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An Oscillatory MHD Convective Flow of Viscoelastic Fluid through Porous Medium Filled in a Rotating Vertical Porous Channel with Heat Radiation

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Abstract ***An oscillatory MHD convective flow of an incompressible, viscoelastic and electrically conducting fluid in a vertical porous channel is analyzed. The two porous plates are subjected to a constant injection and suction velocity as shown in Fig.1. A magnetic field of uniform strength is applied perpendicular to the plates of the channel. The magnetic Reynolds number is assumed to be very small so that the induced magnetic field is negligible. The entire system rotates about an axis perpendicular to planes of the plates. The temperature difference between the plates is high enough to induce the heat due to radiation. A closed form solution of the purely oscillatory flow is obtained. The velocity, temperature and the skin-friction in terms of its amplitude and phase angle have been shown graphically to observe the effects of rotation parameter Ω, suction parameter λ, Grashoff number Gr, Hartmann number M, the pressure A, Prandtl number Pr, Radiation parameter N and the frequency of oscillation ω.***

Keywords: - **Injection/suction, Viscoelastic, convection, magneto hydro magnetic, oscillatory, rotating, radiation.**

I. INTRODUCTION

Magneto hydrodynamic (MHD) convection flows of electrically conducting viscous incompressible fluids in rotating system have gained considerable attention because of its numerous applications in physics and engineering. In geophysics it is applied to measure and study the positions and velocities with respect to a fixed frame of reference on the surface of earth which rotate with respect to an inertial frame in the presence of its magnetic field. The subject of geophysical dynamics nowadays has become an important branch of fluid dynamics due to the increasing interest to study environment. In astrophysics it is applied to study the stellar and solar structure, inter planetary and inter stellar matter, solar storms and flares etc. During the last few decades it also finds its application in engineering. Among the applications of rotating flow in porous media to engineering disciplines, one can find the food processing industry, chemical process industry, centrifugation and filtration processes and rotating machinery. In recent years a number of studies have also appeared in the literature on the fluid phenomena on earth involving rotation to a greater or lesser extent viz. Vidyanidhu & Nigam [1], Gupta [2], Jana & Datta [3]. Mazumder [4] obtained an exact solution of an oscillatory Couette flow in a rotating system. Thereafter Ganapathy [5] presented the solution for rotating Couetteflow. Singh [6] analyzed the oscillatory magneto hydrodynamic (MHD) Couette flow in a rotating system in the presence of transverse magnetic field. Algoa et al studied the radiative and free convective effects of a MHD flow through a porous medium between infinite parallel plates with time-dependent suction. Singh [7] also obtained an exact solution of magneto hydro dynamic (MHD) mixed convection flow in a rotating vertical channel with heat radiation. An unsteady hydro magnetic flow of a viscous, incompressible and electrically conducting fluid in a rotating channel with oscillating pressure gradient is analyzed by Seth & Jana [8]. Seth et al [9] studied an unsteady convective flow within a parallel plate rotating channel with thermal source/sink in a porous medium under slip condition. Chandran et al [10] studied the rotational effect on unsteady hydro magnetic Couette flow. Transient effect on magneto hydro dynamic Couette flow with rotation is investigated by Singh et al [11]. Prasad Rao et al [12] studied combined effect of free and forced convection on MHD flow in a rotating porous channel. Soundalgekar& Pop [13] analyzed hydro magnetic flow in a rotating fluid past an infinite porous plate. Singh & Mathew [14] studied the effects of injection/suction on an oscillatory hydro magnetic flow in a rotating horizontal porous channel. Singh &Garg [15] have also obtained exact solution of an oscillatory free convection MHD flow in a rotating channel in the presence of heat transfer due to radiation. The study of the flows of viscoelastic fluids is important in the fields of petroleum technology and in the purification of crude oils. In recent years, flows of visco-elastic fluids attracted the attention of several scholars in view of their practical and fundamental importance associated with many industrial applications. Literature is replete with the various flow problems considering variety of geometries such as Rajgopal [16], Rajgopal& Gupta [17], Ariel [18], and Pop&Gorla [19]. Hayat et al [20] discussed periodic unsteady flows of a non-Newtonian fluid. Choudhury and Das [21] studied the oscillatory viscoelastic flow in a channel filled with porous medium in the presence of radiative heat transfer. Singh [22] analyzed viscoelastic mixed convection MHD oscillatory flow through a porous medium filled in a vertical channel. Taking the rotating

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frame of reference into account Puri [23] investigated rotating flow of an elastic-viscous fluid on an oscillating plate. Puri & Kulshrestha [24] analyzed rotating flow of non-Newtonian fluids. Rajgopal [25] investigated flow of viscoelastic fluids between rotating disks. Applying quasi linearization to the problem Verma et al [26] analyzed steady laminar flow of a second grade fluid between two rotating porous disks. Hayat et al [27] studied fluctuating flow of a third order fluid on a porous plate in a rotating medium. Hayat et al [28] investigated the unsteady hydro magnetic rotating flow of a conducting second grade fluid. The objective of the present analysis is to study an oscillatory convection flow of an incompressible, electrically conducting and viscoelastic fluid in a vertical porous channel. Constant injection and suction is applied at the left and the right infinite porous plates respectively. The entire system rotates about an axis perpendicular to the planes of the plates and a uniform magnetic field is also applied along this axis of rotation. A general exact solution of the partial differential equations governing the flow problem is obtained and the effects of various flow parameters on the velocity field and the skin friction are discussed in the last section of the paper with the help of figures.

