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Image Decomposition and Compression by using Lifting Scheme

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Abstract*—In this paper an attempt has been made to decompose an image using lifting scheme so as to suite compression. Lifting based wavelets are constructed using Haar, Dubieties, Bi-orthogonal, Cohen-Dubieties- Feauveau (CDF), Symlet wavelets. Set Partitioning In Hierarchical Trees (SPIHT) algorithm was utilized to encode the transformed image, as well as to decode the coded version of transformed image. Compression ratio and PSNR are calculated and compared the results with so called traditional wavelets. It has been observed that the lifting based wavelets have produced better compression results.

Index Terms—**Compression, Decomposition, Design Metric, Lifting.**

I. INTRODUCTION

The suitability of wavelet transforms (WT) for use in image analysis is well established: a representation in terms of the frequency content of local regions over a range of scales provides an ideal framework for the analysis of image features, which in general are of different size and can often be characterized by their frequency domain properties [1]. However, the standard form of wavelet decomposition, based on the translation and scaling of a single mother wavelet [2], has its limitations when considering general analysis problems. Apart from its lack of shift invariance [3], it also necessarily links scale and frequency: the size of a given region determines its representative frequencies within the transform. This latter property seems particularly restrictive given that there is no reason in general to assume that the frequency content of an image region should be related to its size. Thus, although the basic advantages of a wavelet approach are well-founded, the need exists for a form which addresses these limitations. The wavelets are a family of functions generated from a single function by translation and dilation. The general form of these wavelets is described by

$$\Psi_{a,b}(t) - |a|^{-1/2}\,\Psi\left(\frac{t-b}{a}\right) \qquad (1)$$

Ψ is called the mother wavelet and it is used to generate all other members of the family. A common choice for **a** and **b** is $a - 2^m$, $b - 2^n$ where n, m ϵ Z.
This reduces (1) to

$$\Psi_{m,n}(t) - 2^{-m/2}\,\Psi\left(2^{-m}t-n\right) \qquad (2)$$

These wavelets are used in the wavelet transform. The purpose of the wavelet transform is to represent a signal, x(t), as a superposition of wavelets. For special choices of Ψ the signal can be represented as

$$x(t) - \sum_{m,n} c_{m,n}\Psi_{m,n}(t) \qquad (3)$$

$$c_{m,n} - 2^{-m/2}\int x(t)\Psi_{m,n}(t)dt \qquad (4)$$

The purpose of obtaining this description is that it provides a representation of the signal x(t) in terms of both space and frequency localization. In comparison, the Fourier transform is excellent at providing a description of the frequency content of a signal. But if the signal is non-stationary the frequency characteristics vary in space, that is in different regions the signal x(t) may exhibit very different frequency characteristics, the Fourier transform does not take this into account. The wavelet transform on the other hand produces a representation that provides information on both the frequency characteristics and where these characteristics are localized in space. The coefficients $c_{m,n}$ characterizes the projection of x onto the base formed by $\Psi_{m,n}$. For different m $\Psi_{m,n}$ represents different frequency characteristics, n is the translation of the dilated mother wavelet, therefore $c_{m,n}$ represent the combined space-frequency characteristics of the signal. The $c_{m,n}$ are called wavelet coefficients. The rest of the paper is organized as follows.

II. HISTORICAL PERSPECTIVE

From an historical point of view, wavelet analysis is a new method, though its mathematical roots date back to the work of Joseph Fourier in the nineteenth century. Fourier laid the foundations with his theories of frequency analysis, which proved to be enormously important and influential. The attention of researchers gradually turned from frequency-based analysis to scale-based analysis when it started to become clear that an approach measuring average fluctuations at different scales might prove less sensitive to noise. The first recorded mention of what we now call a "wavelet" seems to be in 1909, in a thesis by Alfred Haar. The concept of wavelets in its present theoretical form was first proposed by Jean Morlet and the team at the Marseille Theoretical Physics Center working under Alex Grossmann in France. The methods of wavelet analysis have been developed mainly by Y. Meyer and his colleagues, who have ensured the methods' dissemination. The main algorithm dates back to the work of Stephane Mallat in 1988. Since then, research on wavelets has become international. Such research

