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Some S-Type Spaces of Fourier-Stieltjes Transform

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***Abstract*— There are various integral transform such as Fourier, Laplace, Mellin and Stieltjes etc. But the pair of Fourier-Stieltjes Transform is widely used as a Integral Transform, This pair of transform is very powerful mathematical tool applied in various areas of engineering and sciences with the increasing complexity of engineering problems. We extend our paper [3] by using S-type Spaces, In this paper, Gelfand- Shilov type spaces for Generalized Fourier-Stieltjes transform are defined and Some Topological properties for these spaces are also proved.**

***Index Terms*— Fourier Transform, Generalized function, Stieltjes Transform, Testing function Space, Fourier-Stieltjes Transform.**

I. INTRODUCTION

Integral Transformed had provided a well established and valuable method. The roots of the method can be stressed back to the original work of Oliver Heaviside 1890.Due to wide spread applicability of this method for P.D.E. involving distributional conditions, many of integral transformed extended to generalized function and In last few years, the theory of generalized integral transforms have been of ever increasing interest due to its application in physics especially in quantum field theory, Engineering and pure as well as applied mathematics. It provided new aspect to many mathematical disciplines such as ordinary and partial differential equation, Operational calculus, transformation theory and functional analysis.

There are various integral transforms such as Fourier, Stieltjes etc, When use real life situations it can have far reaching implication about the world around us. The Fourier transformation has become fundamental method in signal processing. This Mathematical tool originally was used for analysis of continuous signal and system [4]. A.H.Zemanian [7] studied different integral transform in distributional generalized sense. The Fourier transform is linear operator that maps functional space to another functional space and decomposes a function into another function of its frequency component [6], [8]. Gelfand-Shilov [5] in their series of basic had studied generalized function and given a several results. The Fourier transform is used in speech processing, radar, filtering, compression of signal, image encryption, Digital watermarking [12], [13], [14]. R.S.Pathak [9], had generalized many integral transform to the distribution of compact support.

Stieltjes transform is most attractive and impressive transform arise s in many problems I applied mathematics, Mathematical physics and Engineering Science. It plays an important role in fluid mechanics, aerodynamics, signal processing and electronics [11].

In the present work, we have generalized Fourier-Stieltjes transform into distributional generalized sense. Plan of the present paper as-We have defined different S-Type Spaces for Fourier-Stieltjes transform using Gelfand-Shilov technique. Some topological properties are also presented.

II. S-TYPE SPACES

Let I_1 $(0 \le t \le \infty, 0 \le x \le \infty)$ be the open sets in $R_+ \times R_+$ and E_+ denotes the class of infinitely differentiable function defined on I.

A. A. -Space:-

It is given by

$$FS_\alpha = \{\emptyset : \emptyset \in E_+ \,/\, \gamma_{k,p,l,q}\emptyset(t,x) = \operatorname{Sup}_{I_1}\left|t^k (1+x)^p D_t^l (xD_x)^q \emptyset(t,x)\right| \le C_{plq} A^k k^{k\alpha}\} \ldots\ldots (2.1)$$

Where, A and C_{plq} depends on testing function space$\emptyset$.

B. A. -Space:-

It is given by

$$FS^\beta = \{\emptyset : \emptyset \in E_+ \,/\, \sigma_{k,p,l,q}\emptyset(t,x) = \operatorname{Sup}_{I_1}\left|t^k (1+x)^p D_t^l (xD_x)^q \emptyset(t,x)\right| \le C_{pqk} B^l l^{l\beta}\} \ldots\ldots (2.2)$$

Where, B and C_{pqk} depends on testing function space$\emptyset$.

C. A. -Space:-

This Space is combination of two condition (2.1) and (2.2)

$$FS_\alpha^\beta = \{\emptyset : \emptyset \in E_+ \,/\, \rho_{k,p,l,q}\emptyset(t,x) = \operatorname{Sup}_{I_1}\left|t^k (1+x)^p D_t^l (xD_x)^q \emptyset(t,x)\right| \le C A^k k^{k\alpha} B^l l^{l\beta}\} \ldots\ldots \quad (2.3)$$

D. A. -Space:-

It is given by

$$FS_\theta = \{\emptyset : \emptyset \in E_+ \,/\, \mathfrak{Z}_{k,p,l,q}\emptyset(t,x) = \operatorname{Sup}_{I_1}\left|t^k (1+x)^p D_t^l (xD_x)^q \emptyset(t,x)\right| \le C_{lkq} A^k k^{k\theta}\} \ldots\ldots\ldots (2.4)$$

Where, A and C_{lkq} depends on testing function space$\emptyset$.

