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The Blasius and Sakiadis Flow in a Nanofluid through a Porous Medium in the Presence Of Thermal Radiation under a Convective Surface Boundary Condition

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Abstract— *The porosity effects on the flow and heat transfer characteristics of the Blasius and Sakiadis flow in a nanofluid in the presence of thermal radiation. The resulting system of nonlinear partial differential equations is solved numerically using an efficient numerical shooting technique with a fourth-order Runge–Kutta scheme. The solutions for the flow and heat transfer characteristics are evaluated numerically for various values of the governing parameters, namely the nanoparticle volume fraction ϕ and the porosity parameter ξ. Three different types of nanoparticles are considered, namely Cu, Al_2O_3 and TiO_2. The variations of dimensionless surface temperature as well as flow and heat-transfer characteristics with the governing dimensionless parameters of the problem, which include the nanoparticles volume fraction ϕ, the thermal radiation parameter N_R and the porosity ξ are graphed and tabulated. Results of some earlier workers have been deduced as special cases. Excellent validation of the present numerical results has been achieved with the earlier Blasius and Sakiadis flow problem of Olanrewaju et al. [37] for local Nusselt number without taking the effect of nanoparticles and porosity.*

Keywords: **Nanofluid, porosity, Blasius and Sakiadis, Thermal radiation, convective surface boundary condition.**

Nomenclature

a	the convective parameter
C_f	Skin friction coefficient
c_p	Specific heat
Ec	Eckert number
f	Dimensionless stream function
k	Thermal conductivity
K	Permeability of the porous medium
k^*	Mean absorption coefficient
m	Surface temperature parameter
N_R	Radiation parameter
Nu_x	Nusselt number
Pr	Prandtl number
q_r	Radiative heat flux
T	Temperature
u, v	Velocity components along x- and y-directions, respectively
x, y	Cartesian coordinates along the plate and normal to it, respectively

Greek symbols

α	Thermal diffusivity
η	Similarity variable
ξ	Porosity parameter
θ	Dimensionless temperature
μ	Effective viscosity
υ	Kinematic viscosity
ρ	Density
σ^*	Stefan–Boltzmann constant
$(\rho C_p)_{nf}$	Heat capacitance of the nanofluid
$(\rho C_p)_f$	Heat capacity of the fluid
$(\rho C_p)_s$	Effective heat capacity of the nanoparticles material
ϕ	Nanoparticles volume fraction

Subscripts

f	Fluid fraction
nf	Nanofluid fraction
s	Solid fraction
w	Condition at the wall
∞	Stream function condition at the infinity

I. INTRODUCTION

Studies related to nanofluids have attracted a great deal of interest in recent time due to their enormous enhanced performance properties, particularly with respect to heat transfer. Nanofluids have novel properties that make them potentially useful in many applications in heat transfer, including microelectronics, fuel cells, pharmaceutical processes, and hybrid-powered engines. Nanoparticle is of great scientific interest as it is an effectively bridge between bulk materials and atomic or molecular structures. Investigations of boundary layer flow and heat transfer of viscous fluids over a flat sheet are important in many manufacturing processes, such as polymer extrusion, drawing of copper wires, continuous stretching of plastic films and artificial fibers, hot rolling, wire drawing, glass-fiber, metal extrusion, and metal spinning. For instance, in their experimental study of drug nanoparticles by antisolvent precipitation, Matteucci et al. [1] revealed that an adequate knowledge of nanoparticles volume fraction is beneficial for designing mixing systems and surfactant stabilizers for forming nanoparticles of poorly water soluble drugs with the potential for high dissolution rates. Nanofluids are engineered colloids made of a base fluid and nanoparticles.

Nanofluids are solid-liquid composite materials consisting of solid nanoparticles or nanofibers, with sizes typically on the order of 1–100 nm, suspended in a liquid. Since the pioneering experimental work of Choi [2] on nanofluids, numerous models have been proposed by different authors to study convective flows of nanofluids and we mention here the papers in Refs. [3–8].

A comprehensive survey of convective transport in nanofluids was made by Buongiorno [9], who revealed that a satisfactory explanation for the abnormal increase in the thermal conductivity and viscosity was yet to be found. Moreover, the boundary layer flow over a moving surface has attracted considerable attention in recent years due to its crucial role in numerous industrial and engineering applications. Very recently, Kuznetsov and Nield [8] have examined the influence of nanoparticles on natural convection boundary-layer flow past a vertical plate, using a model in which Brownian motion and thermophoresis are accounted for. The authors have assumed the simplest possible boundary conditions, namely those in which both the temperature and the nanoparticles fraction are constant along the wall. Further, Nield and Kuznetsov [10] have studied the Cheng and Minkowycz [11] problem of natural convection past a vertical plate, in a porous medium saturated by a nanofluid. The analytical and experimental results for the flow and temperature fields in the boundary layer regime were confirmed by Makinde [12] investigated the free-convection flow with thermal radiation and mass transfer past a moving vertical porous plate. Sakiadis [13] initiated the study of the boundary layer flow over a stretched surface moving with a constant velocity and formulated a boundary-layer equation for two-dimensional and axisymmetric flows. Tsou et al. [14] analyzed the effect of heat transfer in the boundary layer on a continuous moving surface with a constant velocity and experimentally confirmed the numerical results of Sakiadis [13]. Erickson et al. [15] extended the work of Sakiadis [13] to include blowing or suction at the stretched sheet surface on a continuous solid surface under constant speed and investigated its effects on the heat and mass transfer in the boundary layer. The related problems of a stretched sheet with a linear velocity and different thermal boundary conditions in Newtonian fluids have been studied, theoretically, numerically and experimentally, by many researchers, such as Crane [16], Fang [17 , 18], Fang and Lee [19]. The classical problem (i.e., fluid flow along a horizontal, stationary surface located in a uniform free stream) was solved for the first time in 1908 by Blasius [20]; it is still a subject of current research [21 , 22] and, moreover, further study regarding this subject can be seen in most recent papers [23 , 24].

