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Stress and Displacement Analysis of a Rectangular Plate with Central Elliptical Hole

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Abstract- *A static load is defined as a force, which is gradually applied to a mechanical component and which does not change its magnitude or direction with respect to time. Many a times, the presence of holes in the body makes it prone to stress concentration and thus the stress near these stress raisers becomes larger than the nominal stress by a certain amount. This stress can be found by analytical methods or other suitable methods like Finite Element Analysis. FEA divides (descretize) the structure (which is continuous otherwise) into small but finite, well defined, elastic substructures (elements).*

In the present investigation, a continuous elastic plate of size 400mm x 100mm x 10mm and made up of steel with a central elliptical hole of major radius of 10mm has been modelled and descritized using a commercially available finite element solver ANSYS. The minor radius has been determined in each case with the help of the aspect ratio for each case. The plate is descritized with SOLID95 elements which area available in the element library of ANSYS. The model is analyzed for different aspect ratios (0.2, 0.4, 0.6, 0.8 and 1.0) of the elliptical hole by keeping the dimensions of plate fixed and value of maximum equivalent Von-Mises stress and deflection of plate under a constant pressure is determined with the help of ANSYS. Effect of the geometry of hole on the stress distribution around the hole is studied. The results of various analyses have been presented with the help of tables, graphs and nodal plots as provided by ANSYS.

Index terms: **ANSYS, SOLID95, descritization, Von-Mises Stress, FEA.**

I. INTRODUCTION

Design is either to formulate a plan for the satisfaction of a specified need or to solve a problem. If the plan results in the creation of something having a physical reality, then the product must be functional, safe, reliable, competitive, usable, manufacturable and marketable. With the advent of Computer Aided Design, the development of three dimensional (3D) designs from which conventional 2 dimensional orthographic views with automatic dimensioning can be produced.

Mathematical analysis and experimental measurement show that in a loaded structural member, near changes in the section, distributions of stress occur in which the peak stress reaches much larger magnitudes than does the average stress over the section. This increase in peak stress near holes, grooves, notches, sharp corners, cracks, and other changes in section is called stress concentration. The section variation that causes the stress concentration is referred to as a stress raiser.

A rectangular elastic isotropic plate with central elliptical hole under longitudinal static loading, have found widespread applications in various fields of engineering such as aerospace, marine, automobile and mechanical. For design of plates with hole, accurate knowledge of deflection, stresses and stress concentration are required. Stress concentration arises from any abrupt change in geometry of plate under loading. As a result, stress distribution is not uniform throughout the cross section. Failures such as fatigue cracking and plastic deformation frequently occur at points of stress concentration.

II. REVIEW OF LITERATURE

Paul and Rao [1,2] presented a theory for evaluation of stress concentration factor of thick and FRP laminated plate with the help of Lo-Christensen-Wu higher order bending theory under transverse loading. Shastry and Raj [3] have analyzed the effect of fibre orientation for a unidirectional composite laminate with finite element method by assuming a plane stress problem under in plane static loading. Xiwu et al. [4] evaluated stress concentration of finite composite laminates with elliptical hole and multiple elliptical holes based on classical laminated plate theory. Iwaki [5] worked on stress concentrations in a plate with two unequal circular holes. Ukadgaonker and Rao [6] proposed a general solution for stresses around holes in symmetric laminates by introducing a general form of mapping function and an arbitrary biaxial loading condition in to the boundary conditions. Ting et al. [7] presented a theory for stress analysis by using rhombic array of alternating method for multiple circular holes. Chaudhuri [8] worked on stress concentration around a part through hole weakening a laminated plate by finite element method. Peterson [15] has developed good theory and charts on the basis of mathematical analysis and presented excellent methodology in graphical form for evaluation of stress concentration factors in isotropic plates under in-plane loading with different types of abrupt change, but no results are presented for transverse loading. Patle et. al.[16] determined stress concentration factors in plate with oblique hole using FEM. Various angle of holes have been considered to evaluate stress concentration factors at such holes. The stress concentration factors are based on gross area of the plate.