II. BASIC EQUATIONS

Consider the flow of a viscoelastic, incompressible and electrically conducting fluid in a rotating vertical channel. In order to derive the basic equations for the problem under consideration following assumptions are made:

(i) The two infinite vertical parallel plates of the channel are permeable and electrically non-conducting.
(ii) The vertical channel is filled with a porous medium.
(iii) The flow considered is fully developed, laminar and oscillatory.
(iv) The fluid is viscoelastic, incompressible and finitely conducting.
(v) All fluid properties are assumed to be constant except that of the influence of density variation with temperature is considered only in the body force term.
(vi) The pressure gradient in the channel oscillates periodically with time.
(vii) A magnetic field of uniform strength B_0 is applied perpendicular to the plates of the channel.
(viii) The magnetic Reynolds number is taken to be small enough so that the induced magnetic field is neglected.
(ix) Hall effect, electrical and polarization effects are also neglected.
(x) The temperature of a plate is non-uniform and oscillates periodically with time.
(xi) The temperature difference of the two plates is also assumed to be high enough to induce heat transfer due to radiation.
(xii) The fluid is assumed to be optically thin with relatively low density.
(xiii) The entire system (consisting of channel plates and the fluid) rotates about an axis perpendicular to the plates.

Under these assumptions we write hydro magnetic governing equations of motion and continuity in a rotating frame of reference as:

$$\nabla . v = \qquad (1)$$

$$\frac{\partial V}{\partial t} + (V . \nabla V) + 2\Omega \times V = -\frac{1}{\rho}\nabla p^{*} + \vartheta_1 \nabla^2 V + \frac{\vartheta_1}{K^{*}} V + \nabla . \exists + \frac{1}{\rho}(J \times B) + F , \quad 2)$$

$$\rho c_p \left[\frac{\partial T}{\partial t} + (V . \nabla) T\right] = k \nabla^2 T - ' . \qquad (3)$$

In equation (2) the last term on the left hand side is the Coriolis force. On the right hand side of (2) the last term $F (= g\beta T$ accounts for the force due to buoyancy. The second last term is the Lorentz Force due to magnetic field B and is given by

$$J \times B = \sigma(V \times B) \times \qquad (4)$$

and the modified pressure $p^{*} = p' - \frac{\rho}{2}|\Omega \times R|^2$, where R denotes the position vector from the axis of rotation, p' denotes the fluid pressure, J is the current density, and all other quantities have their usual meaning and have been defined from time to time in the text. In the term third from last of equation (2), $\exists$ is the Cauchy stress tensor and the constitutive equation derived by Coleman and Noll [29] for an incompressible homogeneous fluid of second order is

$$\exists = -p_1 I + \mu_1 A_1 + \mu_2 A_2 + \mu_3 A_1^2 . \qquad (5)$$

Here $-p_1 I$ is the interdeterminate part of the stress due to constraint of incompressibility, μ_1, μ_2 and μ_3 are the material constants describing viscosity, elasticity and cross-viscosity respectively. The kinematic A_1 and A_2 are the Rivelen Ericson constants defined as

$$A_1 = (\nabla \overline{V}) + (\nabla \overline{V})^T,$$
$$A_2 = \frac{dA_1}{dt} + (\nabla \overline{V})^T A_1 + A_1 (\nabla \overline{V}),$$

where ∇ denotes the gradient operator and d/dt the material time derivative. According to Markovitz and Coleman [30] the material constants μ_1, μ_3 are taken as positive and μ_2 as negative. The modified pressure $p^{*} = p' - \frac{\rho}{2}|\Omega \times R|^2$, where R denotes the position vector from the axis of rotation, p' denotes the fluid pressure.

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III. PROBLEM FORMULATION

In the present analysis we consider an unsteady flow of a viscoelastic incompressible and electrically conducting fluid bounded by two infinite vertical porous plates distance'd' apart. A coordinate system is chosen such that the X*-axis is oriented upward along the centerline of the channel and Z*-axis taken perpendicular to the planes of the plates lying in $z^* =$ planes. The fluid is injected through the porous plate at $z^* =$ with constant velocity w_0 and simultaneous sucked through the other porous plate at $z^* =$ with the same velocity w_0. The non-uniform temperature of the plate at $z^* =$ is assumed to be varying periodically with time. The temperature difference between the plates is high enough to induce the heat due to radiation. The Z*- axis is considered to be the axis of rotation about which the fluid and the plates are assumed to be rotating as a solid body with a constant angular velocity Ω*. A transverse magnetic field of uniform strength B (0, 0, B_0) is also applied along the axis of rotation. All physical quantities depend on z* and t* only for this problem of fully developed laminar flow. The equation of continuity $\nabla . V =$ gives on integration $w^* =$ .Then the velocity may reasonably be assumed with its components along x*, y*, z* directions as V (u*, v*, w_0). A schematic diagram of the flow problem is shown in figure 1. Following Attia [31] and under the usual Boussinesq approximation and by using the velocity and the magnetic field distribution as stated above the magneto hydrodynamic (MHD) flow in the rotating channel is governed by the following Cartesian equations:

$$\frac{\partial u^*}{\partial t^*} + w_0 \frac{\partial u^*}{\partial z^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} + \vartheta_1 \frac{\partial^2 u^*}{\partial z^{*2}} + \vartheta_2 \frac{\partial^3 u^*}{\partial z^{*2}\partial t^*} + 2\Omega^* v^* - \frac{\sigma B_0^2}{\rho} u^* - \frac{\vartheta_1 u^*}{K^*} + g\beta (T^* - T_1), \quad (5)$$