A study on boundary layer flow of a nanofluid past a stretching sheet with a convective boundary condition was conducted by Makinde and Aziz [25]. Recently, Ahmad et al. [26] presented a numerical study on the Blasius and Sakiadis problems in nanofluids under isothermal condition. Their results revealed that the solid volume fraction affects the fluid flow and heat transfer characteristics of nanofluids.

Moreover, their theoretical studies of nanofluids relied on the assumption that the combined effects of both the viscous dissipation and Newtonian heating on the flow system are negligible. In reality, this may not be the case, since viscous heating of the base fluid is enhanced by the addition of nanoparticles, and the convective heat transfer may also take place at the heated plate surface. The exclusion of viscous dissipation and Newtonian heating in the analysis may affect the outcome of their investigation. To the best of our knowledge, no attempt has been made in the literature to highlight the combined effects of viscous dissipation and Newtonian heating on the thermal boundary layer of nanofluids. Makinde [27] investigate the combined effects of viscous dissipation and Newtonian heating on the boundary layer flow of water-based nanofluids containing two types of nanoparticles such as copper (Cu) and titanium (TiO_2) over a moving surface. The natural convection boundary-layer flow past a vertical cone embedded in a porous medium filled with a non- Newtonian nanofluid in the presence of heat generation or absorption is presented in Hady et al. [28]. A similarity solution of the steady boundary layer flow near the stagnation-point flow on a permeable stretching sheet in a porous medium saturated with a nanofluid and in the presence of internal heat generation/absorption is theoretically studied by Hamad and Pop [29]. Convective flow in porous media has been widely studied in the recent years due to its wide applications in engineering as post accidental heat removal in nuclear reactors, solar collectors, drying processes, heat exchangers, geothermal and oil recovery, building construction (Nield and Bejan [30], Ingham and Pop [31], Vafai [32] and Vadasz [33], etc.). It is well known that conventional heat transfer fluids, including oil, water, and ethylene glycol mixture are poor heat transfer fluids, since the thermal conductivity of these fluids plays an important role on the heat transfer coefficient between the heat transfer medium and the heat transfer surface. An innovative technique for improving heat transfer by using ultra fine solid particles in the fluids has been used extensively during the last several

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years. The derivation of the empirical equations which govern the flow and heat transfer in a porous medium has been discussed in Mahdy and Hady [34], Ibrahim et al. [35], Abdel-Gaied and Eid [36].

The principal aim of this paper is therefore to extend the work of Olanrewaju et al. [37] to report the effects of thermal radiation and Eckert number as well as Prandtl number *Pr* and convective parameter *a* on both Blasius and Sakiadis thermal boundary layers under a convective boundary condition. The focus of this paper is to examine the effect of nanoparticles volume fraction ϕ and porosity parameter ξ for Blasius and Sakiadis flow in a nanofluid. Three different types of nanoparticles are considered, namely Cu, Al_2O_3 and TiO_2, when the base fluid is H_2O.

II. PROBLEM FORMULATION

The governing equations of motion and heat transfer for the classical Blasius and Sakiadis flat-plate flow problem can be summarized by the following boundary value problem as Olanrewaju et al. [37]. Taking into account the porosity term in the momentum equation for nanofluid. The thermo-physical properties of the nanofluid are tabled in Table 1 (see Oztop and Abu-Nada [4]).

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \quad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon_{nf}\frac{\partial^2 u}{\partial y^2}-\frac{\upsilon_{nf}}{K}u, \quad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k_{nf}}{(\rho c_p)_{nf}}\frac{\partial^2 T}{\partial y^2}+\frac{\mu_{nf}}{(\rho c_p)_{nf}}\left(\frac{\partial u}{\partial y}\right)^2-\frac{1}{(\rho c_p)_{nf}}\frac{\partial q_r}{\partial y}. \quad (3)$$

The boundary conditions for the velocity field for the Blasius flat-plate flow problem are :

$$u=v=0; \quad \text{at } y=0; \quad u=U_\infty \quad \text{at } x=0$$
$$u\to U_\infty \text{ as } y\to\infty \quad (4)$$

