost price}) \times (\text{Quantity of Fertilizer}) \\ &= (2,000 - 1,050) x_1 + (2,500 - 1,287.5) x_2 + (3,000 - 1,675) x_3 \\ &= 950 x_1 + 1,212.5 x_2 + 1,325 x_3 \end{aligned}$$

Subject to constraints:

$$0.05x_1 + 0.05x_2 + 0.10x_3 \leq 1,000 \text{ or}$$

$$5x_1 + 5x_2 + 10x_3 \leq 1,000,000 \text{ or}$$

$$10x_1 + 10x_2 + 10x_3 \leq 1,80,000$$

$$0.05x_1 + 0.10x_2 + 0.10x_3 \leq 1,200 \text{ or}$$

$$5x_1 + 10x_2 + 10x_3 \leq 1,20,000$$

$$x_1 \geq 6,000$$

$$x_1, x_2, x_3 \geq 0$$

By introducing slack variables p₁ ≥ 0, p₂ ≥ 0 and the surplus variable p₃ ≥ 0 respectively, the set of constraints of the LPP are converted into the system of equations, and converting the objective function into that of maximization; the linear programming problem is modified as:

$$\begin{aligned} \text{Maximize } z &= 950 x_1 + 1221.5 x_2 + 1325 x_3 \\ &+ 0 p_1 + 0 p_2 + 0 p_3 + 0 p_4 - N A_1 \end{aligned}$$

Subject to the constraints:

$$5x_1 + 5x_2 + 10x_3 + p_1 = 1,00,000$$

$$10x_1 + 10x_2 + 10x_3 + p_2 = 1,80,000$$

$$5x_1 + 10x_2 + 10x_3 + p_3 = 1,20,000$$

$$x_1 - p_4 + A_1 = 6000$$

$$x_1, x_2, p_1, p_2, p_3 \geq 0$$

Initial table:

| Row | Basic | z | y ₁ | y ₂ | y ₃ | P ₁ | P ₂ | P ₃ | P ₄ | A ₁ | RHS |
|----------------|----------------|---|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------|
| R ₁ | z | 1 | -950 | -1221.5 | -1325 | 0 | 0 | 0 | 0 | N | 0 |
| R ₂ | P ₁ | 0 | 5 | 5 | 10 | 1 | 0 | 0 | 0 | 0 | 1,00,000 |

| | | | | | | | | | | | |
|----------------|----------------|---|-----------|----|----|---|---|---|----|---|----------|
| R ₃ | P ₂ | 0 | 10 | 10 | 10 | 0 | 1 | 0 | 0 | 0 | 1,80,000 |
| R ₄ | P ₃ | 0 | 5 | 10 | 10 | 0 | 0 | 1 | 0 | 0 | 1,20,000 |
| R ₅ | P ₄ | 0 | 1 | 0 | 0 | 0 | 0 | 0 | -1 | 1 | 6000 |

↑
↓

where P_1 , P_2 and P_3 denotes the slack variables, P_4 denote the surplus variable, A_1 the artificial variable and R_1, R_2, R_3, R_4 represents the first, second, third and fourth row respectively. From the above table, it is clear that the least negative coefficient of z lies for column y_1 i.e. (-950), that will enter into the basis. The pivot element is 10, as it is most positive term along the column y_1 and the corresponding identity element lies in the P_2 column. Hence we drop P_2 .

First Iteration: Introduce y_1 and drop P_2 .

| Row | Basic | z | y ₁ | y ₂ | y ₃ | P ₁ | P ₂ | P ₃ | P ₄ | A ₁ | RHS |
|----------------|----------------|---|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|-------------|
| R ₁ | z | 1 | 0 | 0 | 0 | 103.5/5 | 56.5/0 | 54.3 | 0 | N | 1,89,36,000 |
| R ₂ | y ₃ | 0 | 0 | 0 | 1 | 1/5 | -1/10 | 0 | 0 | 0 | 2,000 |
| R ₃ | y ₁ | 0 | 1 | 0 | 0 | 0 | 1/5 | -1/5 | 0 | 0 | 12,000 |
| R ₄ | y ₂ | 0 | 0 | 1 | 0 | -1/5 | 0 | 1/5 | 0 | 0 | 4,000 |
| R ₅ | P ₄ | 0 | 0 | 0 | 0 | -2/5 | 1/5 | 1/5 | 1 | -1 | 14,000 |

↑
↓

Since the coefficient of y_2 is the next least negative in the initial row R_1 i.e. (-271.5), that will enter into the basis. The pivot element is 5 and the corresponding identity element which lies in column P_3 . Hence we drop column vector P_3 .