$$\frac{\partial v^*}{\partial t^*} + w_0 \frac{\partial v^*}{\partial z^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial y^*} + \vartheta_1 \frac{\partial^2 v^*}{\partial z^{*2}} + \vartheta_2 \frac{\partial^3 v^*}{\partial z^{*2}\partial t^*} - 2\Omega^* u^* - \frac{\sigma B_0^2}{\rho} v^* - \frac{\vartheta_1 u^*}{K^*}, \quad (6)$$

$$0 = -\frac{1}{\rho}\frac{\partial p^*}{\partial z^*}, \quad (7)$$

$$\frac{\partial T^*}{\partial t^*} + w_0 \frac{\partial T^*}{\partial z^*} = \frac{k}{\rho c_p}\frac{\partial^2 T^*}{\partial z^{*2}} - \frac{1}{\rho c_p}\frac{\partial q}{\partial z^*}. \quad (8)$$

where ρ is the density, ϑ_1 is the kinematic viscosity,ϑ_2 is visco elasticity, p^* is the modified pressure, t^* is the time, σ is the electric conductivity, B_0 is the component of the applied magnetic field along the z^*-axis, g is the acceleration due to gravity, k is the thermal conductivity, c_p is the specific heat at constant pressure and the last term in equation (8) is the radiative heat flux. Following Cogley et al [32] it is assumed that the fluid is optically thin with a relatively low density and the heat flux due to radiation in equation (8) is given by

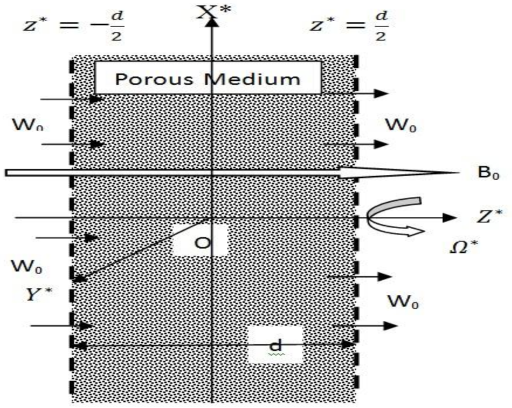

Fig.1. Physical configuration of the flow

$$\frac{\partial q}{\partial z^*} = 4\alpha^2 (T^* - T_1). \quad (9)$$

Where α is the mean radiation absorption coefficient? After the substitution of equation (9) equation (8) becomes

$$\frac{\partial T^*}{\partial t^*} + w_0 \frac{\partial T^*}{\partial z^*} = \frac{k}{\rho c_p}\frac{\partial^2 T^*}{\partial z^{*2}} - \frac{4\alpha^2}{\rho c_p}(T^* - T_1). \quad (10)$$

Equation (7) shows the constancy of the hydrodynamic pressure along the axis of rotation. We shall assume now that the fluid flows under the influence of pressure gradient varying periodically with time in the X^*-axis is of the form

$-\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} = A \cos\omega^* t^*$ and $-\frac{1}{\rho}\frac{\partial p^*}{\partial y^*} = 0$, (11)

where A is a constant.

The boundary conditions for the problem are

$z^* = \frac{d}{2}$: $u^* = v^* = 0$, $T^* = T_1 + (T_2 - T_1)\cos\omega^* t^*$ (12)

$z^* = -\frac{d}{2}$: $u^* = v^* = 0$, $T^* = T_1$, (13)

Where T_0 is the mean temperature and ω^* is the frequency of oscillations.

Introducing the following non-dimensional quantities:

$$\eta = \frac{z^*}{d}, x = \frac{x^*}{d}, y = \frac{y^*}{d}, u = \frac{u^*}{w_0}, v = \frac{v^*}{w_0}, T = \frac{T^* - T_1}{T_2 - T_1}, t = \frac{t^* w_0}{d}, \omega = \frac{\omega^* d}{w_0}, p = \frac{p^*}{\rho w_0^2}, \quad (14)$$

into equations (5), (6) and (10), we get

$$\lambda\left(\frac{\partial u}{\partial t} + \frac{\partial u}{\partial \eta}\right) = -\lambda \frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial \eta^2} + \gamma \frac{\partial^3 u}{\partial \eta^2 \partial t} + 2\Omega v - M^2 u - K^{-1} u + Gr\, T, \quad (15)$$

$$\lambda\left(\frac{\partial v}{\partial t} + \frac{\partial v}{\partial \eta}\right) = -\lambda \frac{\partial p}{\partial y} + \frac{\partial^2 v}{\partial \eta^2} + \gamma \frac{\partial^3 v}{\partial \eta^2 \partial t} - 2\Omega u - M^2 v - K^{-1} v, \quad (16)$$

$$\lambda Pr\left(\frac{\partial T}{\partial t} + \frac{\partial T}{\partial \eta}\right) = \frac{\partial^2 T}{\partial \eta^2} - N^2 T, \quad (17)$$

where '*' represents the dimensional physical quantities,

$\lambda = \frac{w_0 d}{\vartheta_1}$ is the injection/suction parameter,

$\gamma = \frac{\upsilon_2 \lambda}{d^2}$ is the visco-elastic parameter,

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$\Omega = \frac{\Omega^* d^2}{\vartheta_1}$ is the rotation parameter,

$M = B_0 d\sqrt{\frac{\sigma}{\rho\vartheta_1}}$ is the Hartmann number,

$K = \frac{K^*}{d^2}$ is the permeability of the porous medium,

$Gr = \frac{g\beta d^2 (T_2 - T_1)}{\vartheta_1 w_0}$ is the Grash of number,

$Pr = \frac{\rho\vartheta_1 c_p}{k}$ is the Prandtl number,

$N = \frac{2\alpha d}{\sqrt{k}}$ is the radiation parameter,

$\omega = \frac{\omega^* d}{w_0}$ is the frequency of oscillations.

