ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 3, Issue 12, June 2014

The Impact Forces End Face Circular Cylinder

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Abstract: ***In this paper, we consider the transient dynamic problem of elasticity for. Problem of harmonic oscillations occupy the dynamic theory of elasticity is much more modest place in comparison with non-stationary problems. If we consider that in most cases, the sources of excitation wave propagation in elastic media are shock or explosive nature, it becomes clear that in practical applications of dynamic elasticity theory must deal usually with non-stationary problems.***

Keywords: **on-stationary wave, cylinder, impact axial force, Lame equation.**

I. INTRODUCTION

Consider the problem of wave propagation in a semi-infinite circular cylinder radius a under impact axial forces applied on the face area of the cylinder.

II. FORMULATION AND SOLUTION OF THE PROBLEM

In a cylindrical coordinate system associated with the end of the cylinder z, φ, z, the solution of the ax symmetric problem is related to the following system:

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{\partial \sigma_{rz}}{\partial z} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = \rho \frac{\partial^2 u_r}{\partial t^2}$$

$$\frac{\partial \sigma_{rz}}{\partial r} + \frac{\partial_{zz}}{\partial z} + \frac{\partial_{rz}}{r} = \rho \frac{\partial^2 u_z}{\partial t^2} \tag{1}$$

$$\sigma_{rr} = 2\mu\varepsilon_{rr} + \lambda e; \ \sigma_{\theta\theta} = 2\mu\varepsilon_{\theta\theta} + \lambda e; \ \sigma_{zz} = 2\mu\varepsilon_{zz} + \lambda e.$$

$$\sigma_{rz} = 2\mu\varepsilon_{rz}; \ e = \varepsilon_{rr} + \varepsilon_{zz} + \varepsilon_{\theta\theta};$$

$$\varepsilon_{rr} = \frac{\partial u_r}{\partial r}; \quad \varepsilon_{\theta\theta} = \frac{u_r}{r}; \quad \varepsilon_{zz} = \frac{\partial u_r}{\partial z}; \tag{2}$$

$$\varepsilon_{rz} = \frac{1}{2}\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right);$$

$$\varepsilon_{z\theta} = \varepsilon_{r\theta} = 0.$$

Here u_r u_z and in the radial and axial movement of the particle ρ density.

Join this system the initial and boundary conditions; they are as follows:

$$\left.\begin{aligned} u_r = u_z = 0 \\ \frac{\partial u_r}{\partial t} = \frac{\partial u_z}{\partial t} = 0 \end{aligned}\right\} \quad t = 0. \tag{3}$$

$$\left.\begin{aligned} \sigma_{zz} = \sigma_0(r) f(t) \\ u_r = 0 \end{aligned}\right\} \quad z = 0 \tag{4}$$

$$\sigma_{rr} = \sigma_{rz} = 0 \ , \quad r = a; \quad 0 < z < \infty \tag{5}$$

(1) and (2) leads to the equation system ax symmetric:

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ISO 9001:2008 Certified
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$$\begin{cases}(2\mu+\lambda)\dfrac{\partial^2 u_r}{\partial r^2}+(\lambda+\mu)\dfrac{\partial^2 u_z}{\partial r\partial z}+\mu\dfrac{\partial^2 u_r}{\partial r^2}+\dfrac{(\lambda+2\mu)}{r}\dfrac{\partial u_r}{\partial r}-\dfrac{(\lambda+2\mu)}{r^2}u_r=\rho\dfrac{\partial^2 u_r}{\partial t^2}\\[2ex] (\mu+\lambda)\dfrac{\partial^2 u_r}{\partial r\partial z}+(\lambda+2\mu)\dfrac{\partial^2 u_r}{\partial z^2}+\mu\dfrac{\partial^2 u_z}{\partial r^2}+\dfrac{(1+\mu)}{r}\dfrac{\partial u_r}{\partial z}+\dfrac{\mu}{r}\dfrac{\partial u_z}{\partial r}=\rho\dfrac{\partial^2 u_z}{\partial t^2}.\end{cases}$$

