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Analysis and Application of Hydraulic Jump

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Abstract- ***Analytical-statistical solutions of the characteristics in hydraulic jump, including submerged situation with the sluice gate effects, of 2DV flows, such as primary velocity profile for fixed bed, turbulent shear stress profiles and profiles of turbulent kinetic energy and kinetic energy dissipation rate of fixed bed are derived respectively. Then, the comparisons of the analytical results are made with the experimental results and numerical model solutions. Good trends and agreements are obtained, while the applicability of the analytical results are also presented and discussed.***

Index Terms- **Turbulence; hydrodynamics; hydraulic jump; submerged hydraulic jump; primary velocity; turbulent shear stress; turbulent kinetic energy; kinetic energy dissipation rate;**

I. INTRODUCTION

Water bodies of river engineering works can be mastered with good understanding of their self-form geometric shapes and their response to changes in nature and human interference. Naturally, the channel geometries vary not only in the lateral direction but also in the depth one, or said 3-D, while due to the lack of sufficient data to be used to calibrate this 3-D model, the flow situation may be simplified to 2DV flows with the assumption of quite uniformity in the width direction. The idea of plan turbulent wall jets, which possessed very strong turbulence, is chosen to describe and solve the 2-D hydraulic jump flows. The primary velocity profile along the depth direction strongly dominate the turbulent phenomena, such as turbulent shear stress with the expression of turbulent eddy viscosity, turbulent kinetic energy, and the energy dissipation rate, and all the mentioned turbulence will be solved analytically. The typical appearance of a hydraulic jump in a horizontal channel downstream a sluice gate is discussed. It is common practice to consider the length of the jump defined unconventionally as the distance between the toe of the jump and a point downstream where the influence of the jump has become negligible and flow conditions are determined by the characteristics of the channel alone. The condition of flow into the hydraulic jump (inflow Froude number, Fro) depends on the state of flow development of the supercritical stream issuing from underneath the sluice gate. Due to convective acceleration upstream, flow at the point of vena contraction is essentially not rotational with vortex restricted to a layer at the boundary. With increasing distance from the sluice gate, the thickness of the eventually turbulent boundary layer increases, while the potential layer shrinks, until after some length of flow development the boundary layer thickness δ becomes equal to the downstream depth of flow, and the flow is said to be fully developed. The location of hydraulic jump is determined by downstream channel condition. As a consequence, the inflow section of the jump can occur anywhere in the region of developing flow, or in fully developed flow. When the sequent depth is greater than the tail-water depth, it is named R-jump, while S-jump with the tail-water depth greater than the one of sequent depth, called submerged hydraulic jump.

II. ANALYTICAL SOLUTIONS OF THESE TURBULENT ITEMS FOR HYDRAULIC JUMP FLOWS

As mentioned above, the wall jet proposed by 〔Rajaratnam, N (1965, 1976)〕are used to derive the primary velocity and turbulent shear stress profiles, which are then used to express the turbulent eddy viscosity. The 2-DV continuity equation and equation of motion are used to derive the secondary velocity and turbulent kinetic energy. Finally, the 2-D k-ε two-equation model presented by [Rodi (1980)] is used to obtain the energy dissipation rate. The main turbulent quantities of the analytical expressions are as follows [Luo, C. R. (1993)],

A. Mathematical Models of 2DV with width-averaged Equations for Numerical Solutions

These equations of 2DV with width-averaged flow situations (VEST) are shown as:

$$\frac{\partial U}{\partial x}+\frac{\partial W}{\partial z}=0$$

……………………………………. *(1)*

$$\frac{\partial U^2}{\partial x}+\frac{\partial UW}{\partial z}=-g\frac{\partial \xi}{\partial x}+\frac{\tau_{wx}}{\rho B}+\frac{1}{\rho}\frac{\partial(\tau_{xx})}{\partial x}+\frac{1}{\rho}\frac{\partial(\tau_{xz})}{\partial z}+$$

$$+\frac{1}{\rho B}\frac{\partial}{\partial x}\int_{y_1}^{y_2}\rho(U-\overline{U})^2dy+\frac{1}{\rho B}\frac{\partial}{\partial z}\int_{y_1}^{y_2}\rho(U-\overline{U})(W-\overline{W})dy$$