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is particularly active in the United States, where it is spearheaded by the work of scientists such as Ingrid Daubechies, Ronald Coifman, and Victor Wicker Hauser. The following gives latest advancements in this area. Sonja Grgic et. al, in [4] examined a set of wavelet functions (wavelets) for implementation in a still image compression system and to highlighted the benefit of this transform relating to today's methods. The paper discusses important features of wavelet transform in compression of still images, including the extent to which the quality of image is degraded by the process of wavelet compression and decompression. Bushra K. Al-Abudi et. al, in [5] describes a color image compression scheme based on Haar wavelet transform. The vertical and horizontal Haar filters are composed to construct four 2-dimensional filters, such filters applied directly to the image to speed up the implementation of the Haar wavelet transform. Haar wavelet transform was used to map the image to frequency bands, each band was assigned a weighting factor according to its subjective significance. Such weighting factors were included through the computation process of the number of bits required to present the quantized indices of the wavelet coefficients. Brendan Babb et. al, in [6] described the evolution of new wavelet and scaling numbers for optimized transforms for fingerprint compression and reconstruction. Tilo Strutz, in [7] proposes a new design method for wavelet filter banks, which is explained based on a single lifting structure suitable for 9/7 filter pairs. The filters are derived directly, the factorisation of known filters is not necessary. In addition, it is shown that the signal boundaries can be treated with little computational efforts. The modification of the standard design constraints leads to families of related filter pairs with varying characteristics. It includes a filter bank that can be implemented in integer arithmetic without divisions, shows better performance than the standard 9/7 filter bank for lossless image compression and competitive performance when applied in lossy compression. P. Anand Goud et. al, in [8] proposed a simplified mathematical procedure to arrive at the Daubechies filter coefficients for 4 and 6 taps, as it has been found that Daubechies method of finding filter coefficients is a mathematically demanding one for the undergraduate students. So as to overcome this difficulty, the H(z) is operated to obtain the filter coefficients. G. Chenchu Krishnaiah et. al, in [9] proposed two new lifting based wavelet transforms 13/9 and 15/6 which outperforms the well-known 5/3 and 9/7lifting based wavelets.

III. DECOMPOSITION BY LIFTING

The wavelets generated by translations and dilatations of a single or several basic functions are called first generation wavelets (classic wavelets). Since these operations represent algebraic operations in the frequency domain, the basic tool for their construction is the Fourier transform. There are a number of problems, such as problems defined on intervals, curves, surfaces or manifolds, where the Fourier transform cannot be applied, and thus neither can classic wavelets. Classic wavelets also need to be modified when used for solving problems defined by irregular grids or where the inner product with a weight function needs to be used. Wavelets, attached to problems not allowing for translation and dilatation, are called *second generation wavelets.* Coefficients that correspond to these wavelets can depend on the resolution level. It is clear that working with non-constant coefficients is more complex. The basic idea of wavelet transform is to use a correlation existing in most signals in order to construct a good approximation with few addends. A correlation is a typical local property in space (time) and frequency, meaning that neighbouring data and frequencies are far more correlated than those further moved from each other. In transformation with classic wavelets the basic tool for the space (time)-frequency localization is the Fourier transform, which cannot be applied to more complex geometries. However, localization can be performed in the physical domain space, time etc.), which is the essence of the so-called "lifting" algorithm. This algorithm was primarily developed for constructing second generation wavelets, but it is used successfully also for the construction of biorthogonal wavelets [10, 11]. Lifting generalizes the idea of multiresolution to spaces that are not invariant relative to translation and dilatation, thus enabling users to create wavelets according to their needs and to speed up wavelet transform. The basic idea is to use the correlation between neighboring data in the signal. The way in which this is performed shall be illustrated on a simple example of constructing biorthogonal wavelets. Wavelet algorithms are recursive. The output of one step of the algorithm becomes the input for the next step. The initial input data set consists of 2^p elements. Each successive step operates on 2^{p-i} elements, where i = 1 ... p-1. For example, if the initial data set contains 256 elements, the wavelet transform will consist of eight steps on 256, 128, 64, 32, 16, 8, 4, and 2 elements. If element i in step j is being updated, the notation $step_{j,i}$ is used. The forward lifting scheme wavelet transform divides the data set being processed into an even half and an odd half. In the notation below $even_i$ is the index of the i^{th} element in the even half and odd_i is the i^{th} element in the odd half. Viewed as a continuous array the even element would be a[i] and the odd element would be a[i+(p/2)]. Another way to refer to the recursive steps is by their power of two. Here $step_{j-1}$ follows $step_j$, since each wavelet step operates on a decreasing power of two. This is a nice notation, since the references to the recursive step in a summation also correspond to the power of two being calculated.