Using *sin* and *cos* Fourier, z coordinate functions, respectively, u_r and u_z then the *t*- Laplace transform in system (1) - (5) can be reduced to:

$$(\lambda+2\mu)\frac{\partial^2 u_r^{(s)}}{\partial r}+(\lambda+2\mu)\frac{1}{r}\frac{\partial \bar{u}_r^{(s)}}{\partial r}-\left(\mu q^2+\rho p^2\right)\bar{u}_r^{(s)}-(\lambda+2\mu)q\frac{\partial \bar{u}_z^{(c)}}{\partial z}-(\lambda+2\mu)\frac{\bar{u}_r^{s}}{r^2}=0$$

$$(\lambda+\mu)q\frac{\partial u_r^{(s)}}{\partial r}+(\lambda+\mu)q\frac{u_r^{(s)}}{r}+\mu\frac{1}{r}\frac{\partial u_z^{(s)}}{\partial r}-\left[\rho p^2+(\lambda+2\mu)q^2\right]\bar{u}_z^{(s)}=\sigma_0 f(p)$$

Here *p* and *q* respectively parameters Laplace and Fourier transforms.

Choose two new functions by the formulas:

$$\bar{u}_r^{(s)}=\frac{\partial\varphi}{\partial r}-q\frac{\partial\psi}{\partial r}$$

$$\bar{u}_z^{(s)}=q\varphi-\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\psi}{\partial r}\right)$$

(7)

System (6) after applying the changes (7), takes the simple form:

$$\left.\begin{aligned}&(\lambda+2\mu)(B_1\varphi)=q(B_2\psi)\\ &q^2(B_2\psi)-\frac{1}{r}\frac{d}{dr}(B_2\psi)-\frac{d^2(B_2\psi)}{dr^2}=\frac{\sigma_0\tau(p)}{\mu}\end{aligned}\right\}\qquad(8)$$

Here B_1 and B_2 - modified Bessel operators (zero indexes)

$$B_k=\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}-\left(\frac{p^2}{c_k^2}+q^2\right)\quad k=1,2.$$

The second equation of system (8) is also a modified Bessel equation in relation to the function.

$$B_0B_2\psi=-\frac{\sigma_0 f(p)}{\mu};$$

where $B_0=\dfrac{d^2}{dr^2}+\dfrac{1}{r}\dfrac{d}{dr}-q^2.$

Bounded $0\le r<a$ solution of this equation in a function:

$$B_2\psi=D_0I_0(qr)+\frac{\sigma_0\tau(p)}{\mu q^2}\quad(\sigma_0=const)\qquad(9)$$

Substituting in the first equation (8) we obtain:

$$B_1\varphi=\frac{\mu D_0}{\lambda+2\mu}I_0(qr)+\frac{\sigma_0\tau(p)}{(\lambda+2\mu)q}\qquad(10)$$

Solutions of equations (9) and (10) the following:

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$$\left.\begin{aligned}\varphi &= -\frac{\sigma_0 f(p)}{q\nu_1^2(\lambda+2\mu)} + A_0 I_0(\nu_1 r)\\ \psi &= -\frac{\sigma_0 f(p)}{q^2 \nu_2^2 \mu} + B_0 I_0(\nu_2 r)\end{aligned}\right\} \quad (11)$$

where $\nu_k = \sqrt{\frac{p^2}{c_k^2} + q^2}$, к=1,2

It should be noted, $I_0(q,r)$ due to the substitution (7) the displacement field generated by members containing identically zero, so the final decisions (11), these terms are not present. The unknown coefficients are determined from the side conditions $r = a$

$$\left.\begin{aligned}\sigma_{rr} &= 0\\ \sigma_{rz} &= 0\end{aligned}\right\}, \quad r = a;\ z > 0.$$

In transformations, these conditions take the form:

$$\bar{\sigma}_{rr} = \lambda\left(\frac{\partial \bar{u}_r}{\partial r} + \frac{u_r}{r} - q\bar{u}_z\right) + 2\mu\frac{\partial \bar{u}_r}{\partial r} = 0$$

$$\bar{\sigma}_{rz} = \mu\left(q\bar{u}_r + \frac{\partial \bar{u}_z}{\partial r}\right) = 0.$$

Taking into account (7) and (11) the last relations into a system of two linear algebraic equations for determining A_0 and B_0 :

$$A_0\left[(\lambda+2\mu)I_0''(v_1 a) + \frac{\lambda}{a}I_0'(v_1 a) - \lambda q^2 I_0(v_1 a)\right] -$$

$$-B_0\left[(\lambda+2\mu)qI_0''(v_2 a) + \frac{\lambda}{a}qI_0'(v_2 a)] - \lambda q[\frac{1}{a}I_0'(v_2 a) + I_0''(v_2 a)\right] = \frac{\lambda q \sigma_0 f(p)}{v_1^2(\lambda+2\mu)}$$

$$2A_0 q I_0'(v,a) - B_0\left[q^2 I_0'(v_2 a) + \frac{1}{a}I_0'(v_2 a) + \frac{1}{a}I_0''(v_2 a) + I_0'''(v_2 a)\right] = 0 \quad (12)$$