And for the classical Sakiadis flat-plate flow problem :

$$u=U_w, v=0 \text{ at } y=0;$$
$$u\to 0 \text{ as } y\to\infty, \quad (5)$$

The boundary conditions at the plate surface and far into the cold fluid may be written as

$$-k_f\frac{\partial T}{\partial y}(x,0)=h_f[T_f-T(x,0)],$$
$$T(x,\infty)=T_\infty . \quad (6)$$

where u and v are the velocity components of the nanofluid in the *x*- and *y*-directions, respectively. Property ρ_{nf} and μ_{nf} are the density and effective viscosity of the nanofluid, α_{nf} and υ_{nf} are the thermal diffusivity and the kinematic viscosity, k_{nf} the thermal conductivity of the nanofluid, K is the permeability of the porous medium, U_∞ is the constant free stream velocity, U_w is the plate velocity and T is the temperature of the nanofluid inside the thermal boundary layer. which are defined as (see Khanafer et al. [38]):

$$\rho_{nf}=(1-\phi)\rho_f+\phi\rho_s; \quad \mu_{nf}=\frac{\mu_f}{(1-\phi)^{2.5}}; \quad \alpha_{nf}=\frac{k_{nf}}{(\rho C_p)_{nf}};$$
$$(\rho C_p)_{nf}=(1-\phi)(\rho C_p)_f+\phi(\rho C_p)_s; \quad (7)$$
$$\frac{k_{nf}}{k_f}=\frac{(k_s+2k_f)-2\phi(k_f-k_s)}{(k_s+2k_f)+2\phi(k_f-k_s)}.$$

Here ϕ is the solid volume fraction, where μ_f is the viscosity of the basic fluid, ρ_f and ρ_s are the densities of the pure fluid and nanoparticle, respectively, $(\rho c_p)_f$ and $(\rho c_p)_s$ are the specific heat parameters of the base fluid and nanoparticle, respectively, k_f and k_s are the thermal conductivities of the base fluid and nanoparticle, respectively. Using the Rosseland approximation for radiation, the radiative heat flux is simplified as:

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}, \quad (8)$$

where σ^* and k^* are the Stefan–Boltzmann constant and the mean absorption coefficient, respectively. We assume that the temperature differences within the flow such as the term T^4 may be expressed as a linear function of temperature. Hence, expanding T^4 in a Taylor series about a free stream temperature T_∞ and neglecting higher-order terms we get:

$$T^4\cong 4T_\infty^{\ 3}T-3T_\infty^{\ 4}. \quad (9)$$

In view of Eqs. (8) and (9) with Eq. (3):

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\left(\alpha_{nf}+\frac{16\sigma^* T_\infty^{\ 3}}{3(\rho c_p)_{nf}k^*}\right)\frac{\partial^2 T}{\partial y^2}+\frac{\alpha_{nf}\mu_{nf}}{k_{nf}}\left(\frac{\partial u}{\partial y}\right)^2, \quad (10)$$

where $\alpha_{nf}=\frac{k_{nf}}{(\rho c_p)_{nf}}$ is the thermal diffusivity of a nanofluid.

Eq. (10) becomes:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\alpha_{nf}}{k_0}\frac{\partial^2 T}{\partial y^2}+\frac{\alpha_{nf}\mu_{nf}}{k_{nf}}\left(\frac{\partial u}{\partial y}\right)^2. \quad (11)$$

Where $k_0=\frac{3N_R}{3N_R+4}$; $N_R=\frac{k_{nf}k^*}{4\sigma^* T_\infty^{\ 3}}$ as the radiation parameter. It is worth citing here that the classical solution for energy equation, Eq. (11) without thermal radiation and viscous dissipation influence can be obtained from the above equation which reduces to

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\frac{\partial^2 T}{\partial y^2}$$ as $N_R \to \infty$ (i.e $k_0 \to 1$).

Introducing a similarity variable η, a dimensionless stream function f(η) and the non- dimensional temperature $\theta(\eta)$ as:

$$\eta = y\sqrt{\frac{U}{\upsilon_f x}}=\frac{y}{x}\sqrt{\mathrm{Re}_x},\qquad u=Uf'(\eta),$$

$$v=\frac{1}{2}\sqrt{\frac{U\upsilon_f}{x}}\left[\eta f'(\eta)-f(\eta)\right],\quad \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}. \tag{12}$$

where prime denotes differentiation with respect to η and $\mathrm{Re}_x=\frac{Ux}{\upsilon_f}$ is the local Reynolds number. Note that in Eq. (12); $U=U_\infty$ represents Blasius flow, whereas $U=U_w$ indicates Sakiadis flow, respectively. We also assume the bottom surface of the plate is heated by convection from a hot fluid at uniform temperature T_f which provides a heat transfer coefficient h_f .The equation of continuity is satisfied identically. We substitute Eq. (12) into Eqs. (2) and (11) we have:

$$f'''+(1-\phi)^{2.5}\left(1-\phi+\phi\frac{\rho_s}{\rho_f}\right)\frac{ff''}{2}-\xi f'=0 \tag{13}$$

$$\frac{1}{\mathrm{Pr}}\left(\frac{k_{nf}}{k_f}\right)\frac{\theta''}{k_0}+\frac{Ec}{(1-\phi)^{2.5}}f''^2+\frac{1}{2}\left[(1-\phi)+\phi\frac{(\rho c_p)_s}{(\rho c_p)_f}\right]f\theta'=0, \tag{14}$$

Where $\mathrm{Pr}=\frac{\upsilon_f}{\alpha_f}$ is the Prandtl number, $Ec=\frac{u^2}{(c_p)_f(T_w-T_\infty)}$ is the Eckert number.

and $\xi=\frac{\upsilon_f^2\,\mathrm{Re}_x}{U^2K}$ porosity parameter, and the corresponding boundary conditions become:

$$f(0)=0,\ f'(0)=0,\ \theta'(0)=-a[1-\theta(0)],$$
$$f'(\infty)\to 1,\ \theta(\infty)\to 0. \tag{15}$$

For the Blasius flat-plate flow problem, and

$$f(0)=0,\ f'(0)=1,\ \theta'(0)=-a[1-\theta(0)],$$
$$f'(\infty)\to 0,\ \theta(\infty)\to 0. \tag{16}$$

for the Sakiadis case, respectively.