A. Predict Wavelets

Like all lifting scheme wavelets the predict wavelet transform starts with a split step, which divides the data set into odd and even elements. The predict step uses a function that approximates the data set. The difference between the approximation and the actual data replaces the odd elements of the data set. The even elements are left unchanged and become the input for the next step in the transform. The predict step, where the odd value is "predicted" from the even value is described by the equation

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$$odd_{j+1,i} = odd_{j,i} - P(\ even_{j,i}\ )$$

The inverse predict transform adds the prediction value to the odd element. In the inverse transform the predict step is followed by a merge step which interleaves the odd and even elements back into a single data stream. The simple predict wavelets are not useful for most wavelet applications. The even elements that are used to "predict" the odd elements result from sampling the original data set by powers of two (e.g., 2, 4, 8...).

B. The update step

The update step replaces the even elements with an average. This result in a smoother input for the next step of the wavelet transform. The odd elements also represent an approximation of the original data set, which allows filters to be constructed. A simple lifting scheme forward transform is shown in Fig. 1.

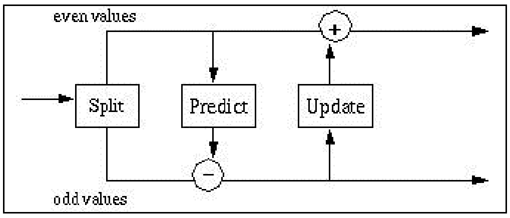

Fig. 1. Lifting Scheme – Forward wavelet tansform

The update phase follows the predict phase. The original value of the odd elements has been overwritten by the difference between the odd element and its even "predictor". So in calculating an average the update phase must operate on the differences that are stored in the odd elements:

$$even_{j+1,i} = even_{j,i} + \boldsymbol{U}(\ odd_{j+1,i}\ )$$

C. Lifting Scheme Haar Transform

In the lifting scheme version of the Haar transform, the prediction step predicts that the odd element will be equal to the even element. The difference between the predicted value (the even element) and the actual value of the odd element replaces the odd element. For the forward transform iteration j and element i, the new odd element, $j+1,i$ would be $odd_{j+1,i} = odd_{j,i} - even_{j,i}$

In the lifting scheme version of the Haar transform the update step replaces an even element with the average of the even/odd pair (e.g., the even element s_i and its odd successor, s_{i+1}):

$$even_{j+1,i} = \frac{even_{j,i} + odd_{j,i}}{2}$$

The original value of the $odd_{j,i}$ element has been replaced by the difference between this element and its even predecessor. Simple algebra lets us recover the original value:

$$odd_{j,i} = even_{j,i} + odd_{j+1,i}$$

Substituting this into the average, we get

$$even_{j+1,i} = \frac{even_{i,i} + even_{j,i} + odd_{j+1,i}}{2}$$

$$even_{j+1,i} = even_{j,i} + \frac{odd_{j+1,i}}{2}$$

The averages (even elements) become the input for the next recursive step of the forward transform. This is shown in Fig. 2, below.

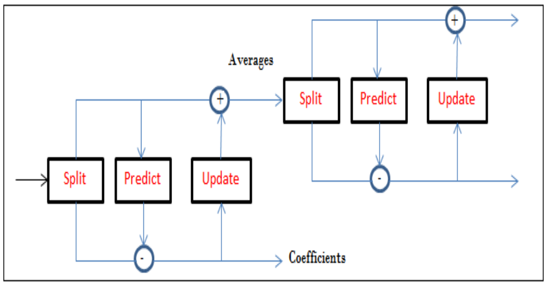

Fig. 2. Two Steps in Lifting forward transform

The number of data elements processed by the wavelet transform must be a power of two. If there are 2^p data elements, the first step of the forward transform will produce 2^{p-1} averages and 2^{p-1} differences. These differences are sometimes referred to as wavelet coefficients. Fig. 3 shows a 4-steps forward wavelet transform on a 16-element data set.