III. MATERIALS AND METHOD

In the present study, the effect of aspect ratio (b/a) of a central elliptical hole on the stress distribution and deflection in a rectangular 3D plate of dimensions 400mm x 100mm x 10mm under longitudinal static load

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of magnitude 10Mpa has been analyzed using finite element method. Due to the presence of elliptical hole in the plate the maximum equivalent Von-Mises stress induced is expected at the corner of major axis of the hole.

Stress concentration is defined as localization of high stresses due to the irregularities present in the component and abrupt changes of the cross-section. In the present analysis, ANSYS has been employed to find out the localized stresses in the plate. The amount of stress concentration due to irregularities in any body is measured by the stress concentration factor (SCF) which is denoted by K_t and given by the generalized relation:

$$K_t = \frac{\text{Highest value of actual stress near discontinuity}}{\text{Nominal stress obtained by elementary equations for gross cross-sectional area}} \quad 1$$

It is important to note here that gross area (area of the plate ignoring the hole) has been considered in this analysis. The gross area approach is helpful for the design engineer in the way that it gives the SCF due to discontinuity based on the applied load. Although the actual SCF is calculated with the help of net cross-sectional area of the plate [17].

The stress concentration factor for an elliptical hole in a rectangular plate is given by the following relation:

$$K_t = 1 + 2\left(\frac{a}{b}\right) \quad 2$$

Where,

a = Major radius of the elliptical hole.

b = Minor radius of the elliptical hole.

According to eq. 1, as the value of b decreases, the value of Stress concentration factor (K_t) increases. It tends to infinity as b approaches zero.

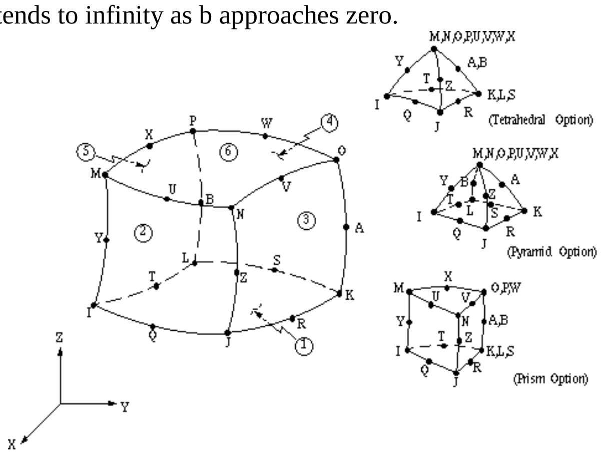


Fig.1. Solid95 element description

The 3D model of the elastic rectangular plate has been created and descretized using modelling and meshing capabilities of ANSYS. The elements used for desretization are SOLID95, which is a higher order version of the 3-D 8-node solid element SOLID45. It can tolerate irregular shapes without as much loss of accuracy. SOLID95 elements are well suited to model curved boundaries. The element is defined by 20 nodes having three degrees of freedom per node: translations in the nodal x, y, and z directions. The element may have any spatial orientation. It has plasticity, creep, stress stiffening, large deflection, and large strain capabilities. The model geometry and various boundary conditions applied are shown in the figure below:

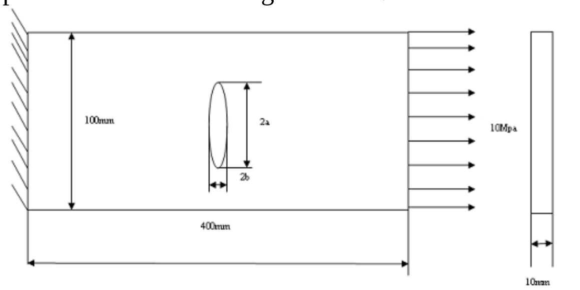


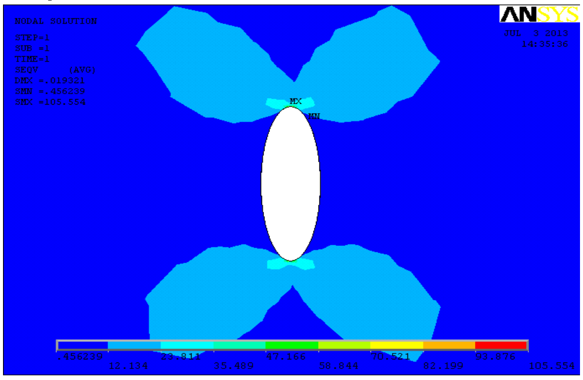
Fig.2. Geometry and boundary conditions of elastic plate with central elliptical hole

The Young's modulus (E) and Poisson's Ratio (v) of the steel plate are taken to be equal to 2.1x105N/mm2 and 0.3 respectively.

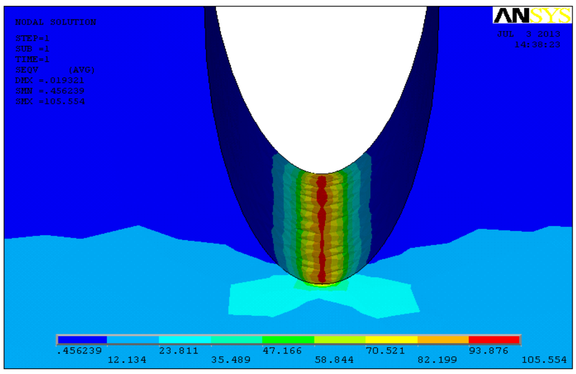
IV. RESULTS AND DISCUSSION

In this section, Maximum Von-Mises stress and displacement for different values of b/a (aspect ratio) of the central elliptical hole in the elastic plate are determined with the help of ANYS. The results are displayed with the help of graphs, tables and figures.

A. Effect of aspect ratio of elliptical hole on Von-Mises stress for b/a = 0.2:



(a)



(b)

Fig.3. (a) and (b). Stress distribution around elliptical hole for b/a=0.2

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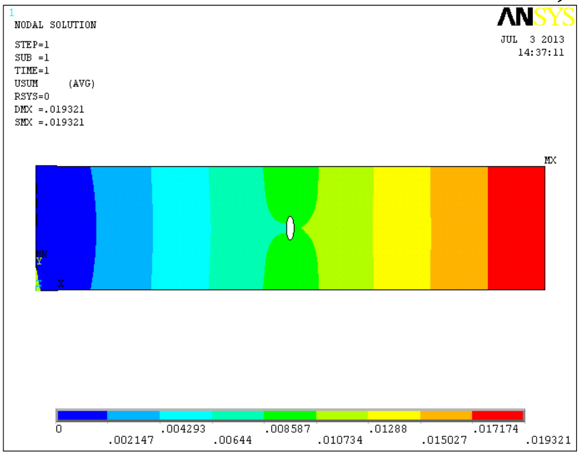
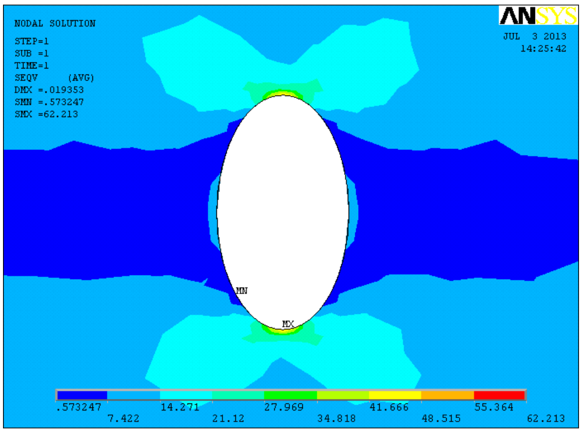