Where $$a=\frac{h_f}{k}\sqrt{\upsilon_f x/U_\infty} \tag{17}$$

For the momentum and energy equations to have a similarity solution, the parameter a must be a constant and not function of x as in Eq. (17). This condition can be met if the heat transfer coefficient h_f is proportional to $x^{-1/2}$

We therefore assume

$$h_f=cx^{-1/2} \tag{18}$$

Where c is constant. Putting Eq. (18) into Eq. (17), we have

$$a=\frac{c}{k}\sqrt{\frac{\upsilon_f}{U_\infty}} \tag{19}$$

Here, a is defined by Eq. (19), the solutions of Eqs. (13)-(16) yield the similarity solutions, however, the solutions generated are the local similarity solutions whenever a is defined as in Eq. (17). The quantities of practical interest in this study are the skin friction coefficient C_f and the local Nusselt number Nu_x which are defined as:

$$C_f=\frac{2\mu_{nf}}{\rho_f(u_w(x))^2}\left(\frac{\partial u}{\partial y}\right)_{y=0},\quad Nu_x=\frac{-k_{nf}\left.\frac{\partial T}{\partial y}\right|_{y=0}x}{k_f(T_w-T_\infty)} \tag{20}$$

Using Eq. (12), quantities (20) can be expressed as:

$$\mathrm{Re}_x^{1/2}C_f=\frac{1}{(1-\phi)^{2.5}}f''(0) \tag{21}$$

$$\mathrm{Re}_x^{1/2}Nu_x=-\frac{k_{nf}}{k_f}\theta'(0) \tag{22}$$

III. RESULTS AND DISCUSSION

In order to get the physical insight into the flow problem, comprehensive numerical computations are conducted for various values of the parameters that describe the flow characteristics and the results are illustrated graphically. The system of non-linear ordinary differential equations (13) and (14) with the boundary conditions (15) and (16) are integrated numerically by means of the efficient numerical shooting technique with a fourth-order Runge–Kutta scheme (MATLAB package). The step size $\eta=0.001$ is used while obtaining the numerical solution with $\eta_{\max}=5$.

We consider three different types of nanoparticles, namely, copper (Cu), alumina (Al_2O_3) and titanium oxide (TiO_2) with H_2O as the base fluid. Table 1 shows the thermo-physical properties of H_2O and the elements Cu, Al_2O_3 and TiO_2. The Prandtl number of the base fluid H_2O is kept constant at 6.2. It is worth mentioning that this study reduces to those of a viscous or regular fluid when $\phi=0$. In order to verify the accuracy of the present method, we have compared our results with those of Olanrewaju et al. [37] for temperature profile $\theta(0)_{Blasius}$ and $\theta(0)_{Sakiadis}$ in the absence of the nanoparticles $\phi=0$ and ξ =0 for different values of a without thermal radiation ($N_R\to\infty$ (i.e. $k_0=1$)) and viscous dissipation term and with thermal radiation