Second Iteration: Introduce y_2 and drop P_3 .

| Row | Basic | z | y ₁ | y ₂ | y ₃ | P ₁ | P ₂ | P ₃ | P ₄ | A ₁ | RHS |
|----------------|----------------|---|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|------------|
| R ₁ | z | 1 | 0 | -271.5 | -375 | 0 | 95 | 0 | 0 | N | 171,00,000 |
| R ₂ | P ₁ | 0 | 0 | 0 | 5 | 1 | -1/2 | 0 | 0 | 0 | 10,000 |
| R ₃ | y ₁ | 0 | 1 | 1 | 1 | 0 | 1/0 | 0 | 0 | 0 | 1,80,000 |
| R ₄ | P ₃ | 0 | 0 | 5 | 5 | 0 | -1/2 | 1 | 0 | 0 | 30,000 |
| R ₅ | P ₄ | 0 | 0 | -1 | 1 | 0 | 1/0 | 0 | 1 | -1 | 12,000 |

↑
↓

Since y_3 has the only negative value, so it will enter into the basis. The pivot element is 5 and corresponding identity element to the pivot element lies in the P_1 . Hence we drop column vector P_1 .

Third Iteration: Introduce y_3 and drops P_1 .

| Row | Basic | z | y ₁ | y ₂ | y ₃ | P ₁ | P ₂ | P ₃ | P ₄ | A ₁ | RHS |
|----------------|----------------|---|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|-------------|
| R ₁ | z | 1 | 0 | 0 | -103.5 | 0 | 67.85 | 54.3 | 0 | N | 1,87,29,000 |
| R ₂ | P ₁ | 0 | 0 | 0 | 5 | 1 | -1/2 | 0 | 0 | 0 | 10,000 |
| R ₃ | y ₁ | 0 | 1 | 0 | 0 | 0 | 1/5 | -1/5 | 0 | 0 | 12,000 |
| R ₄ | y ₂ | 0 | 0 | 1 | 1 | 0 | -1/10 | 1/5 | 0 | 0 | 6,000 |
| R ₅ | P ₄ | 0 | 0 | 0 | 2 | 0 | 0 | 1/5 | 1 | -1 | 18,000 |

Since all the elements in the initial row are positive, the current solution is optimal. Therefore the optimum basic feasible solution is

$x_1 = 12,000$, $x_2 = 4,000$, $x_3 = 2,000$ and
 Max. $z = 1,89,36,000$.

VII. CONCLUSION

We observed that the optimum solution obtained in less iteration or at the most equal iterations by our modified technique, than usual simplex method. This technique is very easy to apply on numerical problems, reduces the computational work and also it gives more accuracy in giving improved optimum solution. Therefore this KKL method is more powerful for solving Linear programming problem as compare to conventional simplex method.

REFERENCES

- [1] Beale E M L (1955): Cycling in the dual Simplex algorithm, Nav. Res. logist Q.2, 269-75.
- [2] Charnes, A (1952): Optimality and degeneracy in liner programming, Econometrical 20, 160-170.
- [3] Casetti, E (1972): Competitive spatial equilibrium by mathematical programming and alternative theoretical frame of reference. Geograph.Anal.4, 368-372.
- [4] Dantzig G. B. (1951): Maximization of linear function of variables subject to linear inequalities in 21 ed. Koopman cowls commission monograph 13, John Wiley and Sons, Inc., New York.
- [5] Gass S. I. (1964): Linear programming, McGraw-Hill Book Co. Inc., New York.

AUTHOR BIOGRAPHY



Dr. N.W. Khobragade For being M.Sc in statistics and Maths he attained Ph.D. He has been teaching since 1986 for 27 years at PGTD of Maths, RTM Nagpur University, Nagpur and successfully handled different capacities. At present he is working as Professor. Achieved excellent experiences in Research for 15 years in the area of Boundary value problems and its application. Published more than 180 research papers in reputed journals. Fourteen students awarded Ph.D Degree and four students submitted their thesis in University for award of Ph.D Degree under their guidance.