$$\frac{\partial WU}{\partial x}+\frac{\partial W^2}{\partial z}=-g\frac{\partial \xi}{\partial z}+\frac{\tau_{wz}}{\rho B}+\frac{1}{\rho}\frac{\partial(\tau_{xz})}{\partial x}+\frac{\partial(\tau_{zz})}{\partial z}+$$

$$+\frac{1}{\rho B}\frac{\partial}{\partial x}\int_{y_1}^{y_2}\rho(U-\overline{U})(W-\overline{W})dy+\frac{1}{\rho B}\frac{\partial}{\partial z}\int_{y_1}^{y_2}\rho(W-\overline{W})^2dy$$

....

(2).

$$U\frac{\partial \xi}{\partial x}-W=0$$

……………………………………….. *(3).*

$$k_w=\frac{u_{*w}^2}{\sqrt{C_\mu}};\qquad \varepsilon_w=\frac{u_{*w}^3}{\kappa z_w}$$

…………………………*(4).*

If the surface shear stress is sufficiently large, then Eq. (4) is substituted by:

$$k_s=\frac{u_{*s}^2}{\sqrt{C_\mu}};\qquad \varepsilon_s=\frac{\left(k_s\sqrt{C_\mu}\right)^{3/2}}{\kappa\left[z_s+ah\left(1-\frac{u_{*s}^2}{k_s\sqrt{C_\mu}}\right)\right]}$$

………………………………………… *(5).*

The k_s in Eq .(5) limits the length scale near the surface and thus increases ε. The values of mean shear stresses for both the wall and the side boundary are given as follows,

$$\tau_{wx} = \frac{\rho g U}{C_c^2}\sqrt{U^2 + W^2} \qquad \text{…………………………………………(6).}$$

And

$$\tau_{wz} = \frac{\rho g W}{C_c^2}\sqrt{U^2 + W^2} \qquad \text{…………………………………………(7).}$$

B. Analytical Primary Velocity Profile

The idea of plane turbulent wall jets is used to solve analytical solution of velocity profile for hydraulic jump flow. The primary velocity profiles are obtained by using 2-D continuity equation and equation of motion in primary flow direction with neglecting wall friction items and assuming the vertical velocity smaller after comparing with the primary one. The turbulent viscosity is obtained from dividing the turbulent shear stress by velocity gradient. Turbulent kinetic energy, k, is from 2-D momentum equation while ε is from the simple depth-averaged k-ε relationship. The definition of each symbol is shown in Figures. 1 and 2.

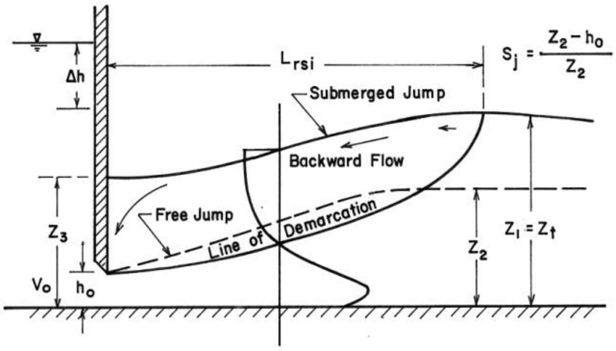


Fig 1: Submerged hydraulic jump (from N. Rajaratnam, 1965 ）

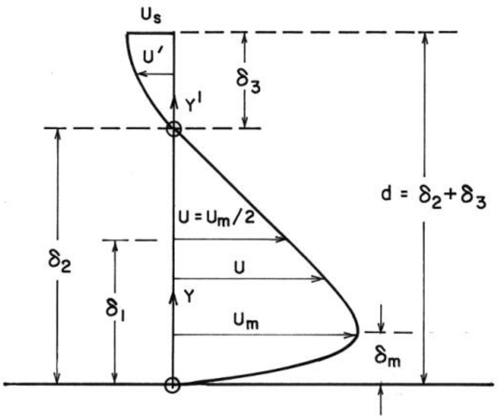


Fig 2: Submerged hydraulic jump (from N. Rajaratnam, 1965)