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parameter. The comparisons in all the above cases are found to be in excellent agreement, as shown in Table 2 and Table 3. Table 4 depicts the skin friction at the surface $|f''(0)|$ and the rate of heat transfer $-\theta'(0)$ for various values of porosity parameter ξ with $\phi = 0.1$, $\Pr = 6.2$, $Ec = 2.0$, a =1.0 and $N_R = 5.0$ for different types of nanoparticles when the base fluid is H_2O (Sakiadis flow). It is clear that as the porosity parameter ξ increase, the skin friction $|f''(0)|$ increases , the rate of heat transfer $-\theta'(0)$ increases and the Cu nanoparticles are the highest skin friction than the other follow by TiO_2 and next Al_2O_3 as shown in Table 4. Figures 1–12 depict the influence of different parameters on the velocity and the temperature profiles as well as the local skin friction coefficient and the Nusselt number. A selected set of graphical results presented in figures 1–12 will give a good understanding of the influence of different parameters on the velocity and the temperature profiles as well as the skin friction coefficient and the Nusselt number. Figures 1 and 2 show the effect of nanoparticles volume fraction ϕ on the nanofluid velocity and temperature profile, respectively, in the case of nanoparticles are Cu and the base fluid is H_2O ($\Pr = 6.2$) when $\phi = 0, 0.05, 0.1, 0.15, 0.2$ with $Ec = 2.0$,a =1.0, $\xi = 0.0$ and $N_R = 5$. It is clear that the nanoparticles volume fraction increases, the nanofluid velocity increases and the temperature increases for the Blasius flow but as the nanoparticles volume fraction increases, the nanofluid velocity decreases and the temperature increases for the Sakiadis flow. These figures illustrate this agreement with the physical behavior. When the volume of nanoparticles increases the thermal conductivity increases and then the thermal boundary layer thickness increases. The nanofluid velocity profile is the highest at the moving plate surface and decreases gradually to the free stream zero value satisfying the far field boundary condition. Figures 3 and 4 present the dimensionless temperature and velocity profiles for $\xi = 0.0, 0.1, 0.5, 1.0, 1.5, 2.0$ when $Ec = 2.0$, $\Pr = 6.2$,a =1.0, $\phi = 0.1$ and $N_R = 5$ .The thermal boundary layer thickness increases with the increase of the porosity parameter ξ , while the velocity profile decreases with the increase of the porosity parameter ξ, for both Blasius and Sakiadis flow. Figures 5-8 show the variation of the shear stress in terms of the skin-friction coefficient and the rate of heat transfer in terms of the reduced Nusselt number for different values of nanoparticles volume fraction ϕ ,with $Ec = 2.0$, $\Pr = 6.2$, a =1.0, $\xi = 0.0$ and $N_R = 5$,for different types of nanoparticles,(Cu- H_2O, Al_2O_3- H_2O and TiO_2- H_2O) for both Blasius and Sakiadis flow. Figure 5 illustrates the effect of parameter variation on the local skin friction coefficient in case of Blasius flow. The local skin friction coefficient increases with the increase in the nanoparticle volume fraction ϕ in case of (Cu- H_2O) while it increases near the plate and then decreases at some distance from the plate in case of (Al_2O_3- H_2O and TiO_2- H_2O). It is interesting to note that the Cu- H_2O nanofluid produces is highest skin friction coefficient than the TiO_2- H_2O nanofluid and Al_2O_3- H_2O nanofluid. A similar trend is noticed in figure 6 in case of Sakiadis flow. Figures 7 and 8 illustrate the effects of various parameters on the heat transfer rate at the moving plate surface for both Blasius and Sakiadis flow. As the values of the nanoparticle volume fraction ϕ increase, an increase in the Nusselt number is observed. Generally, the heat transfer rate at the moving plate surface for the Cu- H_2O nanofluid are highest than the TiO_2- H_2O nanofluid and Al_2O_3- H_2O nanofluid as the working nanofluid. Figures 9 and 10 illustrate the skin friction at the surface $|f''(0)|$ for various values of porosity parameter ξ with $\phi = 0.1$, $\Pr = 6.2$, $Ec = 2.0$, a=1.0 and $N_R = 5.0$ for different types of nanoparticles when the base fluid is H_2O. It can be noticed that the numerical values of $|f''(0)|$ decrease with an increase in the porosity parameter ξ for different kinds of nanofluids in case of the Blasius flow as shown in figure 9, while it increases in case of the Sakiadis flow, as shown in figure 10 and Table 4. In case of the Blasius flow, It is found that the rate of heat transfer $-\theta'(0)$ decreases with the increase on porosity parameter ($\xi < 0.5$). On the other hand ($\xi > 0.5$) the rate of heat transfer $-\theta'(0)$ increases with the increase on ξ ,as shown in figure 11 for different types of nanoparticles with $\phi = 0.1$, $\Pr = 6.2$, $Ec = 2.0$, a=1.0 and $N_R = 5.0$. Finally, We observe the remarkable effect of porosity parameter ξ on the rate of heat transfer $-\theta'(0)$, i.e. the rate of heat transfer $-\theta'(0)$ increases with the increase of porosity parameter ξ for different types of nanoparticles with $\phi = 0.1$, $\Pr = 6.2$, $Ec = 2.0$, a=1.0 and $N_R = 5.0$ in a case of the Sakiadis flow as shown as figure 12 and Table 4.

Table 1: Thermo-physical properties of fluid and nanoparticles (Oztop and Abu-Nada [4]):

Physical properties	Fluid phase (water)	*Cu*	*Al_2O_3*	*TiO_2*
$C_p\,(J/kgK)$	4179	385	765	686.2
$\rho\,(kg/m^3)$	997.1	8933	3970	4250
$k\,(W/mK)$	0.613	401	40	8.9538

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$\beta \times 10^5 (K^{-1})$	21	1.67	0.85	0.9

Table 2: Values of $\theta(0)_{Blasius}$ with φ=0 and ξ =0 for different values of a without thermal radiation and viscous dissipat term. Parenthesis indicates results from Ref. [37].

A	Pr = 0.72	Pr = 10	Pr = 0.1
0.05	0.144666 (0.144661)	0.06425538 (0.064255)	0.2535732 (0.2535732)
0.20	0.4035216 (0.403522)	0.21548348 (0.215484)	0.5760672 (0.5760672)
0.60	0.6699470 (0.669915)	0.45175776 (0.451759)	0.8030175 (0.8030175)
1.00	0.77182147 (0.771822)	0.57865501 (0.578656)	0.8717014 (0.8717044)
10.0	0.97128527 (0.971285)	0.93212755 (0.932127)	0.9854953 (0.9854953)
20.0	0.98543349 (0.985433)	0.96487165 (0.964871)	0.9926946 (0.9926946)

Table 3: Values of $\theta(0)_{Blasius}$ and $\theta(0)_{Sakiadis}$ with φ=0 and ξ =0 for different values of *a*, Pr, and N_R in the absent of viscous dissipation parameter. Parenthesis indicates results from Ref. [37].

