ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 3, Issue 12, June 2014

Manufacturing process with disruption under Quadratic Demand for Deteriorating Inventory

Nita H. Shah, Dushyant Kumar G. Patel and Digesh Kumar B. Shah
Department of Mathematics, Gujarat University, Ahmedabad-380009, Gujarat, India
Department of Mathematics, Govt. Poly. for Girls, Ahmedabad-380015, Gujarat, India
Department of Mathematics, L. D. College of Engg., Ahmedabad-380015, Gujarat, India

Abstract*— Each and every supply chain or logistics structure is subject to disruption. This disruption may be due to uncertainty or unintended events like instrument break down, unavailability of raw materials, some crisis, natural calamities or strikes. This article first establishes the production time when there is no disruption and afterwards when system gets disrupted. After disruption in the production, we investigate whether it has been resulted in any amount of shortages or not. To maintain the goodwill of the company, an additional replenishment is suggested if there are any shortages. For disrupted production system, disruption time is calculated. Also in the case of shortages, when to replenish and how much to replenish that is also shown in the article. Moreover, quadratic demand for products is analyzed in the present article. This type of demand initially increases with time up to some extent and then it starts to decrease. In addition, the units in the inventory system are subject to deterioration at a constant rate. Numerical example and related graphical studies are given to validate the results. Effects of variations in inventory parameters on production time are shown for manufacturers to take advantageous decisions.

***Keywords*—Production Time, Disruption Time, Quadratic Demand, Deterioration, Shortages, Inventory**

I. INTRODUCTION

Every production system is subject to disruption and the companies are always keen to concentrate on this issue. This disruption may be due to unplanned events like instrument break down, unavailability of raw materials, unavailability of electricity, some crisis, strikes or natural calamities like earthquakes, flood etc. Lin and Kroll [5] discussed the production system under an imperfect production system subject to random breakdowns. Lin and Gong [6] derived optimal production time during normal and disrupted production periods for constant demand. Chung et al. [2] developed an economic production quantity model for deteriorating items with stochastic machine unavailability time and shortages. Sheu and Chen [9] developed an model to determine economic production quantity and level of preventive maintenance for an imperfect production process. Khedlekar [4] discussed a disruption production model with exponential demand. Wee [10] defined deterioration as decay, damage, spoilage, evaporation, obsolescence, pilferage, loss of utility or loss of marginal value of commodities that reduces utility. Food grains, fruits and vegetables, beverages, medicines, radioactive chemicals, fertilizers and pesticides, volatile liquids like spray and perfumes, passenger seats are examples of deteriorating products, whose usefulness decreases during the regular storage. Raafat [7], Shah and Shah [8], Goyal and Giri [3], Bakker et al. [1] formulated a review of literature on deteriorating inventory model. Most of the above cited articles considered constant demand, which is not always seen in the market. This paper takes into consideration quadratic demand. This type of demand initially increases with time for some time and then decreases. This article first finds the optimal production time when there is no disruption. Once any disruption comes into picture, production gets affected. In such situation, if there are any shortages then to overcome the shortages new spot replenishment is done by the company to take care of the customers. For disrupted production system, disruption time and production time are calculated. Also in the case of shortages, time for placing new order and quantity to be ordered are also computed. Inventory model deals with units which deteriorate at constant rate. Numerical example is studied and graphical conclusions are exhibited.

II. NOTATIONS AND ASSUMPTIONS

A. Notations

P	Production Rate
H	Time horizon
$R(t)$	Demand rate
$I(t)$	Inventory level at any instant of time t, $0 \le t \le T$
θ	Deterioration rate, $0 \le \theta \le 1$
a	Scale demand $(a > 0)$
b	Linear rate of change of demand with respect to time $(0 \le b < 1)$
c	Quadratic rate of change of demand $(0 \le c < 1)$
Q_r	Ordering quantity when shortages occur
T_p	Production stops at this time when there is no disruption
T_d	Production System disrupts at this time
$T_p^{\,d}$	Production stops at this time when there is disruption in the system
T_r	Time of placing order to overcome shortages

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 3, Issue 12, June 2014

B. Assumptions

1. The inventory system has single item.
2. The items in inventory deteriorate at a constant rate.
3. We consider demand rate to be function of time as

$$R(t) = a\left(1+bt-ct^2\right) \qquad (1)$$

where $a>0$ is scale demand, $0\le b<1$ denotes linear rate of change of demand with respect to time, $0\le c<1$ denotes quadratic rate of change of demand.