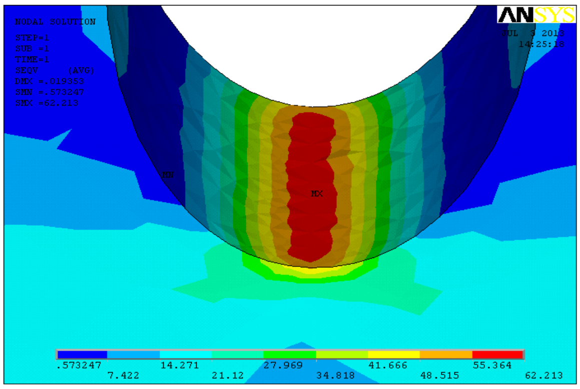
Fig.4. Displacement due to the elliptical hole of aspect ratio (b/a) = 0.2

It is clear from the above figures (3 and 4) that the maximum Von-Mises stress for a central elliptical hole on a rectangular plate for aspect ratio of 0.2 is 105.554 MPa and the maximum deflection of 0.019321 mm. As expected, the maximum stress again occurs at the corner of major axis on both sides. The stress concentration factor comes out to be 10.555.

B. Effect of aspect ratio of elliptical hole on Von-Mises stress for b/a = 0.4:



(a)



(b)

Fig.5 (a) and (b). Stress distribution around elliptical hole for b/a=0.4

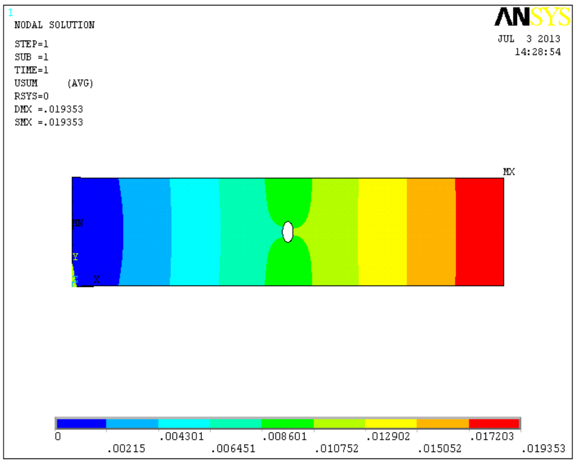
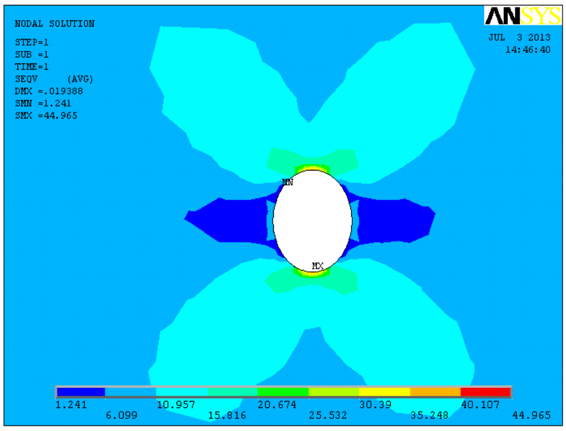


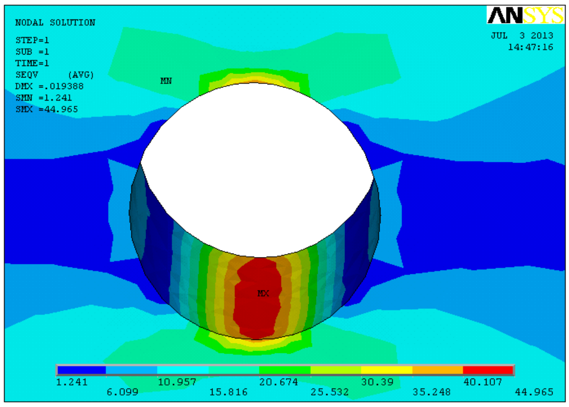
Fig.6. Displacement due to the elliptical hole of aspect ratio (b/a) = 0.4

It is clear from the above figures (5 and 6) that the maximum Von-Mises stress for the central elliptical hole on a rectangular plate for aspect ratio of 0.4 is 62.213 MPa and the maximum deflection of 0.019353 mm. As expected, the maximum stress occurs at the corner of major axis on both sides. The stress concentration factor comes out to be 6.221.