a	Pr	N_R	$\theta(0)_{Blasius}$	$\theta(0)_{Sakiadis}$
0.1	5	0.7	0.19957262 (0.19957406)	0.13807554 (0.13807609)
0.5	5	0.7	0.55489539 (0.55489763)	0.44474442 (0.44474556)
1.0	5	0.7	0.71373999 (0.71374169)	0.61567210 (0.61567320)
10	5	0.7	0.96143950 (0.96143981)	0.94124369 (0.94124394)
20	5	0.7	0.98034071 (0.98034087)	0.96973264 (0.96973278)
1	0.72	0.7	0.83312090 (0.83312107)	0.84297889 (0.84297896)
1	1.0	0.7	0.81555442 (0.81555469)	0.81785939 (0.81785952)
1	5	0.7	0.71373999 (0.71374169)	0.61567210 (0.61567320)
1	10	0.7	0.66300933 (0.66301284)	0.51639983 (0.51639994)
1	100	0.7	0.47592523 (0.47592614)	0.23747300 (0.23747971)
5	5	0.7	0.92574241 (0.92574298)	0.88900881 (0.88900927)
5	5	5	0.90376641 (0.90376783)	0.83172598 (0.83172654)
5	5	10	0.90044294 (0.90044458)	0.82284606 (0.82284675)
5	5	100	0.89700190 (0.89700322)	0.81361422 (0.81361511)

Table 4: Values related to the skin friction and $-\theta'(0)$ for different values of ξ with φ =0.10 ,Pr=6.2 , Ec=2.0 , *a*=1.0 and N_R =5.

ξ	$-f''(0)_{Sakiadis}$			$-\theta'(0)_{Sakiadis}$		
	Cu	Al_2O_3	TiO_2	Cu	Al_2O_3	TiO_2
0.0	0.521335736	0.443360657	0.448117021	0.552371143	0.339160866	0.370598060
0.1	0.602412377	0.536859170	0.540759501	0.707558805	0.514104489	0.550352049
0.5	0.864340428	0.822323869	0.824731368	1.285627932	1.145488218	1.200813914
1.0	1.113768215	1.082626404	1.084393613	1.918057163	1.812997271	1.890499613
1.5	1.318154533	1.292424678	1.293880638	2.482202797	2.399667893	2.497646636
2.0	1.495281886	1.472895534	1.474160859	2.999210239	2.933744080	3.051002415
2.5	1.653724347	1.633655563	1.634789138	3.481100035	3.429730492	3.565414205
3.0	1.798350101	1.780006422	1.781042215	3.935275473	3.896141624	4.049606757
4.0	2.057452808	2.041545285	2.042443201	4.778866415	4.760698271	4.948297997
5.0	2.287469582	2.273232560	2.274036045	5.555393750	5.555211892	5.775514721
7.0	2.689205205	2.677165763	2.677844845	6.961436921	6.991907014	7.274490816
8.0	2.869076810	2.857813434	2.858449089	7.607564042	7.651565660	7.963982361
9.0	3.038326500	3.027706273	3.028305454	8.223129842	8.279788842	8.621332129
10.0	3.198638158	3.188562318	3.189130821	8.812097073	8.880688809	9.250663862

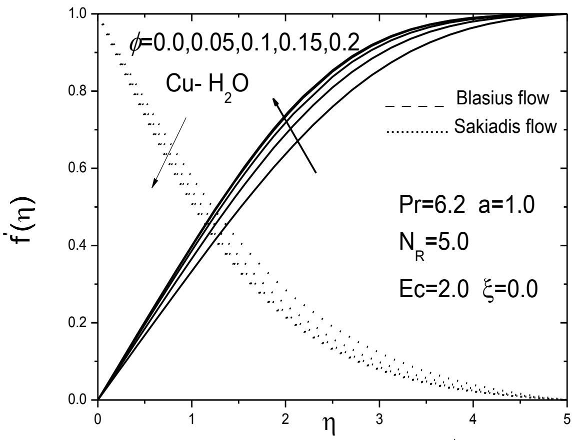

Figure 1 Effect of nanoparticle volume fraction ϕ on velocity distribution $f'(\eta)$ in the case of Cu- H_2O

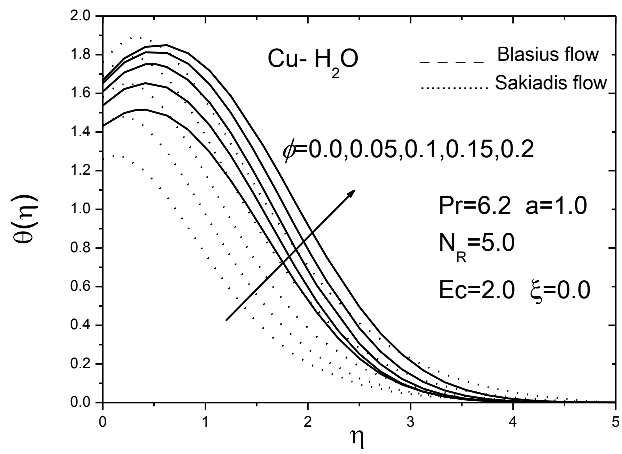

Figure 2 Effect of nanoparticle volume fraction ϕ on temperature profiles $\theta(\eta)$ in the case of Cu- H_2O