4. Production rate is constant.

III. MATHEMATICAL MODEL

A. Model: I Production without disruption

In this model, we optimize production system run, when production is smooth *i.e.* no disruption. Here, production continues till time T_p. Then production stops and inventory level reaches to zero at cycle time H due to demand and deterioration as shown in fig. 1.

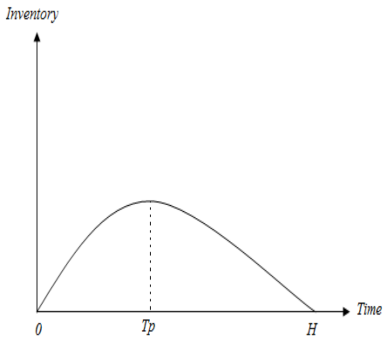

Fig.1 Production system without disruption

$$\frac{dI_1(t)}{dt} + \theta I_1(t) = P - a\left(1+bt-ct^2\right);\ 0\le t\le T_p$$

with boundry condition $I_1(0)=0.$ (2)

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = -a\left(1+bt-ct^2\right);\ T_p\le t\le H$$

with boundry condition $I_2(H)=0$ (3)

Then solution of differential equation (2) and (3) is given by

$$I_1(t) = \frac{1}{\theta^3}\left(\begin{array}{l}\left((a-P)\theta^2-\theta ab-2ac\right)e^{-\theta t}\\ +\left(P-a\left(1+bt-ct^2\right)\right)\theta^2\\ +a(b-2ct)\theta+2ac\end{array}\right) \qquad (4)$$

$$I_2(t) = \frac{1}{\theta^3}\left(a\left(\begin{array}{l}\left(\begin{array}{l}\left(1+bH+cH^2\right)\theta^2-2c\\ -\theta(b-2cH)\end{array}\right)e^{\theta(H-t)}\\ -\left(1+bt-ct^2\right)\theta^2+(b-2ct)\theta+2c\end{array}\right)\right) \qquad (5)$$

According to figure 1, $I_1(T_p)=I_2(T_p)$, which gives

$$T_p = \frac{1}{ac\theta H^2 - aH(b\theta+2c) - \theta P}\left(\begin{array}{l}\left(\begin{array}{l}\left(a^2cH^4(b\theta+c)\right)\\ -2\left(a\left((b-\theta)c+\frac{1}{2}b^2\theta\right)+c\theta P\right)aH^3+\\ aH^2(b\theta+2c)(P-2a)+P^2\end{array}\right)^{1/2}\\ +ac\theta H^3 - aH^2(b\theta+c) - a\theta H - p\end{array}\right)$$

B. Model: II Production with disruption

In this case, we optimize production system run when production is disrupted due to unplanned event. Due to this, production rate is changed by ΔP. Production begins disrupted at the time T_d. Production will be increased or decreased as per positive or negative ΔP respectively.

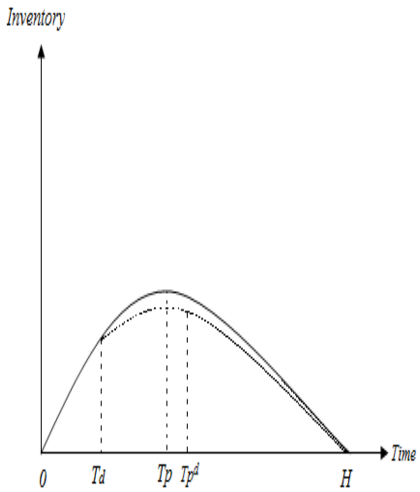

Fig.2 Production system with disruption

Then the governing differential equations are

$$\frac{dI_1(t)}{dt} + \theta I_1(t) = P - a\left(1+bt-ct^2\right);\ 0\le t\le T_d$$

with boundry condition $I_1(0)=0.$ (6)