(a) Forward flows

$$\frac{U}{U_{\max}} = \exp\left[-0.905(\eta - 0.125)^2\right] \qquad \text{…………………. (8).}$$

$$\frac{U_{\max}}{V_0} = 5.395\left(\frac{x'}{h_0} + 11.2\right)^{-0.555} \qquad \text{…………………….. ….(9).}$$

$$\frac{\delta}{h_0} = 0.0678\left(\frac{x'}{h_0} + 11.2\right) \qquad \text{………………………………….(10).}$$

$$x = x_0 + x'$$

$$x_0 = \left(\frac{h_0}{2}\right)\cot\theta$$

$$\theta = 1.94^{o} \qquad \text{…………………………………………(11).}$$

Where η =dimensionless boundary layer displacement=z/Δ; $2.0 \leqq \eta \leqq 2.5$. The distance, x, the location to be analysed, includes the distance from the virtual origin x_0, which is the length for translating the flow from developing to fully developed flow case and generally is neglected for calculating velocity values but it is included for concerning calculation of the length of submerged hydraulic jumps. And there is a core angle, 3.8°, and the opening of the sluice gate is h_0. V_0 is the inflow velocity. The inflow Reynold number (R_e) is less than 50,000 when $F_{ro} \leqq 1.0$, while $F_{ro} > 2.0$, $R_e > 500{,}000$.

(b) Backward flows

$$\frac{U'}{U_s} = f(\eta') = -1.122\,(\eta')^2 + 2.098\,\eta' \qquad \text{………………(12).}$$

$$\frac{U_s}{V_0} = -0.27\sin(\pi\,\alpha) \qquad \text{………………………………………(13).}$$

with η'=z'/δ3, and δ3= h–2.5δ, the velocity distribution in the backward flow is in Figure. 3, here, Us is the surface velocity of adverse flow.

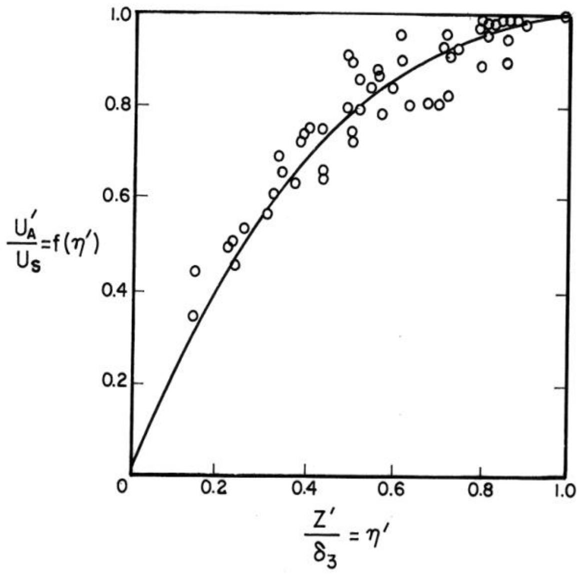


Fig 3: Velocity distribution in the backward Flow (from N. Rajaratnam, 1965 ）

and α=x/L_{rsi}, where Lrsi is the length of hydraulic jump, and it could be presented as $L'_{rsi} + x_0$, furthermore, L'rsi is,

$$\frac{L'_{rsj}}{z_c} = \frac{3.31}{\left[\left(\frac{z_t - z_3}{z_3}\right) + \left(\frac{1}{F_{r0}}\right)\right]^{0.885}} \qquad \text{………………..(14)}$$

$$z_c = \sqrt[3]{\frac{(V_0 h_0)^2}{g}}; \qquad F_{r0} = \frac{V_0}{\sqrt{g h_0}} \qquad \text{………………(15)}$$

$S_j=(Z_t-Z_2)/Z_2$ ………….. *(16)*

Where Z_2 is the subcritical sequent depth without submerged hydraulic jump happening.