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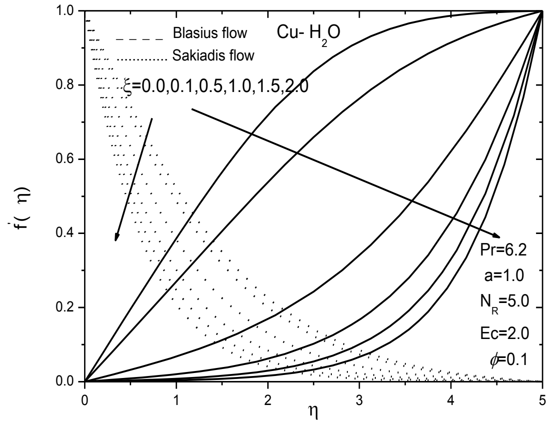

Figure 3 Effect of porosity parameter ξ on velocity distribution $f'(\eta)$ in the case of Cu- H_2O

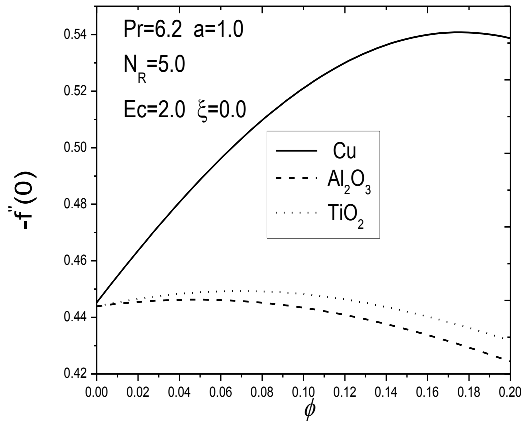

Figure 6 Effect of nanoparticle volume fraction ϕ on skin friction coefficient $f''(\eta)$ for different types of nanoparticles (Sakiadis flow)

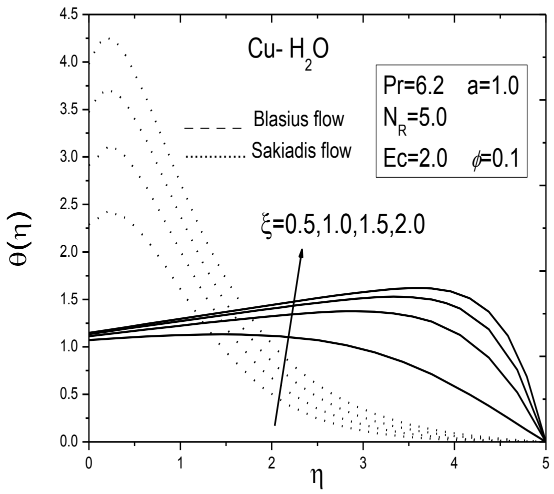

Figure 4 Effect of porosity parameter ξ on temperature profiles $\theta(\eta)$ in the case of Cu- H_2O

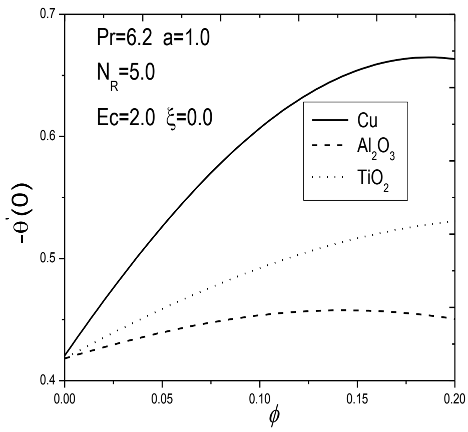

Figure 6 Effect of nanoparticle volume fraction ϕ on heat transfer rate $-\theta'(0)$ for different types of nanoparticles (Blasius flow)

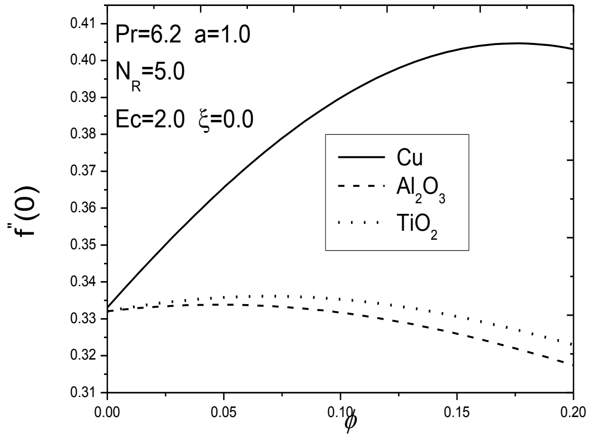

Figure 5 Effect of nanoparticle volume fraction ϕ on skin friction coefficient $f''(\eta)$ for different types of nanoparticles (Blasius flow)

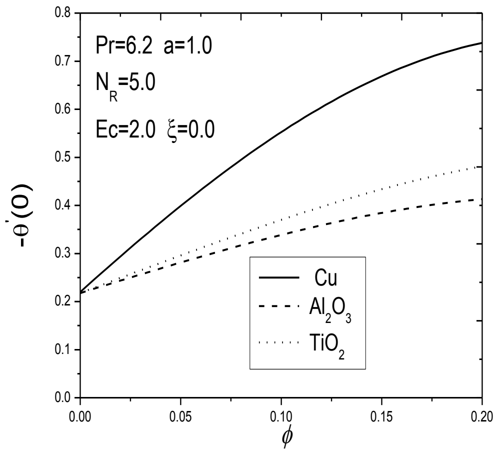

Figure 8 Effect of nanoparticle volume fraction ϕ on heat transfer rate $-\theta'(0)$ for different types of nanoparticles (Sakiadis flow)