Determinant of this system is the

$$F = v_2\left\{\left[\frac{c_1^2}{c_2^2}v_1^2 - \left(\frac{c_1^2}{c_2^2} - 2\right)q^2\right](q^2 + v_2^2)I_1(av_2)I_0(v_1 a) + 2\frac{p^2}{c_2^2}\frac{v_1}{a} \times\right.$$

$$\left.\times I_1(v_1 a)I_1(v_2 a) + 4q^2 v_1 v_2 I_1(v_1 a)I_0(v_2 a)\right\} \quad (13)$$

The system (12) has the following solution:

$$\left.\begin{aligned}A_0 &= \frac{\lambda\sigma_0 q}{(\lambda+2\mu)v_1^2 F}\left(q^2 + v_2^2\right)v_2 I_1(v_2 a)\\ B_0 &= \frac{2\lambda\sigma_0 q^2}{v_1(\lambda+2\mu)F}I_1(v_1 a)\end{aligned}\right\} \quad (14)$$

The final decision in the transformations of the form:

$$\varphi = -\frac{\sigma_0}{\xi(\lambda+2\mu)v_1^2} + A_0 I_0(v_1 r)$$

$$\psi = -\frac{\sigma_0}{\xi^2 \mu v_2^2} + B_0 I_0(v_2 r) \quad (15)$$

Where and are given by formulas (14).

ISSN: 2277-3754
ISO 9001:2008 Certified
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Where and are given by formulas (14). Should now return to the actual coordinates, which requires reproduce inverse transformations (15). Judge by the expressions (14) and (13), this operation is fraught with difficult, and to this day is known only to the asymptotic solution of the problem with $t \to \infty$.

It can be shown that the F function is defined by (13) and included in the denominator in (14), q has no real zeros in the complex plane is the imaginary axis. $\operatorname{Re} p = 0$ On the imaginary $p = ik$ axis is replaced by the equation.

$$F = 0 \qquad (16)$$

Reduces to the well known equation for the Pochhammer vibration frequencies of an infinitely long circular cylinder during the propagation of longitudinal waves. First, this equation was obtained and studied in [4], further analytical and numerical study of the equation were carried out in a number of authors [3, 6].

From these studies it follows that equation (16) q for each valid $\operatorname{Re} p = 0$ on the axis has infinite number of roots, $p_m = if_m(q)$ $(m = \pm 1, \pm 2 \ldots)$ and they are all simple and complex conjugate pairs $(p_{-m} = -p_m)$. But here we will not deal with finding these roots, we apply the method developed in [5] for solving the transient dynamics of a rectangular timber, and in the particular case of the layer.

For simplicity, we demonstrate this method for finding the particle velocity to the central axis, which is given by the transformations:

$$\bar{\dot{u}}_r = -\frac{\sigma_0}{v_1^2}\frac{c_2^2}{c_1^2} + \frac{\lambda\sigma_0 c_2^2}{c_1^2}\left[\frac{q^2\left(q^2+v_2^2\right)v_2}{v_1^2 F} I_1\left(v_1 r\right) I_0\left(v_1 r\right) - \frac{2q^2 v_2^2}{v_1 F} I_1\left(v_1 a\right) I_0\left(v_2 r\right)\right];$$

(17)

Modification to the form:

$$\bar{\dot{u}}_z = -\frac{\sigma_0}{v_1^2(\lambda+2\mu)} + \frac{\lambda\sigma_0 q^2}{(\lambda+2\mu)}\left[\frac{q^2+v_2^2}{v_2^3}\frac{I_1\left(av_2\right)}{I_0\left(v_2 a\right)} \times \frac{1}{v_1^2}\frac{I_0\left(v_1 r\right)}{I_0\left(v_1 a\right)} - 2\frac{1}{v_1}\frac{I_1\left(v_1 a\right)}{I_0\left(v_1 a\right)} \times \frac{1}{v_2^2}\frac{I_0\left(v_2 r\right)}{I_0\left(v_2 a\right)}\right] \times \frac{v_2^4}{F^*}$$

(18)