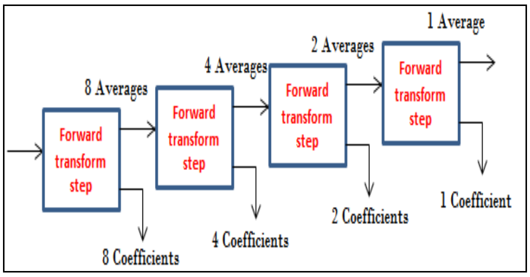

Fig. 3. 4 steps of a 16 element wavelet transform

The split phase that starts each forward transform step moves the odd elements to the second half of the array, leaving the even elements in the lower half. At the end of the transform step the odd elements are replaced by the differences and the even elements are replaced by the averages. The even elements become the input for the next step, which again starts with the split phase. One of the elegant features of the lifting scheme is that the inverse transform is a mirror of the forward transform which is shown is Fig. 4. In the case of the Haar transform, additions are substituted for subtractions and subtractions for additions. The merge step replaces the split step.

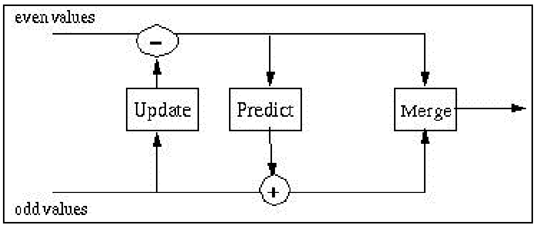

Fig. 4. Inverse Lifting scheme

IV. SIMULATION RESULTS

In this section the simulation results are presented. Lifting based wavelets are constructed using haar, daubechies, biorthogonal, Cohen-Daubechies- Feauveau (CDF), Symlet

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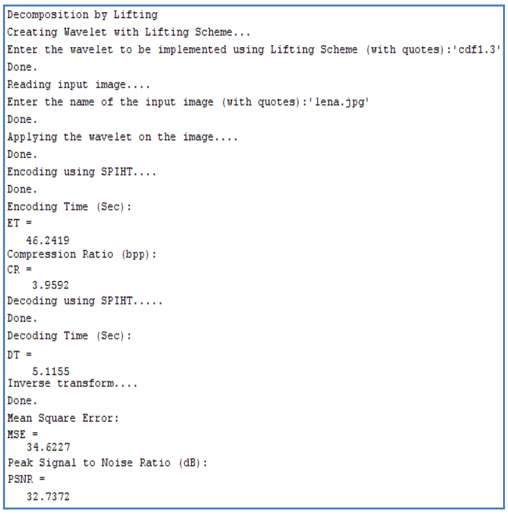
wavelets. The Fig. 5 shows a sample of output on command window of MATLAB.



```
Decomposition by Lifting
Creating Wavelet with Lifting Scheme...
Enter the wavelet to be implemented using Lifting Scheme (with quotes):'cdf1.3'
Done.
Reading input image....
Enter the name of the input image (with quotes):'lena.jpg'
Done.
Applying the wavelet on the image....
Done.
Encoding using SPIHT....
Done.
Encoding Time (Sec):
ET =
   46.2419
Compression Ratio (bpp):
CR =
    3.9592
Decoding using SPIHT.....
Done.
Decoding Time (Sec):
DT =
    5.1155
Inverse transform....
Done.
Mean Square Error:
MSE =
   34.6227
Peak Signal to Noise Ratio (dB):
PSNR =
   32.7372
```

Fig. 5. Screen Shot of output in Command Window of MATLAB

The table I shows the performance of different lifting based wavelets on different images. Table 1 is shown in Appendix The Fig. 6 plots the results in the table I. From the figure, one can observe that the performance is different for different images. The PSNR in the case of 'cameraman' image is above 35dB with all the lifting based wavelets, whereas in the case of 'mandril' image it is just around 25dB. Table II gives the average PSNR, CR and PSRN * CR (PC) values. From the above table, it can be observed that the CR is around 3.25bpp and PSNR 30dB with almost all lifting based wavelets. The PC is above 100 in all the cases. This is very good performance compared to that of traditional wavelets of which the PC values are plotted in the Fig. 7 [12].