C. Analytical Turbulent Shear Stress; Turbulent Kinetic Energy; Energy Dissipation Rate

The differentiation of the combination of Eqs. (8), (9), (10), and (11), we can have the new resultant equation as following,

$$\frac{dU_1}{dz} = -(1.81/\delta)(z/\delta - 0.125)U_1 \qquad \text{…………………..(17)}$$

The other differentiation of the combination of Eqs. (12) and (13), another new resultant equation is obtained,

$$\frac{dU_2}{dz} = -0.27\,V_0 \sin(\pi\,\alpha)\left[\left(-2.244z/\delta_3^2\right) + \left(2.098/\delta_3\right)\right] \qquad \text{……………………………(18).}$$

$$\nu_{tz} = \frac{u_*^2\left(1 - \frac{z}{h}\right)}{\left(\frac{dU_1}{dz} + \frac{dU_2}{dz}\right)} = \frac{(\tau_{xz}/\rho)}{(\partial U/\partial z)}$$

$$= \delta^4\left(\frac{x'}{h_0} + 11.2\right)^{0.555}\left[\frac{u_*^2(h + 0.37\delta)}{9.3z^3 h V_0} - \frac{u_*^2}{9.3z^2 h V_0}\right] \qquad \text{……………………………. (19).}$$

The velocity profile of in vertical direction is expressed as:

$$W = \left(\frac{0.017\delta V_0}{h_0}\right)\left(\frac{x'}{h_0} + 11.2\right)^{-1.665} erf\left[0.951\left(\frac{z}{\delta} - 0.125\right)\right] + 0.66\,V_0 \cdot$$

$$\left(\frac{x'}{h_0} + 11.2\right)^{-0.555}\left[\frac{0.34z^3}{\delta^3} + \frac{0.244z^4}{\delta^4} - \frac{z^2}{16\delta^2} - \frac{0.181z^5}{\delta^5}\right] +$$

$$+ 0.27\,V_0 \sin(\pi\,\alpha)\left[\frac{0.18z^2}{\delta_3} - \frac{0.13z^3}{\delta_3^3}\right] - 0.27V_0\, erf\left[\frac{1.25z}{h} - 0.12\right] +$$

$$- 0.17\,V_0\left[\frac{0.77z^3}{h_0^3} + \frac{0.73z^4}{h_0^4} - \frac{0.71z^5}{h_0^5} - \frac{0.11z^2}{h_0^2}\right] - 0.27\,V_0 \sin(\pi\,\alpha) \cdot$$

$$\left[\frac{0.18z^2}{(h - 1.9h_0)^2} - \frac{0.13z^3}{(h - 1.9h_0)^3}\right] \qquad \text{……………………… (20)}$$

Where

$$\frac{z}{h_0} = \frac{z_3}{h_0} + \left(\frac{1.48\,L_{rsj}}{h_0}\right)\ln\left[csc(\pi\,\alpha) - \cot(\pi\,\alpha)\right] \qquad \text{……(21)}$$

and

$$\bar{k} = \frac{1}{2}\left\{0.6\,h_0 V_0^2\left(\frac{\nu h}{R V_0 h_0}\right)^{1/4}\left[\left(\frac{2.95}{h_0}\right)\bar{P}_*^{0.334} + \left(\frac{1.5}{h_0} - \frac{6Z}{h h_0}\right)P_*^{-0.666} + \right.\right.$$

$$\left(\frac{30Z}{h_0^2} - \frac{101Z}{h_0^2} - \frac{118Z^2}{h h_0^2}\right)P_*^{-1.666} + \left(\frac{542Z^2}{h_0^3}\right)P_*^{-2.666} +$$

$$\left.\left. - \left(\frac{11596Z^3}{h_0^4}\right)P_*^{-3.666} + \left(\frac{93224Z^4}{h_0^5}\right)P_*^{-4.666}\right] - U^2\right\}$$

……………………………… *(22)*

Where

$$P_* = \left(\frac{X}{h_0} + 11.2\right)$$

$$\bar{\varepsilon} = C_\mu\left(\frac{\bar{k}^2}{\bar{\nu}_t}\right) \qquad \text{…………………………………. (23)}$$

$$\overline{\tau_{xz}} = \rho\,\bar{\nu}_t\left(\frac{\partial U}{\partial z}\right) \qquad \text{………………………………………. (24)}$$

III. COMPARISONS OF THE RESULTS BETWEEN ANALYTICAL AND NUMERICAL SOLUTIONS

A. Comparisons of primary velocity profiles for fixed bed

By comparing the analytical results in using Eqs. (8), (9), (12), and (13), among the numerical results by the VEST hydrodynamic models with standard parameter values, and the experimental ones from Tran Thuc (1991) with F_{ro}= 0.90, S_j=0.43 are plotted in Figure 4. Nice agreements among them are obtained for magnitudes of the forward velocities within the region of submerged hydraulic jump. The analytical results of backward velocities are larger than the ones from the numerical methods in this region.