E. A. -Space:-

It is given by

$$FS^\eta = \{\emptyset : \emptyset \in E_+ \,/\, \Psi_{k,p,l,q}\emptyset(t,x) = \operatorname{Sup}_{I_1}\left|t^k (1+x)^p D_t^l (xD_x)^q \emptyset(t,x)\right|$$

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$\leq C_{lkp} B^q q^{q\eta}\}$ ……… (2.5)

Where, B and C_{lkp} depends on testing function space $\emptyset$.

F. A. I-Space:-

This Space is combination of two condition (2.4) and (2.5)

$$FS_{\vartheta}^{\eta} = \{\emptyset : \emptyset \in E_+ \ / \ \xi_{k,p,l,q}\emptyset(t,x) = \mathrm{Sup}_{I_1}\left|t^k(1+x)^p D_t^l (xD_x)^q \emptyset(t,x)\right| \leq C A^k k^{k\vartheta} B^q q^{q\eta}\} \ldots\ldots\ldots (2.6)$$

III. S-TYPE SUB-SPACE

A. A. FS-Space:-

It is given by

$$FS_{\alpha,m} = \{\emptyset : \emptyset \in E_+ \ / \ \gamma_{k,p,l,q}\emptyset(t,x) = \mathrm{Sup}_{I_1}\left|t^k(1+x)^p D_t^l (xD_x)^q \emptyset(t,x)\right| \leq C_{lpq}(m+\delta)^k k^{k\alpha}\} \ldots.. \quad (3.1)$$

For any $\delta > 0$, Where, m is constant depends on testing function space $\emptyset$.

B. A. FS-Space:-

It is given by

$$FS^{\beta,n} = \{\emptyset : \emptyset \in E_+ \ / \ \sigma_{k,p,l,q}\emptyset(t,x) = \mathrm{Sup}_{I_1}\left|t^k(1+x)^p D_t^l (xD_x)^q \emptyset(t,x)\right| \leq C_{kpq}(n+\epsilon)^l l^{l\beta}\} \ldots\ldots (3.2)$$

For any $\epsilon > 0$, Where, n depends on testing function space $\emptyset$

C. A. FS-Space:-

This Space is combination of two condition (3.1) and (3.2)

$$FS_{\alpha,m}^{\beta,n} = \{\emptyset : \emptyset \in E_+ \ / \ \rho_{k,p,l,q}\emptyset(t,x) = \mathrm{Sup}_{I_1}\left|t^k(1+x)^p D_t^l (xD_x)^q \emptyset(t,x)\right| \leq C(m+\delta)^k k^{k\alpha}(n+\epsilon)^l l^{l\beta}\} \ldots\ldots \quad (3.3)$$

For any $\delta > 0, \epsilon > 0$ and for given $m > 0, n > 0$.

D. A. FS- Space:-

It is given by

$$FS_{\vartheta,m} = \{\emptyset : \emptyset \in E_+ \ / \ \mathfrak{z}_{k,p,l,q}\emptyset(t,x) = \mathrm{Sup}_{I_1}\left|t^k(1+x)^p D_t^l (xD_x)^q \emptyset(t,x)\right| \leq C_{lkq}(m'+\delta')^p p^{p\vartheta}\} \ldots\ldots\ldots \quad (3.4)$$

For any $\delta' > 0$, Where, m' is constant depends on testing function space $\emptyset$.

E. A. FS-Space:-

It is given by

$$FS^{\eta,n'} = \{\emptyset : \emptyset \in E_+ \ / \ \Psi_{k,p,l,q}\emptyset(t,x) = \mathrm{Sup}_{I_1}\left|t^k(1+x)^p D_t^l (xD_x)^q \emptyset(t,x)\right| \leq C_{lkp}(n'+\epsilon')^q q^{q\eta}\} \ldots\ldots\ldots (3.5)$$

For any $\epsilon' > 0$, Where, n' is constant depends on testing function space $\emptyset$.