Here

$$F^* = v_2\left\{\left[\frac{c_1^2}{c_2^2}v_1^2 - \left(\frac{c_1^2}{c_2^2} - 2\right)q^2\right]\left(q^2+v_2^2\right)\frac{I_1(av_2)}{I_0(av_2)} + 2\frac{p^2}{c_2^2}\frac{v_1}{a}\frac{I_1(v_1 a)}{I_0(v_1 a)}\frac{I_1(v_2 a)}{I_0(v_2 a)} + 4q^2 v_1 v_2 \frac{I_1(v_1 a)}{I_0(v_1 a)}\right\}$$

Following [2] function can be $\frac{v_2^4}{F^*}$ expanded in a series:

$$\frac{v_2^4}{F^*} = \sum_{n=1}^{\infty} a_n \frac{1}{v_2^n},$$

Whose members are the image and the Laplace and Fourier, satisfied with solutions for small values, t it is sufficient to retain the first few terms:

$$\frac{v_2^4}{F^*} = \frac{1}{v^2}\left[1 + \frac{c_1}{c_2}\frac{1}{av_2} + \frac{1}{v_2^2}\left[q^2\left(1+\frac{c_2^2}{c_1^2}\right) + \frac{c_1^2}{c_2^2}\frac{1}{a^2}\right.\right.$$

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ISO 9001:2008 Certified
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Expression in the square brackets of formula (18) as can be seen $\operatorname{Re} p > 0$, is a geomorphic function in the half disappearing into infinity at $|p| = R_n \to \infty$, as well as having simple poles at the axis $\operatorname{Re} p = 0$. Therefore, you can apply a second decomposition theorem [1]. Whereby

$$f(t) = \sum_{P_k} res_{P_k} f^*(p)e^{pt}$$

$f^*(p) \overset{0}{\underset{0}{=}} L(f(t))$ and wherein it has the above properties.

Please note that for small values in the t expression (18) is dominated by the first term:

$$-\frac{\sigma_0}{\lambda+2\mu}\frac{1}{v^2} \overset{LF_0}{\underset{0}{=}} \sqrt{\frac{\pi}{2}}\frac{\sigma_0 c_1}{\lambda+2\mu} H\left(t-\frac{z}{c_1}\right) \qquad (19)$$

Because another member of the roughly estimated t by two degrees to a smaller value,

$$\overset{\circ}{W} \approx t^2 O(t) + O(t)$$

for small values t.

Now give the inverse functions of images included in the solutions (18), in their derivation was used [1].

1) $$\frac{q^2+v_2^2}{v_2^3}\frac{I_1(v_2 a)}{I_0(av_2)} \overset{L}{\doteq} c_2 a\sin(qc_2 t) - 2\sum_{k=0}^{\infty}\frac{c_2}{a}\frac{\left(q^2-\frac{\alpha_k^2}{a^2}\right)\sin\left(c_2\sqrt{q^2+\frac{\alpha_k^2}{a^2}}\,t\right)}{\alpha_k^2\sqrt{q^2+\frac{\alpha_k^2}{a^2}}};$$

2) $$\frac{1}{v_i^2}\frac{I_0(v_i r)}{I_0(v_i a)} \overset{L}{\doteq} \frac{c_i}{q}\sin(qc_i t) - 2\sum_{k=0}^{\infty}\frac{J_0\left(\alpha_k \frac{r}{a}\right)}{J_1(\alpha_k)}\frac{\sin\left(c_i\sqrt{q^2+\frac{\alpha_k^2}{a^2}}\,t\right)}{\alpha_k\sqrt{q^2+\frac{\alpha_k^2}{a^2}}}$$

3) $$\frac{1}{v_1}\frac{I_1(v_1 a)}{I_0(v_1 a)} \overset{L}{\doteq} \frac{2c_1}{a}\sum_{k=0}^{\infty}\frac{\sin\left(c_1\sqrt{q^2+\frac{\alpha_k^2}{a^2}}\,t\right)}{\sqrt{q^2+\frac{\alpha_k^2}{a^2}}}$$

4) $$\frac{1}{v_2} \doteq c_2 J_0(c_2 qt)$$

5) $$\frac{1}{v_2^3} \doteq \frac{\sqrt{\pi}c_2^2 t}{q\Gamma(3/2)} I_1(c_2 qt)$$

Here α_k the zeros $J_0(x)$ and

$$v_i = \sqrt{\frac{p^2}{c_i^2}+q^2};\ i=1,2.$$

Solution (19) is a flat longitudinal wave that starts with the end site and propagating with the speed.

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ISSN: 2277-3754
ISO 9001:2008 Certified
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Volume 3, Issue 12, June 2014

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