The boundary conditions in the dimensionless form become

$$\eta = \tfrac{1}{2}: \quad u = v = 0,\ T = \cos\omega t, \qquad (18)$$

$$\eta = -\tfrac{1}{2}: \quad u = v = 0,\ T = 0. \qquad (19)$$

For the oscillatory internal flow we shall assume that the fluid flows under the influence of a non-dimension pressure gradient varying periodically with time in the direction of X-axis only which implies that

$$-\frac{\partial p}{\partial x} = A\cos \text{ and } \quad -\frac{\partial p}{\partial y} =. \qquad (20)$$

IV. SOLUTION OF THE PROBLEM

Now combining equations (15) and (16) into single equation by introducing a complex function of the form F = u + iv and with the help of equation (20), we get

$$\lambda\left(\frac{\partial F}{\partial t} + \frac{\partial F}{\partial \eta}\right) = -\lambda\frac{\partial p}{\partial x} + \frac{\partial^2 F}{\partial \eta^2} + \gamma\frac{\partial^3 F}{\partial \eta^2 \partial t} - (M^2 + K^{-1} + 2i\Omega)F + Gr\,T, \qquad (21)$$

with corresponding boundary conditions as

$$\eta = \tfrac{1}{2}: \quad F = 0,\ T = \cos\omega t, \qquad (22)$$

$$\eta = -\tfrac{1}{2}: \quad F = 0,\ T = 0. \qquad (23)$$

In order to solve equation (21) and (17) under boundary conditions (22) and (23) it is convenient to adopt complex notations for the velocity, temperature and the pressure as under:

$$F(\eta, t) = F_0(\eta)e^{i\omega t},\ T = \theta_0(\eta)\,e^{i\omega t},\ -\frac{\partial y}{\partial x} = Ae^{i\omega t} \qquad (24)$$

The solutions will be obtained in terms of complex notations, the real part of which will have physical significance.

The boundary conditions (22) and (23) in complex notations can also be written as

$$\eta = \tfrac{1}{2}: \quad F = 0,\ T = e^{i\omega t}, \qquad (25)$$

$$\eta = -\tfrac{1}{2}: \quad F = 0,\ T = 0. \qquad (26)$$

Substituting expressions (24) in equations (21) and (17), we get

$$(1 + i\omega\gamma)\frac{d^2 F_0}{d\eta^2} - \lambda\frac{dF_0}{d\eta} - (M^2 + K^{-1} + 2i\Omega + i\omega\lambda)F_0 = \lambda A - Gr\,\theta_0 \qquad (27)$$

$$\frac{d^2\theta_0}{d\eta^2} - \lambda Pr\frac{d\theta_0}{d\eta} - (N^2 + i\omega\lambda Pr)\theta_0 = 0. \qquad (28)$$

The transformed boundary conditions reduce to

$$\eta = \tfrac{1}{2}: \quad F_0 = 0,\ \theta_0 = 1, \qquad (29)$$

$$\eta = -\tfrac{1}{2}: \quad F_0 = 0,\ \theta_0 = 0. \qquad (30)$$

The solution of the ordinary differential equation (27) under the boundary conditions (29) and (30) gives the velocity field as

$$F(\eta, t) = \left[\begin{array}{c} \frac{\lambda A}{l}\left\{1 + \frac{e^{m\eta}\sinh\frac{n}{2} - e^{n\eta}\sinh\frac{m}{2}}{\sinh\left(\frac{m-n}{2}\right)}\right\} + \\ \frac{Gr}{4\sinh\left(\frac{m-n}{2}\right)\sinh\left(\frac{r-s}{2}\right)}\left\{\begin{array}{l}\left(\frac{e^{\frac{r-s}{2}}}{C_1} - \frac{e^{-\frac{r-s}{2}}}{C_2}\right)\left(e^{m\eta - \frac{n}{2}} - e^{n\eta - \frac{m}{2}}\right) \\ + \left(\frac{C_1 - C_2}{C_1 C_2}\right)\left(e^{m\eta + \frac{n}{2}} - e^{n\eta + \frac{m}{2}}\right)e^{-\frac{\lambda Pr}{2}}\end{array}\right\} \\ - \frac{Gr}{2\sinh\left(\frac{r-s}{2}\right)}\left(\frac{e^{r\eta - \frac{s}{2}}}{C_1} - \frac{e^{s\eta - \frac{r}{2}}}{C_2}\right)\end{array}\right]e^{i\omega t}, \qquad (31)$$

where $l = (M^2 + K^{-1} + 2i\Omega + i\omega\lambda)$,

$C_1 = (1 + i\omega\gamma)r^2 - \lambda r - l$,

$C_2 = (1 + i\omega\gamma)s^2 - \lambda s - l$,

$m = \frac{\lambda + \sqrt{\lambda^2 + 4l(1 + i\omega\gamma)}}{2}$,

$n = \frac{\lambda - \sqrt{\lambda^2 + 4l(1 + i\omega\gamma)}}{2}$,

$r = \frac{\lambda Pr + \sqrt{\lambda^2 Pr^2 + 4(N^2 + i\omega\lambda Pr)}}{2}$,

$s = \frac{\lambda Pr - \sqrt{\lambda^2 Pr^2 + 4(N^2 + i\omega\lambda Pr)}}{2}$.