C. Effect of aspect ratio of elliptical hole on Von-Mises stress for b/a = 0.6:



(a)



(b)

Fig. 7. (a) and (b). Stress distribution around elliptical hole for b/a=0.6

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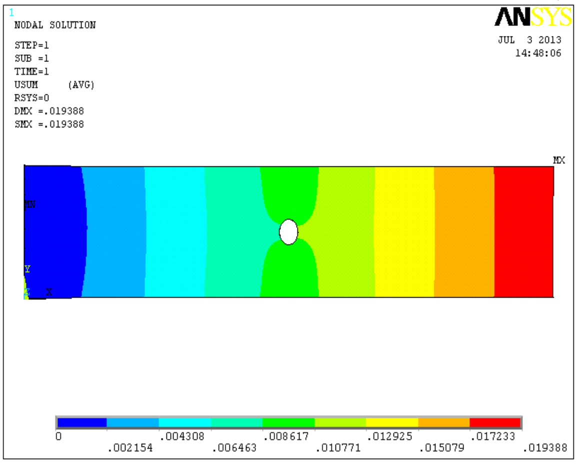
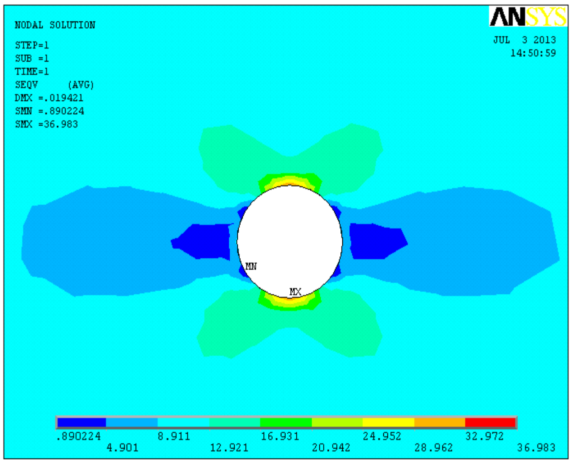


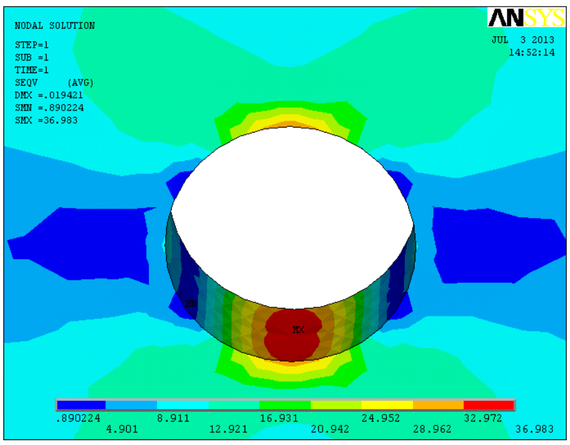
Fig. 8. Displacement due to the elliptical hole of aspect ratio (b/a) = 0.6

The maximum Von-Mises stress for this case with an aspect ratio of 0.6 is 44.965 MPa and the maximum deflection of 0.019388 mm. The stress concentration factor comes out to be 4.496.