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = P + \Delta P - a\left(1+bt-ct^2\right);\ T_d\le t\le T_p^{\,d} \qquad (7)$$

$$\frac{dI_3(t)}{dt} + \theta I_3(t) = -a\left(1+bt-ct^2\right);\ T_p^{\,d}\le t\le H;\ I_3(H)=0$$

(8)

$I_1(t)$ remains same as per equation (4).

Solving Equation (7) using the boundary condition $I_1(T_d)=I_2(T_d)$, we get

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 3, Issue 12, June 2014

$$I_2(t)=\frac{e^{-\theta(t-T_d)}}{\theta^3}\left(\left(\left(\begin{array}{l}\left(P+\Delta P-a\left(1+bt-ct^2\right)\right)\theta^2\\+a\theta\left(b-2ct\right)+2ac\end{array}\right)e^{\theta(t-T_d)}\\+\left(\left(a-P\right)\theta^2-\theta ab-2ac\right)e^{-\theta T_d}\right)-\theta^2\Delta P\right) \tag{9}$$

Solving equation (8) using boundary condition $I_3(H)=0$, we get

$$I_3(t)=\frac{1}{\theta^3}\left(ae^{-\theta\left(t-T_p{}^d\right)}\left(\begin{array}{l}\left(\left(-1-Hb+cH^2\right)\theta^2+\left(b-2cH\right)\theta+2c\right)e^{\theta\left(H-T_p{}^d\right)}+\\\left(\left(-1-bt+ct^2\right)\theta^2+\left(b-2ct\right)\theta+2c\right)e^{\theta\left(t-T_p{}^d\right)}\\+2\theta^2\left(1+bT_p{}^d-c\left(T_p{}^d\right)^2\right)+\left(4cT_p{}^d-2b\right)\theta-4c\end{array}\right)\right) \tag{10}$$

If production system satisfies the demand of items after disruption then we find optimal production time $T_p{}^d$ such that at time H, entire stock will be sold-out and inventory level will be zero.

Using Condition $I_2\left(T_p{}^d\right)=I_3\left(T_p{}^d\right)$, we can obtain production time $T_p{}^d$.

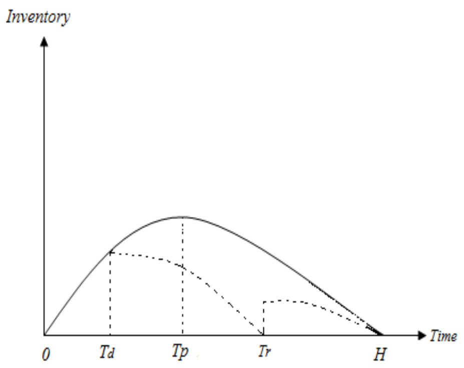

Fig.3 Production system after disruption

and if production system after disruption does not satisfy the demand of items then there will be shortages in the system. In this situation, we will find the replenishment time T_r of placing the order and respective order quantity Q_r.

We can obtain replenishment time T_r using the condition $I_2(T_r)=0$.

At obtained T_r, we can find order quantity Q_r using $Q_r=I_3(T_r)$.

IV. NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS

Example: 1 Taking values $H=1$ year, $P=350$ units, $\Delta P=-200$ units, $T_d=0.25$ year, $a=200$, $b=0.5$, $c=0.15$, $\theta=15\%$,

we have

$T_p=0.7069788488$, $T_p{}^d=0.8940309846$, $Q_r=16.52950962$ and $T_r=0.7205812350$.

Following figures show sensitivity of inventory parameters.