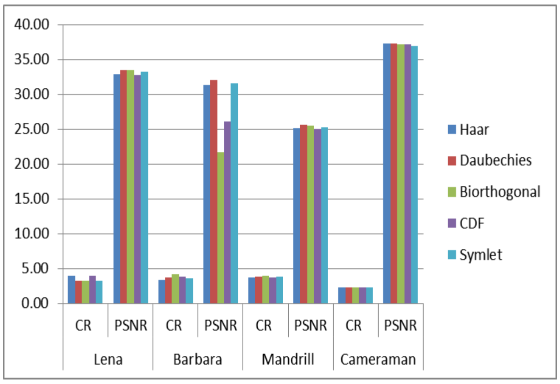


Fig. 6. Performance of Lifting based Wavelets

TABLE II. Average PSNR, CR and PC of Lifting Wavelets

	PSNR	CR	PC
Haar	31.64	3.34	105.62
Daubechies	32.10	3.29	105.58
Biorthogonal	29.46	3.43	100.97
CDF	30.26	3.44	104.07
Symlet	31.74	3.26	103.44

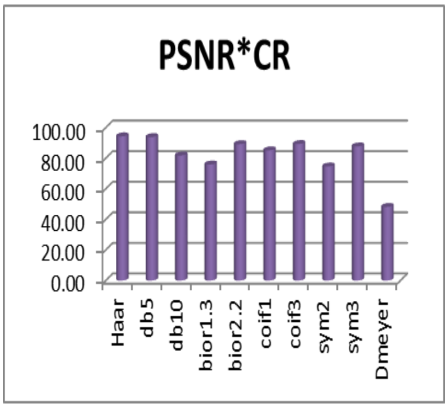


Fig. 7. PSNR*CR Values Of Traditional Wavelets

V. CONCLUSION

This paper presents the lifting version of various classical wavelets. It has been observed that the compression performance was improved compared to that of classical wavelets. By using the design metric PC, the above techniques are compared.

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AUTHOR'S PROFILE

M. Santhosh, born on the 1st of July 1975. She received B.Tech degree from Sri Venkateswara University, Tirupathi, India in 1997. M. Tech degree from Maharaj Sayaji Rao University, Baroda in 1999. She has more than 13 years of teaching experience. She started her career as Assistant Professor in AITS, Rajampet in 1999 and presently working as Associate Professor in Stanley Stephen College of Engineering & Technology, Kurnool.

Dr. B. Stephen Charles (India) born on the 9th of August 1965. He received Ph.d in Electronics & Communication Engineering from JNTU, Hyderabad in 2001. He received his B.Tech in ECE from Nagarjuna University, India in 1986. He started his carrier as Asst. Prof. in Karunya institute of technology during 1989 to 1993, later worked as Asso. Prof. in K. S. R. M. College of Engg till 2001. After that he worked as Principal of St. John's College of Engineering & Technology during 2001 to 2007 and now he is the Secretary, Correspondent and Principal in Stanley Stephen College of Engineering & Technology, Kurnool. He has more than 25 years of teaching and research experience. He published more than 40 research papers in national and international journals and more than 30 research papers in national and international conferences. He is a member of Institute of Engineers and ISTE.

Dr. M.N. Giri Prasad received his B.Tech from J.N.T.U, Anantapur, Andhrapradesh, India in 1982. M.Tech degree from SVU, Tirupati, Andhra Pradesh, India in 1994 and Ph.D degree from J.N.T.U, Hyderabad, in 2003. Presently he is working as a HOD of the Department of Electronics and Communication at J.N.T.U College of Engineering Anantapuramu, Andhrapradesh, India. He has more than 25 years of teaching and research experience. He has published more than 50 papers in national and international journals and more than 30 research papers in national and international conferences. His research areas are Wireless Communications and Biomedical instrumentation, digital signal processing, VHDL coding and evolutionary computing. He is a member of ISTE, IE & NAFEN.

APPENDIX

TABLE I. Performance of Lifting based Wavelets

	Lena		Barbara		Mandrill		Cameraman	
Lifting Wavelets	CR	PSNR	CR	PSNR	CR	PSNR	CR	PSNR
Haar	3.98	32.83	3.4	31.3	3.69	25.13	2.28	37.31
Daubechies	3.26	33.46	3.7	32.06	3.88	25.61	2.32	37.28
Bi-orthogonal	3.28	33.43	4.18	21.71	3.91	25.54	2.33	37.18
CDF	3.96	32.74	3.86	26.14	3.67	25	2.27	37.18
Symlet	3.24	33.21	3.64	31.6	3.84	25.23	2.31	36.91