D. Effect of aspect ratio of elliptical hole on Von-Mises stress for b/a = 0.8:



(a)



(b)

Fig. 9 (a) and (b). Stress distribution around elliptical hole for b/a=0.8

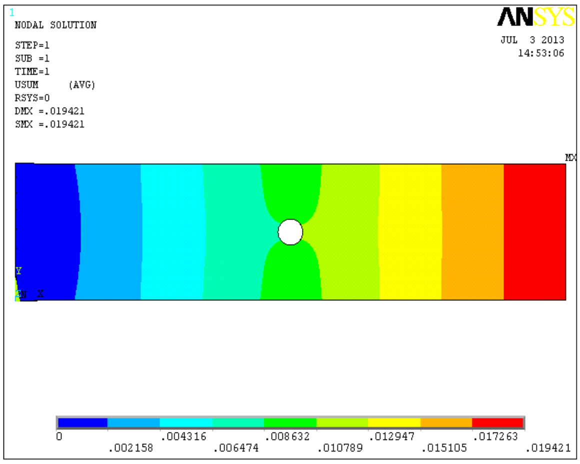
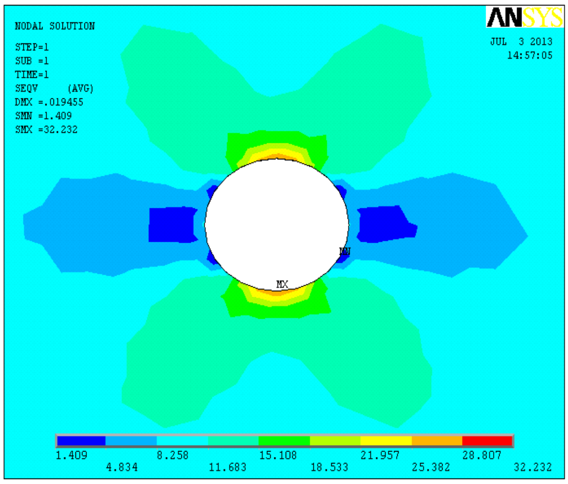


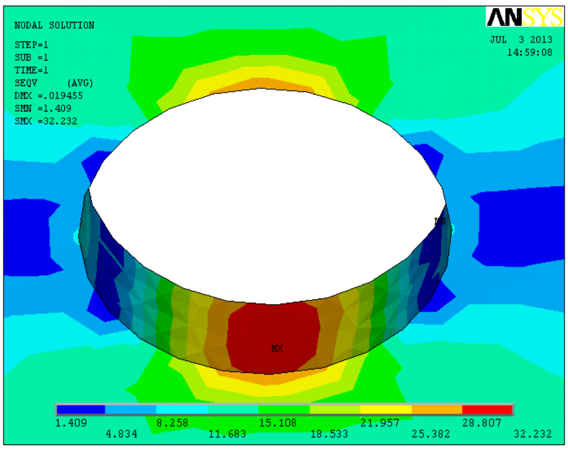
Fig.10. Displacement due to the elliptical hole of aspect ratio (b/a) = 0.8

In this case, the maximum Von-Mises stress and Displacement are found to be 36.983 MPa and 0.019421 mm respectively. The stress concentration factor for this case is calculated as 3.698.

E. Effect of aspect ratio of elliptical hole on Von-Mises stress for b/a = 1.0:



(a)



(b)

Fig.11 (a) and (b). Stress distribution around elliptical hole for b/a=1.0

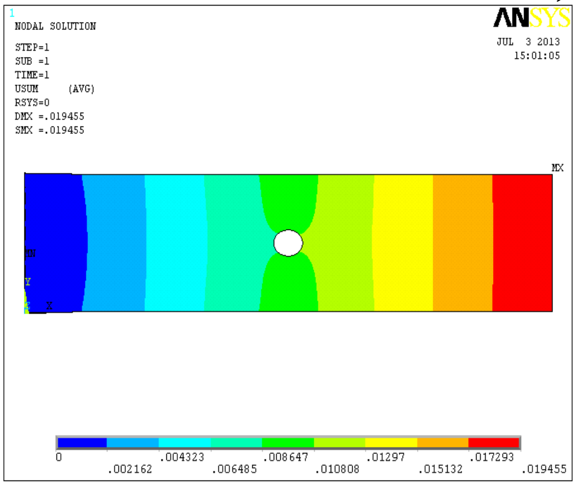


Fig.12. Displacement due to the elliptical hole of aspect ratio (b/a) = 1.0

In this case, the maximum Von-Mises stress and Displacement are found to be 32.232 MPa and 0.019455 mm respectively. The stress concentration factor for this case is calculated as 3.223.The behaviour of the plate under the given boundary conditions is presented with the help of graphs below.