Similarly, the solution of equation (28) for the temperature field can be obtained under the boundary conditions (29) and (30) as

$$T(\eta, t) = \left(\frac{e^{r\eta - \frac{s}{2}} - e^{s\eta - \frac{r}{2}}}{2\sinh\left(\frac{r-s}{2}\right)}\right)e^{i\omega t}. \qquad (32)$$

From the velocity field obtained in equation (31) we can get the skin-friction τ_L at the left plate (η = -0.5) in terms of its amplitude $|F|$ and phase angle φ as

$$\tau = |F|\cos(t + \varphi), \text{ with} \qquad (33)$$

$$F = F_r + iF_i = \left[\begin{array}{c} \frac{\lambda A}{l}\left(\frac{me^{-\frac{m}{2}}\sinh\frac{n}{2} - ne^{-\frac{n}{2}}\sinh\frac{m}{2}}{\sinh\left(\frac{m-n}{2}\right)}\right) + \\ \frac{Gr}{4\sinh\left(\frac{m-n}{2}\right)\sinh\left(\frac{r-s}{2}\right)}\left\{\begin{array}{l}\left(\frac{e^{\frac{r-s}{2}}}{C_1} - \frac{e^{-\frac{r-s}{2}}}{C_2}\right)(m - n)e^{-\frac{\lambda}{2(1+i\omega\gamma)}} + \\ \left(\frac{C_1 - C_2}{C_1 C_2}\right)\left(me^{-\frac{m-n}{2}} - ne^{\frac{m-n}{2}}\right)e^{-\frac{\lambda Pr}{2}}\end{array}\right\} \\ - \frac{Gr}{2\sinh\left(\frac{r-s}{2}\right)}\left(\frac{r}{C_1} - \frac{s}{C_2}\right)e^{-\frac{\lambda Pr}{2}}\end{array}\right]. \qquad (34)$$

The amplitude is $|F| = \sqrt{F_r^2 + F_i^2}$ and the phase angle is $\varphi = \tan^{-1}\frac{F_i}{F_r}$. (35)

Similarly the Nusselt number Nu in terms of its amplitude $|H|$ and the phase angle ψ can be obtained from equation (32) for the temperature field as

$$q = |H|\cos(t + \psi), \qquad (36)$$

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Where $Hr + i\,Hi = \frac{(r-s)e^{-\frac{r+s}{2}}}{2\sinh\left(\frac{r-s}{2}\right)}$, (37)

Where the amplitude $|H|$ and the phase angle ψ of the rate of heat transfer are given as

$$|H| = \sqrt{Hr^2 + Hi^2}, \psi = \tan^{-1}\frac{Hi}{Hr}. \quad (38)$$

The temperature field, amplitude and phase of the Nusselt number need no further discussion because these have already been discussed in detail by Singh [33].

V. RESULT & DISCUSSION

The problem of oscillatory magneto hydrodynamic convective and radiative MHD flow in a vertical porous channel is analyzed. The viscoelastic, incompressible and electrically conducting fluid is injected through one of the porous plates and simultaneously removed through the other porous plate with the same velocity. The entire system (consisting of Porous channel plates and the fluid) rotates about an axis perpendicular to the plates. The closed form solutions for the velocity and temperature fields are obtained analytically and then evaluated numerically for different values of parameters appeared in the equations. To have better insight of the physical problem the variations of the velocity, the skin-friction in terms of its amplitude and phase angle are evaluated numerically for different sets of the values of injection/suction parameter λ, viscoelastic parameter γ, rotation parameter Ω, Hartmann number M, the permeability parameter K, Grash of number Gr, Prandtl number Pr, radiation parameter N, pressure gradient P, and the frequency of oscillations ω. These numerical values are then shown graphically to assess the effect of each parameter for the two cases of rotation Ω = 5 (small) and Ω = 15 (large). The velocity variations are presented graphically in Figs. 2 to 11 for γ=0.2, λ=0.5, Gr=5, M=2, K=0.2, Pr=0.7, N=1, A=5, ω=5 and t=0 except the ones shown in these figures.