B. Comparisons of Turbulent Shear Stress and Energy Dissipation Rate for fixed bed

The inflow velocity V0 can decide the Froude number while Zt setting the situation whether submerged hydraulic jump happening or not. By combining using Eqs. (14), (15), (16), (17), (18), (19), (23) , and (24) the analytical results of turbulent shear stress and energy dissipation rate are constructed for such a case of F_{ro}= 0.90, Sj=0.43, and again comparing with the numerical results of the VEST hydrodynamic models with standard parameter values from Tran Thuc (1991) . Good trends can be seen in Figures 5 and 6.

IV. APPLICATIONS OF ANALYTICAL SOLUTIONS

The analytical results also can be used for the situations of supercritical flows with submerged hydraulic jump situation. The comparisons for the case of primary velocity and the turbulent shear stress profiles along the flow direction for Fr_0= 5.49, S_j=0.63 are presented in Figures 7 and 8. The analytical results are also compared with the experimental results of Long et al (1990, 1991). It can be seen that the analytical solutions are valid for this case.

V. DISCUSSION

The gradients of the convective items in flow direction and the wall friction ones in the Navier-Stoke equations are small and negligible by comparing with the other items in the x-momentum and z- momentum equations for the analytical solutions.

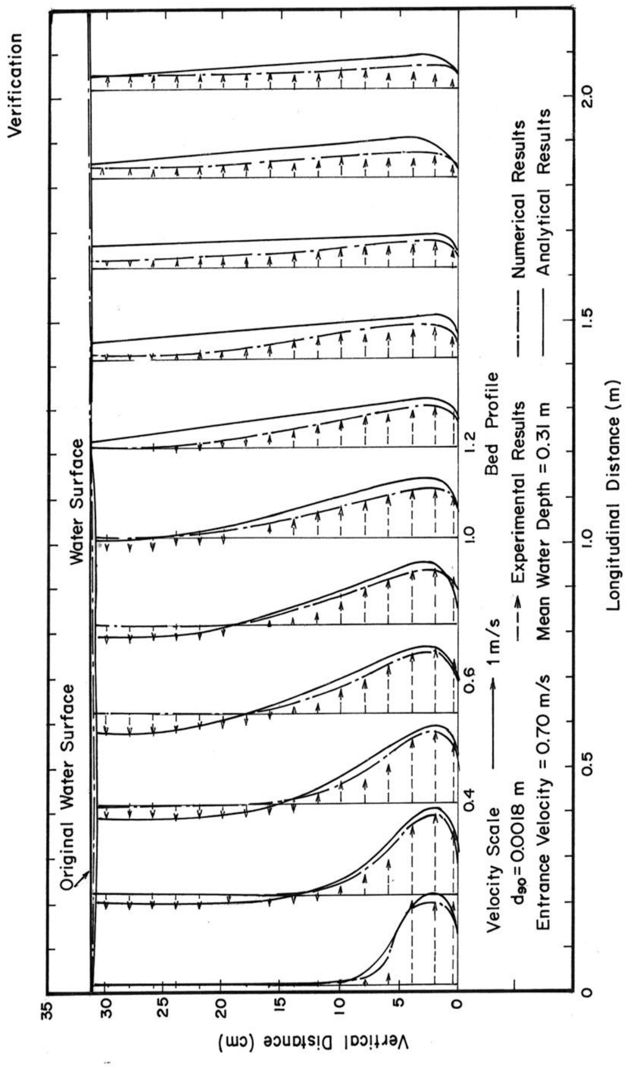


Fig 4: Comparison of 2-DV analytical results and numerical primary velocity with the VEST model, with sluice gate and fixed bed condition, Fro= 0.90, Sj=0.43

The primary velocity profile is compared, too,with the results of Lin (2009), which has the form in sine and exponential function from the laboratory data and by considering the descriptions and results from Liu et al (2004), Ryu et al (2005), and Lin et al (2008, 2009) The analytical results were used to compare the VEST-numerical models with standard parameter values and the experimental results in order to prove the reliability and applicability on the distributions of the turbulent quantities of the analytical solutions. Because the analytical solutions are based on the strong turbulence model, the plane turbulent wall jet, thus, the analytical results are valid for this string turbulent region of the upper half length. The standard k-ε turbulence model has limitation when it is applied to strongly recirculating flows. What the maximum bubble velocities are and if they are coincident with the maximum water velocities, these will become another interesting topics.