F. A. FS-Space:-

This Space is combination of two condition (3.4) and (3.5)

$$FS_{\vartheta,m'}^{\eta,n'} = \{\emptyset : \emptyset \in E_+ \ / \ \xi_{k,p,l,q}\emptyset(t,x) = \mathrm{Sup}_{I_1}\left|t^k(1+x)^p D_t^l (xD_x)^q \emptyset(t,x)\right| \leq C(m'+\delta')^p p^{p\vartheta}(n'+\epsilon')^q q^{q\eta}\} \ldots. (3.6)$$

IV. DIFFERENT TYPES S-TYPE SPACES

A. A. F-Space:-

It is given by

$$F^{\nu}S_{\alpha} = \{\emptyset : \emptyset \in E_- \ / \ i_{k,p,l,q}\emptyset(t,x) = \mathrm{Sup}_{I_2}\left|(-t)^k(1+x)^p D_t^l (xD_x)^q \emptyset(t,x)\right| \leq C_{plq} A^k k^{k\alpha}\} \ldots\ldots\ldots\ldots (4.1)$$

The smooth function $\emptyset(t,x)$ defined on $I_2 \ (-\infty \leq t \leq 0, 0 \leq x \leq \infty)$ is in $F^{\nu}S_{\alpha}$ if $\emptyset^{\nu}(t,x) = \emptyset(-t,x)$ is in FS_{α}.

B. A. F-Space:-

It is given by

$$F^{\nu}S^{\beta} = \{\emptyset : \emptyset \in E_- \ / \ j_{k,p,l,q}\emptyset(t,x) = \mathrm{Sup}_{I_2}\left|(-t)^k(1+x)^p D_t^l (xD_x)^q \emptyset(t,x)\right| \leq C_{plq} B^l l^{l\beta}\} \ldots\ldots \quad (4.2)$$

C. A. F'-Space:-

This Space is combination of two condition (4.1) and (4.2)

$$F^{\nu}S_{\alpha}^{\beta} = \{\emptyset : \emptyset \in E_- \ / \ \mu_{k,p,l,q}\emptyset(t,x) = \mathrm{Sup}_{I_2}\left|(-t)^k(1+x)^p D_t^l (xD_x)^q \emptyset(t,x)\right| \leq C A^k k^{k\alpha} B^l l^{l\beta}\} \ldots\ldots (4.3)$$

Where A, B and C depends on testing function $\emptyset$.

D. A. -Space:-

It is given by

$$FS_{\vartheta}^{\nu} = \{\emptyset : \emptyset \in E_- \ / \ \lambda_{k,p,l,q}\emptyset(t,x) = \mathrm{Sup}_{I_2}\left|t^k[-(1+x)]^p D_t^l (-x)^q (D_x)^q \emptyset(t,x)\right| \leq C_{lkq} A^p p^{p\vartheta}\} \ldots\ldots\ldots (4.4)$$

Where, A and C_{lkq} depends on testing function space $\emptyset$.

E. A. F-Space:-

It is given by

$$FS^{\nu,\eta} = \{\emptyset : \emptyset \in E_- \ / \ \Psi'_{k,p,l,q}\emptyset(t,x) = \mathrm{Sup}_{I_2}\left|t^k[-(1+x)]^p D_t^l (-x)^q (D_x)^q \emptyset(t,x)\right| \leq C_{lkp} B^q q^{q\eta}\} \ldots\ldots.. (4.5)$$

Where, B and C_{lkp} depends on testing function space $\emptyset$.

F. A. F-Space:-

This Space is combination of two condition (4.4) and (4.5)

$$FS_{\vartheta}^{\nu,\eta} = \{\emptyset : \emptyset \in E_- \ / \ \mathfrak{z}'_{k,p,l,q}\emptyset(t,x) = \mathrm{Sup}_{I_1}\left|t^k(1+x)^p D_t^l (xD_x)^q \emptyset(t,x)\right| \leq C A^p p^{p\vartheta} B^q q^{q\eta}\} \ldots\ldots. \quad (4.6)$$

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Where A, B and C are constant depending on $\emptyset$.

Unless specified otherwise, the space introduced through 2.1 to 4.6 will henceforth be considered equipped with their natural hausdorff locally convex topologies to be denoted respectively by-

$T_\alpha, T^\beta, T_\beta^\alpha, T_\vartheta, T^\eta, T_\vartheta^\eta, T_{\alpha,m}, T^{\beta,n}, T_{\alpha,m}^{\beta,n}, T_{\vartheta,m}, T^{\eta,n}$,

$T_{\vartheta,m}^{\eta,n}, T_\alpha^v, T^{v,\beta}, T_\alpha^{v,\beta}, T_\vartheta^v, T^{v,\eta}, T_\vartheta^{v,\eta}$

Those topologies are respectively generated by the total families of semi norm-

$\{\gamma_{k,p,l,q}\}, \{\sigma_{k,p,l,q}\}, \{\rho_{k,p,l,q}\}, \{\mathfrak{Z}_{k,p,l,q}\}, \{\Psi_{k,p,l,q}\}$,