Fig.2 illustrates the variation of the velocity with the increasing rotation of the system. It is quite obvious from this figure that velocity goes on decreasing with increasing rotation Ω of the entire system. The velocity profiles initially remain parabolic with maximum at the centre of the channel for small values of rotation parameter Ω and then as rotation increases the velocity profiles flatten. For further increase in Ω (= 15) the maximum of velocity profiles no longer occurs at the centre but shift towards the walls of the channel. It means that for large rotation there arise boundary layers on the walls of the channel. The variations of the velocity profiles with the viscoelastic parameter γ are presented in Fig.3. It is very clear from this figure that the velocity decreases with the increase of viscoelastic parameter for both the cases of small (Ω = 5) and large (Ω = 15) rotations of the channel. However, for both these cases of rotation, the velocity increases significantly with increasing injection/suction parameter λ as is revealed in Fig.4. The variations of the velocity profiles with the Grashof number Gr are shown in Fig.5. Although the velocity for large rotation (Ω=15) is much less than the velocity for small rotation (Ω=5) but in both these cases velocity increases with the increasing Grash of number Gr. The maximum of the velocity profiles shifts towards right half of the channel due to the greater buoyancy force in this part of the channel because of the presence of hotter plate. The variation of the velocity profiles with Hartmann number M is presented in Fig.6. For small Ω (=5) and large Ω (=15) both the velocity goes on increasing with increasing M. This means that the backward flow caused by the rotation of the system is resisted by the increasing Lorentz force due to increasing magnetic field strength and the visco elasticity of the fluid. Fig.7.depicts the variations of the velocity with the permeability of the porous medium K. It is observed from the figure that the velocity decrease with the increase of K for both small (Ω=5) and large (Ω=15) rotations of the system. We also find from Fig.8 and Fig.9 respectively that with the increase of Prandtl number Prand the radiation parameter N the velocity decreases for small (Ω=5) and large (Ω=15) rotations both. From Fig.10, it is evident that the velocity goes on increasing with the increasing favorable pressure gradient P (> 0). The velocity profiles for small rotation (Ω=5) remain parabolic with maximum at the centerline. But for large rotation (Ω=15) the velocity profiles flatten. The effects of the frequency of oscillations ω on the velocity are exhibited in Fig.11. It is noticed that velocity decreases with increasing frequency ω for either case of channel rotation large or small.

The skin-friction τ_L in terms of its amplitude $|F|$ and phase angle φ has been shown in Figs. 12 and 13 respectively. The effect of each of the parameter on $|F|$ and φ is assessed by comparing each curve with dotted curves in these figures. In Fig.12 the comparison of the curves IV, V, VII and X with dotted curve I indicates that the amplitude increases with the increase of injection/suction parameter λ, the Grashof number Gr, permeability of the porous medium K and the pressure gradient parameter A. Similarly the comparison of other curves II, III, VI, VIII and IX with the dotted curve I reveals that the skin-friction amplitude decreases with the increase of rotation parameter Ω, viscoelastic parameter γ, Hartmann number M, Prandtl number Pr and the radiation parameter N. It is obvious that $|F|$ goes on decreasing with increasing frequency of oscillations ω. Fig. 13 showing the variations of the phase angle φ of the skin-friction it is clear that there is always a phase lag because the values of φ remains negative throughout. Here again the comparison of curves II, III, V and VII with the dotted curve (---) I indicates that the phase lag increases with the increase of rotation parameter Ω, visco elastic parameter γ, Grash of number Gr and permeability of the porous medium K. The comparison of other curves like IV, VI, VIII, IX and X with the dotted curve (---) I shows that the phase lag decreases with the increase of injection/suction parameter λ, Hartmann number M, pertaining to registration on the topic Prandtl number Pr, radiation parameter N and the pressure gradient parameter A. Phase lag goes on increasing with increasing frequency

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of oscillations ω but the trend reverses slightly for large frequency.

VI. CONCLUSIONS

An oscillatory hydro magnetic convective flow of viscous incompressible and electrically conducting fluid in a vertical porous channel is investigated when the entire system consisting of channel plates and the fluid rotates about an axis perpendicular to the plates. A closed form solution of the problem is obtained. It is found that with the increasing rotation of the channel the velocity decreases and the maximum of the parabolic velocity profiles at the centre of the channel shifts towards the walls of the channel. The velocity increases with the increase of the injection/suction parameter, Grash off number and the pressure gradient. However, the velocity decreases with the increase of radiation parameter, Prandtl number and the frequency of oscillation. The increase of Hartmann number i.e. increasing magnetic field strength resists the deceleration of the flow due to rotation and the visco elasticity of the fluid. The amplitude increases with the increase of injection/suction parameter λ, the Grash off number Gr, and the pressure gradient parameter A. There is always a phase lag of the skin friction.

NOMENCLATURE

B_0	magnetic field applied
c_p	specific heat at constant pressure
g	gravitational force
Gr	Grash of number
k	thermal conductivity
M	Hartmann number
N	heat radiation parameter
P	a constant
p	pressure
Pr	Prandtl number
t	time variable
T	fluid temperature
T_0	constant temperature
u, v, w	velocity components along X, Y, Z- directions
w_0	injection/suction velocity
x, y, z	variables along X, Y, Z-directions

Greek symbols

α	Mean radiation absorption coefficient
β	Coefficient of volume expansion
λ	injection/suction parameter
ω	Frequency of oscillations
ϑ_1	Kinematic viscosity
ρ	Fluid density
Ω	rotation parameter
τ_L	Skin-friction at the left wall
φ	Phase angle of the skin-friction
θ_0	Mean non-dimensional temperature
*	Superscript representing dimensional quantities

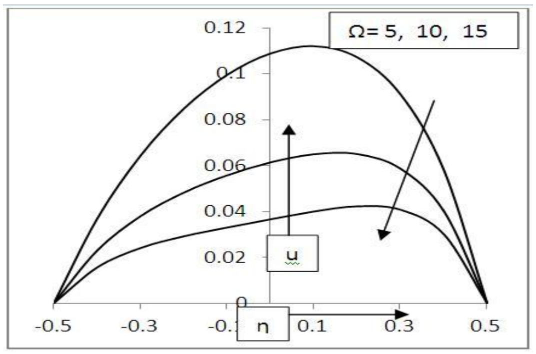

Fig.2.Velocity variations with Ω.