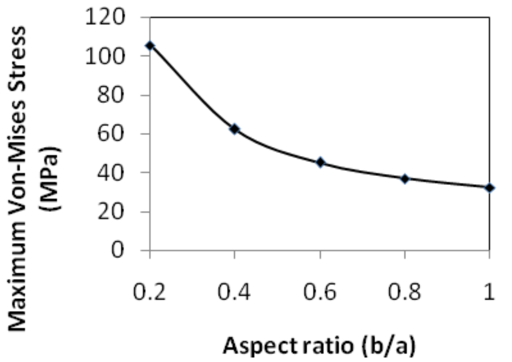


Fig.13. Effect of aspect ratio of elliptical hole on the maximum stress

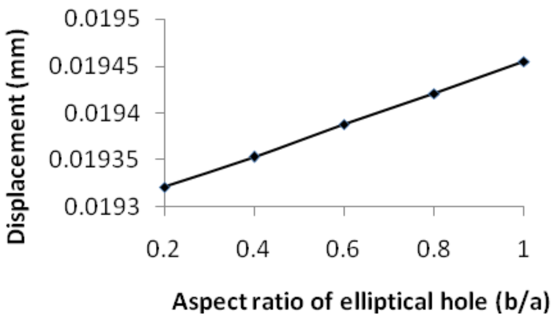


Fig.14. Effect of aspect ratio of elliptical hole on the maximum displacement

The graph below shows the comparison between the theoretical value and value calculated by ANSYS. The theoretical value is obtained with the help of eq.2, whereas the value calculated by ANSYS is obtained with the help of eq.1, which is based upon the gross cross-sectional area of the rectangular plate.

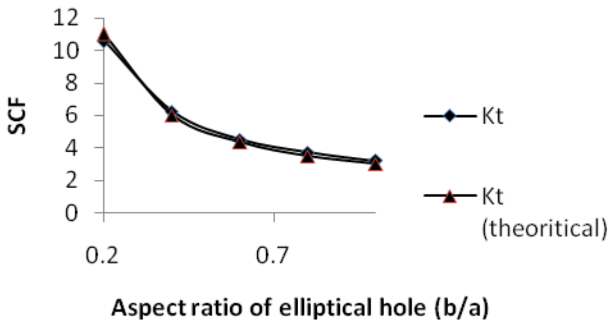


Fig.15. Comparison between the theoretical SCF and value of SCF calculated using ANSYS

The following table shows the percentage change in the maximum Von-Mises stress and deflection of the plate as the aspect ratio is varied.

Table.1: Percentage change in Maximum Von-Mises stress and Maximum deflection

S. No.	Aspect ratio (b/a)	Percentage change in maximum Von-Mises stress	Percentage change in maximum deflection
1	0.2-0.4	41.06	0.1656
2	0.2-0.6	57.4	0.3467
3	0.2-0.8	64.962	0.5175
4	0.2-1	69.4	0.6935

V. CONCLUSION

The following conclusions can be drawn from the above analysis:

- ANSYS provides a very suitable way of determining stresses induced in any body.
- ANSYS obtained results are not very accurate as compared to the analytical results but they can be used for the simulation of complex geometries.
- As compared to the analytical method which only gives the numerical value of stress, ANSYS gives a more intuitive feel to the designer by displaying stress contours throughout the plate.
- As the aspect ratio of the elliptical hole is increased to 1 (keeping the major diameter constant), the SCF decreases to a minimum of 3.223 (for gross cross-sectional area), which is very close to the analytical result of 3 for a circular hole.
- As expected, the maximum deflection occurs when the aspect ratio of elliptical hole is 1. i. e., when the ellipse is a circle.
- The maximum stress concentration occurs at the corners of elliptical hole in all the cases.

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