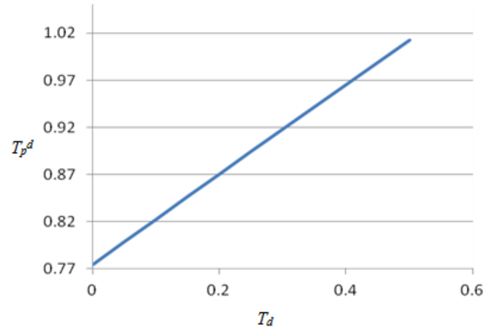

Fig. 4 T_d Vs. $T_p{}^d$

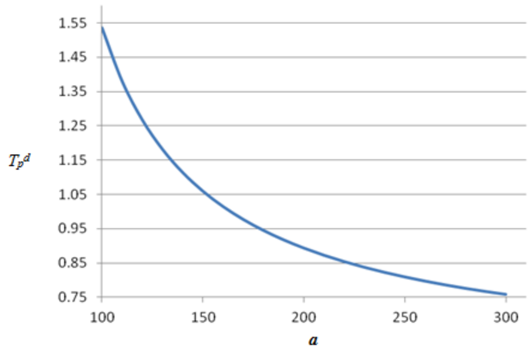

Fig. 5 a Vs. $T_p{}^d$

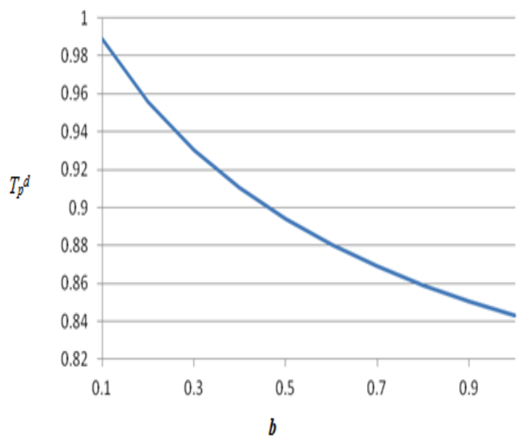

Fig. 6 b Vs. $T_p{}^d$

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 3, Issue 12, June 2014

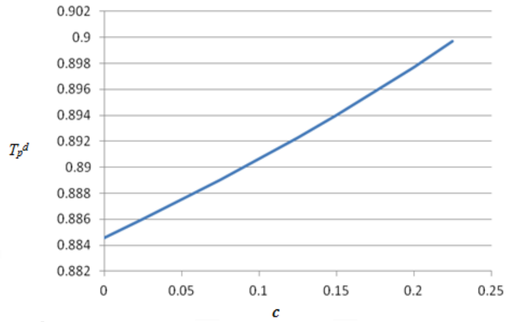

Fig. 7 c Vs. T_p^d

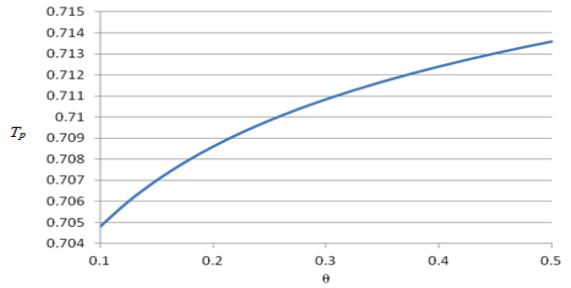

Fig.8 θ Vs. T_p

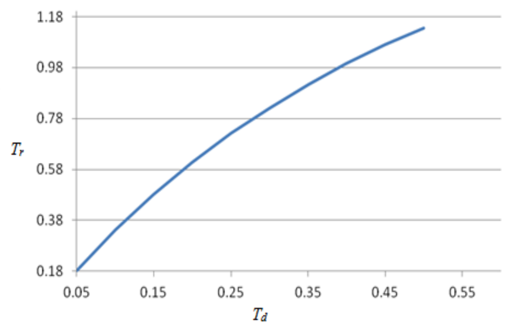

Fig. 9 T_d Vs. T_r

From fig.4 we can observe that T_p^d is directly proportional to T_d. It is evident from fig. 5, fig. 6 and fig. 7 that increase in scale demand a and linear rate of change of demand b decrease T_p^d while increase in quadratic rate of change of demand c increases T_p^d. Fig 8 shows the obvious fact that larger deterioration rate increase production time when there is no disruption. From fig. 9, we can say that in case of shortages, instantaneous replenishment time gets extended according to increase in disruption time.