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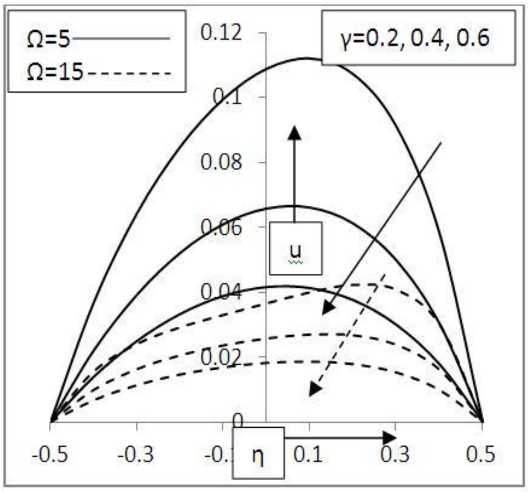

Fig.3.Velocity variations with γ.

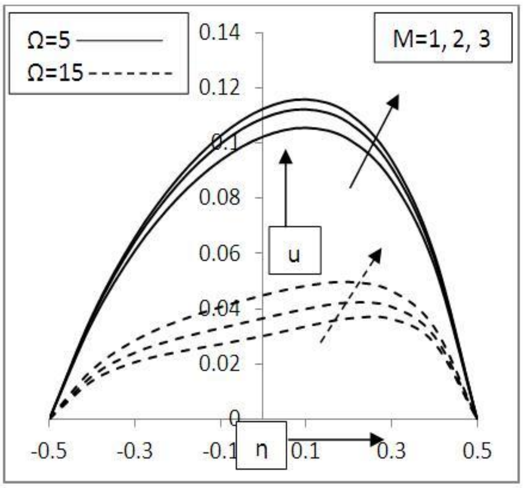

Fig. 6.Velocity variations with M.

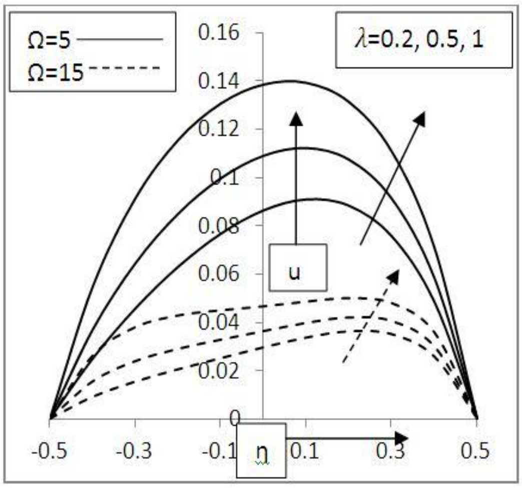

Fig.4.Velocity variations with λ.

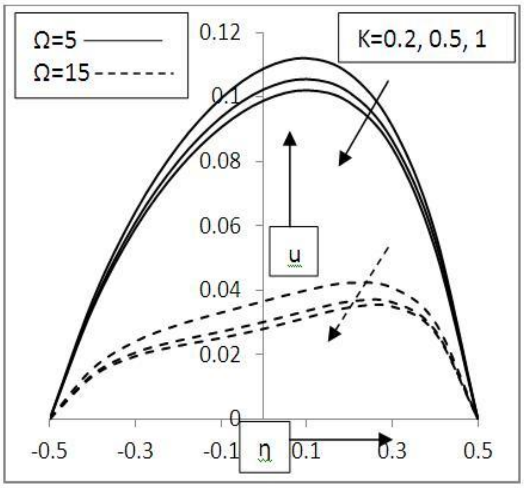

Fig.7.Velocity variations with K

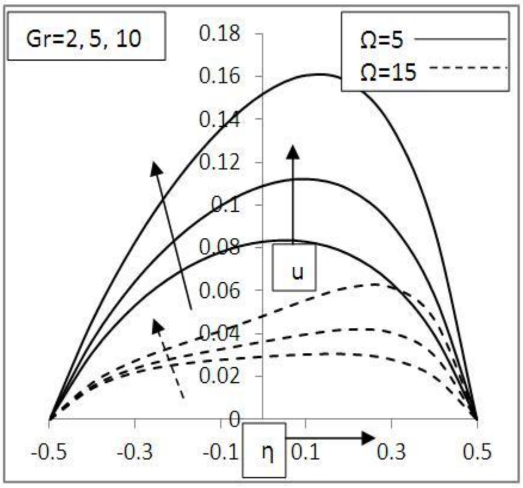

Fig.5.Velocity variations with Gr.

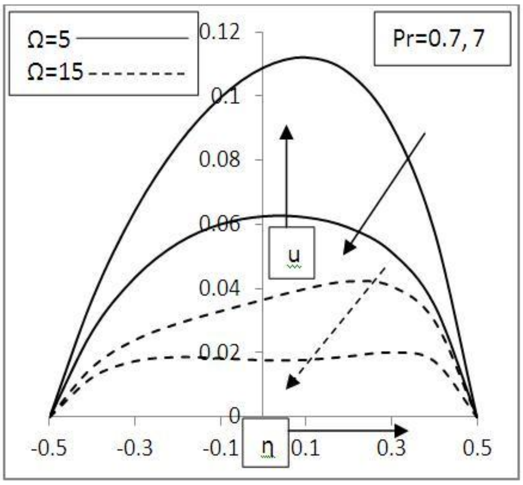

Fig.8.Velocity variations with Pr.