$\{\xi_{k,p,l,q}\}, \{\gamma_{k,p,l,q}\}, \{\sigma_{k,p,l,q}\}, \{\rho_{k,p,l,q}\}, \{\mathfrak{Z}_{k,p,l,q}\}$,

$\{\Psi_{k,p,l,q}\}, \{\xi_{k,p,l,q}\}, \{i_{k,p,l,q}\}, \{j_{k,p,l,q}\}, \{\mu_{k,p,l,q}\}$,

$\{\Psi'_{k,p,l,q}\}, \{\mathfrak{Z}_{k,p,l,q}\}$

V. RESULT ON TOPOLOGICAL PROPERTIES

A. *Theorem: - A. $(FS_\alpha$ is a Frechet space.*

Proof:-As the Family $\mathcal{D}_\alpha$ of semi norms $\{\gamma_{k,p,l,q}\}_{k,q,l=0}$ generating T_α is countable, it suffices to completeness of the space (FS_α, T_α).

Let us consider a Cauchy sequence $\{\emptyset_n\}$ in FS_α. Hence, for a given $\epsilon > 0$, there exists a $N = N_{l,q}$ such that for $m, n \geq N$.

$$\gamma_{k,p,l,q}(\emptyset_m - \emptyset_n) = \mathrm{Sup}_{I_1}\left|t^k(1+x)^p D_t^l (xD_x)^q (\emptyset_m - \emptyset_n)\right| < \epsilon \ldots\ldots\ldots\ldots\ldots (5.1)$$

In particular, for $l = q = 0$ for $m, n \geq N$

$$\mathrm{Sup}_{I_1}\left|t^k(1+x)^p D_t^l (xD_x)^q (\emptyset_m - \emptyset_n)\right| < \epsilon \ldots\ldots\ldots\ldots\ldots (5.2)$$

Consequently, for fixed (t, x) in I_1, $\{\emptyset_m(t,x)\}$ is numerically Cauchy Sequence. Let $\emptyset(t,x)$ be the point wise limit of $\{\emptyset_m(t,x)\}$.

Using 3.1.2, we can easily deduced that $\{\emptyset_m\}$ converges to $\emptyset$ uniformly on I_1. Thus $\emptyset$ is continuous.

Moreover, repeated use of 3.1.1 for different values of k, l, q yields that $\emptyset$ is smooth i.e $\emptyset \in E_+$.

Further from 3.1.1 we get-

$$\gamma_{k,p,l,q}(\emptyset_m) \leq \gamma_{k,p,l,q}(\emptyset_m) + \epsilon \quad \text{for all } m \geq N$$
$$\leq C_{plq} A^k k^{k\alpha} + \epsilon$$

Taking $m \to \infty$ and ϵ is arbitrary we get-

$$\gamma_{k,p,l,q}(\emptyset) = \mathrm{Sup}_{I_1}\left|t^k(1+x)^p D_t^l (xD_x)^q \emptyset(t,x)\right|$$
$$\leq C_{plq} A^k k^{k\alpha} + \epsilon$$

Hence $\emptyset \in FS_\alpha$ and it is the T_α limit of $\emptyset_m$ by 3.1.1 again. This proves the completeness of FS_α and our proof is complete.

B. *Theorem: - The mapping A defined above is Topological isomorphism from space A. $(F^v S_\alpha, T$ onto the spaceA. $(FS_\alpha$.*

Proof:-

Let $\emptyset \in FS_\alpha$ and $\gamma_{k,p,l,q} \in D_\alpha$

$$\gamma_{k,p,l,q}(\emptyset^v)$$
$$= \mathrm{Sup}_{I_2}\left|t^k(1+x)^p D_t^l (xD_x)^q \emptyset(-t,x)\right|$$
$$= \mathrm{Sup}_{I_2}\left|t^k(1+x)^p D_t^l (xD_x)^q \emptyset(-t,x)\right|$$
$$= \gamma_{k,p,l,q}(\emptyset)$$

Thus, $\emptyset$ is continuous.

As the isomorphic chapter of R and R^{-1} is trivial, Hence the result follows.

For the space $F^v S^\beta, F^v S_\alpha^\beta, F^v S_\alpha, F^v S_{\alpha,m}, F^v S^{\beta,n}$, $F^v S_{\alpha,m}^{\beta,n}$, we have similar result obtained.

VI. CONCLUSION

In this paper, we had generalized the Fourier-Stieltjes transform in the Distributional Generalized Sense. We had discussed S-type Spaces for Fourier-Stieltjes transform using Gelfand-Shilov technique and also proved some Topological properties of S-type Spaces.

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