V. CONCLUSIONS

The production system experiences different types of interruptions. This study determines optimal production time initially when system runs smoothly without disruption and afterwards when system has some sort of disruption due to any reason. It is also taken care that if shortages occur due to disruption then how much new instantaneous replenishment should be made. Quadratic demand for deteriorating inventory units is studied. Graphical analysis is provided for a numerical example. Managers can take decisions from the results e.g. what should be the optimal production time if deterioration rate changes or disruption starts or demand changes. Future research can be done by incorporating various demand structures like stock dependent demand, price sensitive demand or fuzzy demand. Different preservation technologies can also be implemented to reduce deterioration.

REFERENCES

[1] Bakker, M., Riezebos, J. and Teunter, R. H., "Review of inventory systems with deterioration since 2001," European Journal of Operational Research. 221, 125-138, 2012.

[2] Chung, C. J., Widyadana, G. A. and Wee, H. M. "Economic production quantity model for deteriorating inventory with random machine unavailability and shortage," International Journal of Production Research. 49: 883-902, 2011.

[3] Goyal, S. K. and Giri, B. C., "Recent trend in modeling of deteriorating inventory," European Journal of the Operational Research Society. 134, 1-16, 2001.

[4] Khedlekar, U. K., "A disruption production model with exponential demand," International Journal of Industrial Engineering Computations, 3, 607-616, 2012.

[5] Lin, G.C. and Kroll, D. E., "Economic lot sizing for an imperfect production system subject to random breakdowns," Engineering Optimization. 38(1) 73-92, 2006.

[6] Lin, G.C and Gong D.C., "On a production-inventory model for deteriorating items subject to an imperfect process," Journal of the Chinese Institute of Industrial Engineers, 24(4), 319-326, 2007.

[7] Raafat, F., "Survey of literature on continuously deteriorating inventory models". Journal of the Operational Research Society. 40, 27-37, 1991.

[8] Shah, Nita H. and Shah, Y. K., "Literature survey on inventory models for deteriorating items," Economic Annals. 44, 221-237, 2000.

[9] Sheu, S. H. and Chen, J. A., "Optimal lot-sizing problem with imperfect maintenance and imperfect production," International Journal of Systems Science. 35(1): 69-77, 2004.

[10] Wee, H. M., "Economic production lot size model for deteriorating items with partial back ordering," Computers & Industrial Engineering. 24, 449-458, 1993.

AUHTOR'S PROFILE

Dr. Nita H. Shah is a professor in the Department of Mathematics, Gujarat University, Ahmedabad, India. She received her Ph.D. in inventory control management, operations research. Currently, she is engaged in research in inventory control and management, supply chain management, forecasting and information technology and information systems, neural networks, sensors and image processing. She has more than 275 papers published in international and national journals. She is author of four books. She is serving as a member of the editorial board of Investigation Operational, Journal of Social Science and Management, International Journal of Industrial Engineering and Computations and Mathematics Today.

Mr. Dushyantkumar G. Patel is a lecturer in mathematics at Government Polytechnic for Girls, Ahmedabad. He has published 12 research papers in international journals viz. Mexican Journal of Operations Research, ISRN Operations Research, International Journal of Mathematics and Mathematical sciences (IJMMS) etc. His area of interest includes inventory models.

Mr. Digeshkumar B. Shah is an assistant professor in the Department of Mathematics and Humanities, L.D. College of Engineering, Ahmedabad. He has published 15 research papers in international journals viz. International Journal of System Science (IJSS), OPSEARCH, Journal of Mathematical Modelling and Algorithms in Operations Research (JMMA), British Journal of Applied Science & Technology, etc. His areas of interest include operations research and mostly inventory models.