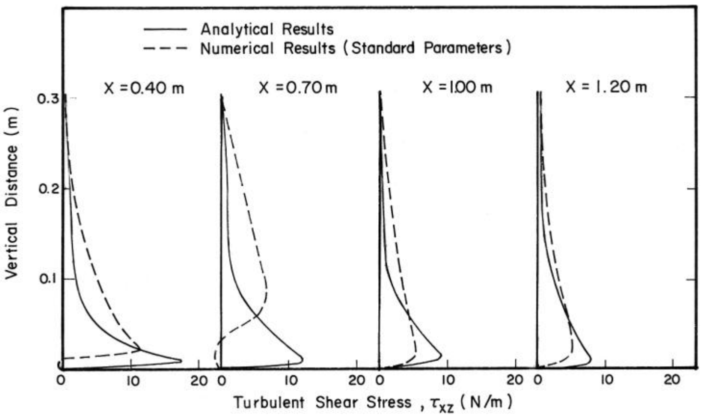


Fig 5: Comparisons of Turbulent shear stress between analytical and numerical results at different positions along flow from sluice gate

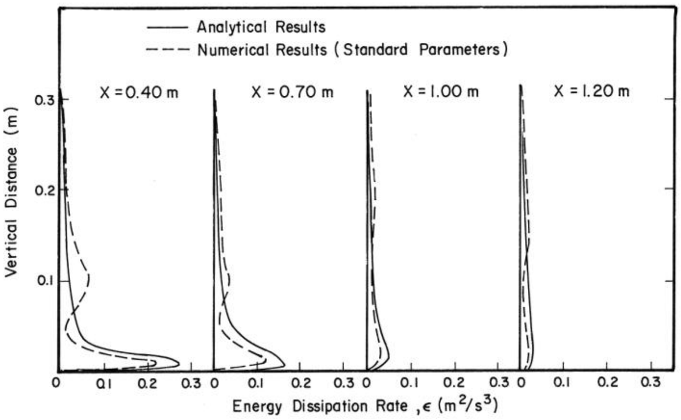
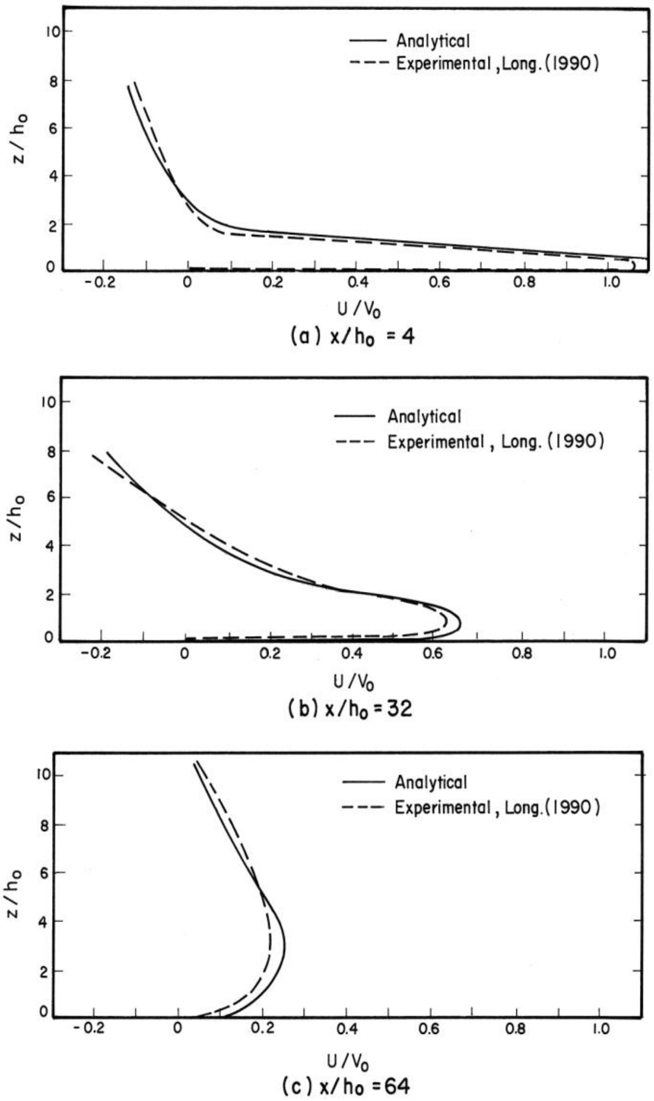


