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# Non-Survival of Massive Scalar Field with Constant Deceleration Parameter in Robertson-Walker Universe

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***Abstract*— The study of Einstein's field equations describing Robertson-Walker cosmological models with massive scalar field and perfect fluid representing the matter has been made. The problem has been investigated with constant deceleration parameter in the Robertson-Walker Universe. It has been found that there occurs the non-survival of massive scalar field interaction with the perfect fluid in the Robertson-Walker Universe under the specified constraint.***

## I. INTRODUCTION

The attention of various workers has been drawn towards the study of scalar field interacting with perfect fluid distribution in General Theory of Relativity. Hawking and Ellis [1] have shown that the flat Robertson-Walker model for a massive scalar field can be reduced to a steady state model as time, t tends to infinity. Gursey [2] has derived that spin-zero gravitation is responsible for gravitational attraction. Roy and Rao [3] have investigated the massive scalar field cannot be a source of axially symmetric gravitational field. Berman and Gomide [4] had studied the problem of cosmological models with constant deceleration parameter by considering Robertson-Walker metric. Singh and Singh[5] studied the problem of massive scalar field interacting with viscous fluid involving bulk viscosity by considering Robertson-Walker metric.

In the present paper we have investigated the problem of massive scalar field interacting with perfect fluid involving constant deceleration parameter for Robertson-Walker metric. We have obtained exact solutions of the problem and discussed the physical and geometrical properties of the exact solutions obtained.

## II. FIELD EQUATIONS AND THEIR SOLUTIONS

The Einstein field equations in the most general form are given by

$$G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij} + \Lambda g_{ij} = -\kappa\left[T_{ij} + T'_{ij}\right] \qquad (1)$$

where the energy momentum tensor of perfect fluid distribution and massive scalar field are given by

$$T_{ij} = (\rho + p)U_i U_j - pg_{ij} \qquad (2)$$

where $\rho$, p and $U_i$, are the mass density, the pressure and four- velocity vector respectively and

$$T'_{ij} = \frac{1}{4\pi}\left[V,_i V,_j - \frac{1}{2}g_{ij}(V,_a V^{,a} - M^2V^2)\right] \qquad (3)$$

The scalar V satisfies the Klein-Gordon equation

$$g^{ij}V_{;ij} + M^2V = \sigma \qquad (4)$$

Where $\sigma$ is the source density and M is related to the mass of zero-spin particle defined by

$$M = m/\hbar \text{ , where } \hbar = \frac{h}{2\pi} \quad ,$$

h, being the Planck's constant.

Where a comma or semicolon followed by a subscript denotes partial differentiation or a covariant differentiation respectively. A dot and dash over a letter denotes partial differentiation with respect to time.

We also have

$$g^{ij}U_iU_j = 1 \qquad (5)$$

To solve the field equations we assume the space-time associated with this distribution to be spherically symmetric with maximally symmetric three dimensional subspaces whose metric have positive Eigen values and arbitrary curvature. The assumption leads to the metric of the form (Weinberg [ ] page 403)

$$ds^2 = dt^2 - R^2(t)\left\{\frac{dr^2}{1-Kr^2} + r^2d\theta + r^2\sin^2\theta d\varphi^2\right\} \qquad (6)$$

where 't' is the cosmic time; R(t) , the scale factor of the universe; and K, the curvature index which takes up the value +1, 0 and -1.This metric is identical with Robertson-Walker metric of Cosmology.

For the stress energy tensors, $T_{ij}$ and $T'_{ij}$ as mentioned above the field equations (1) becomes

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$$G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij} + \Lambda g_{ij} = -\kappa\left[\begin{array}{l}(\rho+p)U_iU_j - pg_{ij} \\ +\frac{1}{4\pi}\left\{V_{,i}V_{,j} - \frac{1}{2}g_{ij}(V_{,a}V^{,a} - M^2V^2)\right\}\end{array}\right] \quad (7)$$

Considering a co-moving co-ordinate system, we obtain

$$U_1 = U_2 = U_3 = 0 \text{ and } U_4 = 1 \quad (8)$$

For the metric (6) the surviving field equations are

$$G_{11} \equiv K + R_{,4}^2 + 2RR_{,44} - \Lambda R^2 = -\kappa\left[pR^2 + \frac{1}{8\pi}\left\{(1-Kr^2)V_{,1}^2 + R^2(V_{,4}^2 - M^2V^2\right\}\right] \quad (9)$$

$$G_{22} \equiv K + R_{,4}^2 + 2RR_{,44} - \Lambda R^2 = -\kappa\left[pR^2 - \frac{1}{8\pi}\left\{(1-Kr^2)V_{,1}^2 - R^2(V_{,4}^2 - M^2V^2)\right\}\right] \quad (10)$$

$$G_{33} = G_{22} \quad (11)$$

$$G_{44} \equiv 3(K + R_{,4}^2) - \Lambda R^2 = -\kappa\left[pR^2 + \frac{1}{8\pi}\left\{(1-Kr^2)V_{,1}^2 + R^2(V_{,4}^2 + M^2V^2)\right\}\right] \quad (12)$$

and

$$G_{14} = 0 = V_{,1}V_{,4} \quad (13)$$

where

$$V_{,2} = 0 = V_{,3} \quad V_{,4} = \frac{\partial V}{\partial t} \text{ and } R_{,4} = \frac{\partial R}{\partial t}$$

The Klein- Gordan equation (4) becomes

$$V_{,44} + \frac{3R_{,4}}{R}V_{,4} + M^2V = \sigma\ , \quad (14)$$

When $\sigma = 0$, and $\Lambda \neq 0$

By taking the deceleration parameter

$$q = -\frac{R\ddot{R}}{\dot{R}^2} = -m\ , \quad (15)$$

where m is a constant and greater than zero .
which on integration yields the solution

$$R = (a+bt)^n \quad (16)$$

Where $n = \frac{1}{1-m}$ and a,b being the arbitrary constants.
Using equation (16) in (9), we obtain

$$\kappa p = \Lambda - \frac{n^2b^2}{(a+bt)^2} - \frac{2n(n-1)b^2}{(a+bt)^2} - \frac{\kappa}{8\pi}(V_{,4}{}^2 - M^2V^2) \quad (17)$$

Using equation (16) in (12), we obtain

$$\kappa\rho = \frac{3n^2b^2}{(a+bt)^2} - \Lambda - \frac{\kappa}{8\pi}(V_{,4}{}^2 + M^2V^2) \quad (18)$$

Using (27) in (14), we obtain

$$V_{,44} + \frac{3nb}{a+bt}V_{,4} + M^2V = 0 \quad (19)$$

The equation (19) is integrable if

$$P_2 - P_1' + P_0'' = 0 \quad (20)$$

where $P_0 = (a+bt), P_1 = 3nb$, and $P_2 = M^2(a+bt)$.

From equation (20), we get

M = 0.

## III. Conclusion

This implies that the mass parameter becomes zero when the deceleration parameter is considered to be a constant. Hence there occurs the non-survival of massive scalar field interacting with the perfect fluid in the Robertson-Walker Universe under the specified constraints.

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