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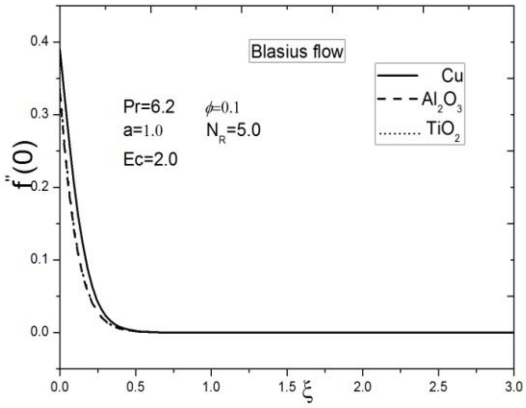

Figure 9 Effects of porosity parameter ξ on skin friction coefficient for different types of nanoparticles

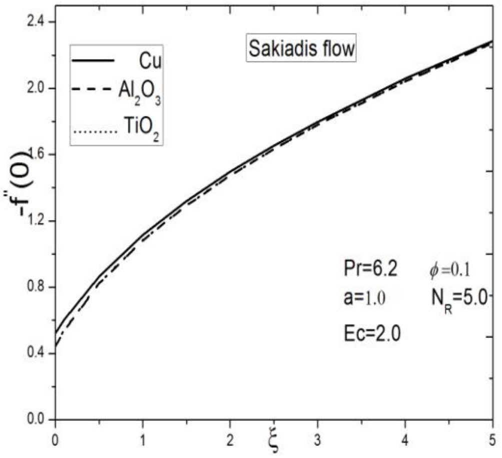

Figure 10 Effects of porosity parameter ξ on skin friction coefficient for different types of nanoparticles

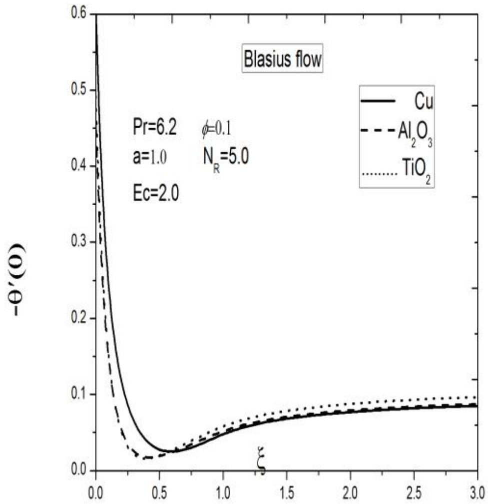

Figure 11 Effects of porosity parameter ξ on heat transfer rate for different types of nanoparticles

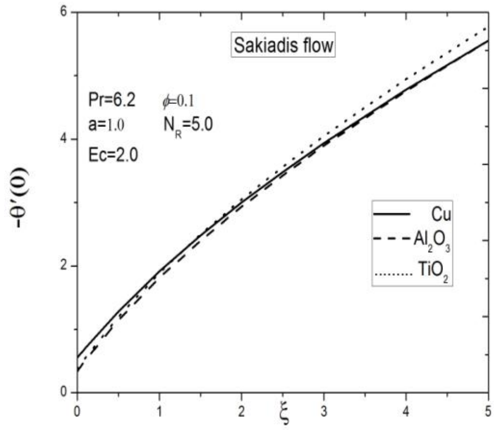

Figure 12 Effects of porosity parameter ξ on heat transfer rate for different types of nanoparticles

IV. CONCLUSION

The porosity effects on the flow and heat transfer characteristics of the Blasius and Sakiadis flow in a nanofluid in the presence of thermal radiation. The resulting system of nonlinear partial differential equations is solved numerically using an efficient numerical shooting technique with a fourth-order Runge–Kutta scheme (MATLAB package). The solutions for the flow and heat transfer characteristics are evaluated numerically for various values of the governing parameters, namely the nanoparticle volume fraction ϕ and the porosity parameter ξ . Three different types of nanoparticles are considered, namely Cu, Al_2O_3 and TiO_2. The variations of dimensionless surface temperature as well as flow and heat-transfer characteristics with the governing dimensionless parameters of the problem, which include the nanoparticles volume fraction ϕ , the thermal radiation parameter N_R and the porosity parameter ξ are graphed and tabulated.

- In the case of Blasius flow the velocity profile increases while it decreases in the case of Sakiadis flow when the solid volume fraction ϕ increases. The rise of the solid volume fraction ϕ leads to increase of the temperature distribution in both cases.
- An increment in the porosity parameter ξ yields a decreasing in the velocity profile and an increment in the temperature distribution in both cases, this leads to a rapid reduction in the heat transfer rates.
- The skin friction coefficient increases as the nanoparticle volume fraction ϕ increases when the nanoparticle are Cu, but decreases when the nanoparticle are Al2O3 and TiO_2 in Blasius and Sakiadis flow.

- The skin friction coefficient is fast decreases as the porosity parameter ξ increases in the case of Blasius flow. On the contrary of Sakiadis flow.
- The heat transfer rate is decreases as the porosity parameter ξ increases in the case of Blasius flow. In contrast of Sakiadis flow as we expected.

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