Fig 6: Comparisons of energy dissipation rate between analytical and numerical results at different positions along flow from sluice gate

VI. CONCLUSION

The analytical results for fixed bed express very nice agreements with the experimental results from Tran Thuc, especially the backward flow within the submerged hydraulic jump region. Only very small portions of this backward flow effect are shown from the VEST numerical model in hydrodynamics in Figure 4. In the region of submerged hydraulic jumps, the analytical method is better than the numerical one. After the zone of submerged hydraulic jump, due to the continuous developments of boundary layers, the analytical results show greater free

surface velocities. Later, a short distance from the end of submerged hydraulic jumps, the velocity profiles from analytical results recover to the ones from the experimental or numerical ones.



(a) $x/h_0 = 4$

(b) $x/h_0 = 32$

(c) $x/h_0 = 64$

Fig 7: Comparisons of primary velocity profile between analytical and experimental result for Fro= 5.49, Sj=0.63 of submerged hydraulic jump.

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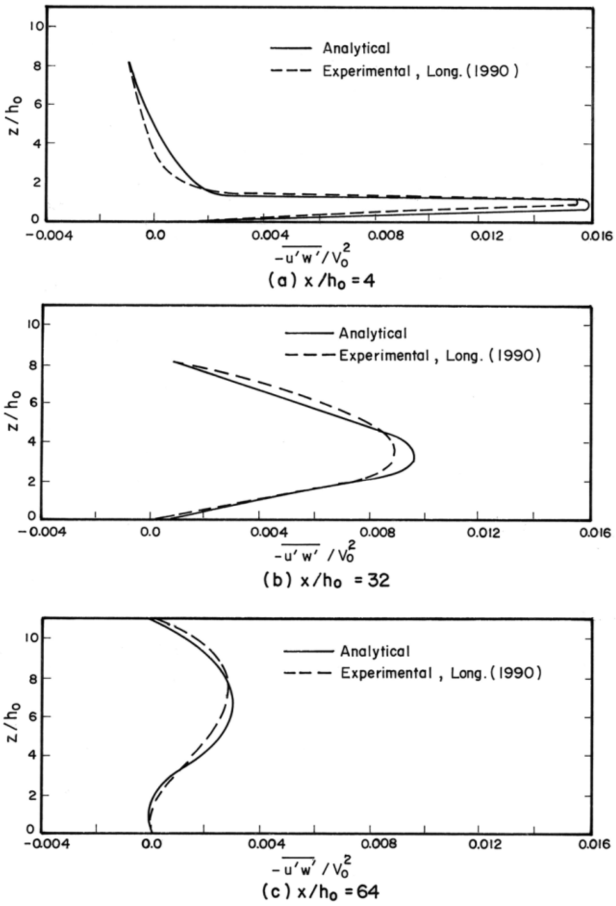

(a) $x/h_0 = 4$

(b) $x/h_0 = 32$

(c) $x/h_0 = 64$

Fig 8: Comparisons of turbulent shear stress profile between analytical and experimental result for Fro=5.49, Sj=0.63 of submerged hydraulic jump

The analytical results on turbulent shear stress and energy dissipation rate in Figure 5 and 6 are little larger than the ones from the VEST numerical model with standard parameter values. The information gives us the hints that the standard parameter values is needed to be modified for the subcritical flow situations in order to be suited for the strong turbulence flow situation. The very good predictions for the case of turbulence characteristics of primary velocity and turbulent shear stress profiles are obtained after comparing the analytical results with the ones from experimental methods for supercritical flow conditions of Fro less than 6.

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