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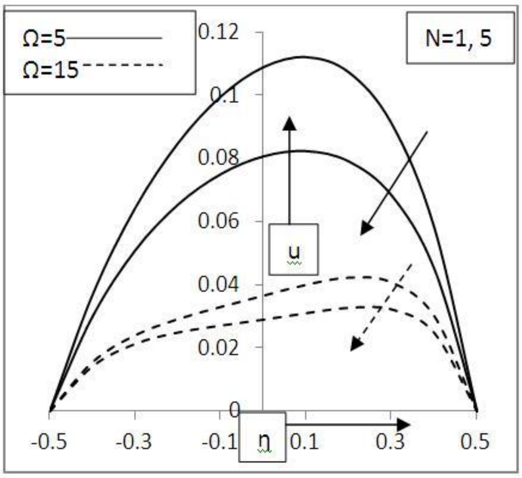

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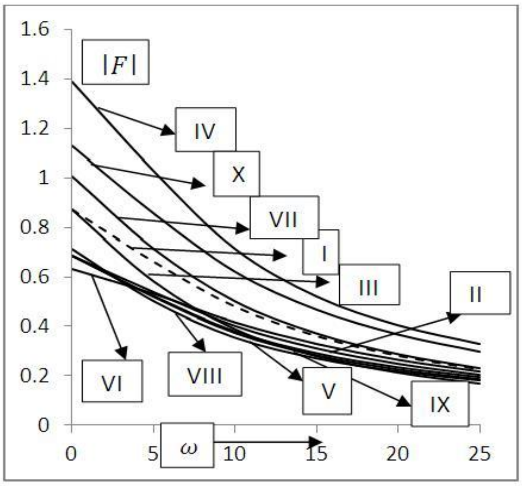

Fig.12. Amplitude of the skin friction.

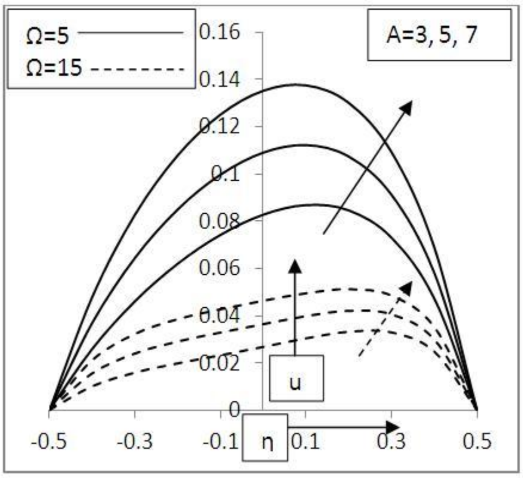

Fig.10.Velocity variations with A.

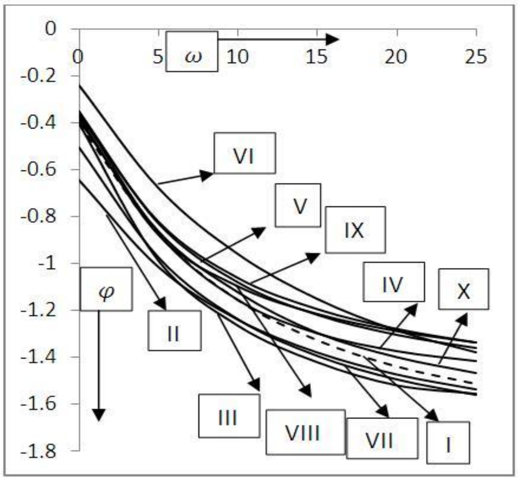

Fig.13. Phase of the skin friction.

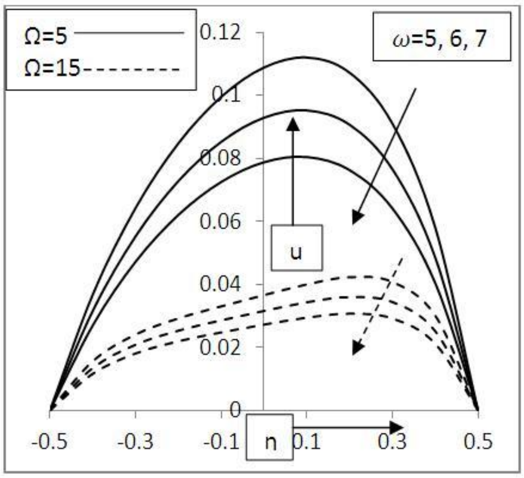

Fig.11.Velocity variations with ω.

Table 1. Values of parameters plotted in Figs.12 & 13.

Ω	γ	λ	Gr	M	K	Pr	N	A	
5	0.2	0.5	5	2	0.2	0.7	1	5	I(---)
10	0.2	0.5	5	2	0.2	0.7	1	5	II
5	0.3	0.5	5	2	0.2	0.7	1	5	III
5	0.2	1.0	5	2	0.2	0.7	1	5	IV
5	0.2	0.5	1	2	0.2	0.7	1	5	V
5	0.2	0.5	5	4	0.2	0.7	1	5	VI
5	0.2	0.5	5	2	1.0	0.7	1	5	VII
5	0.2	0.5	5	2	0.2	7.0	1	5	VIII
5	0.2	0.5	5	2	0.2	0.7	5	5	IX
5	0.2	0.5	5	2	0.2	0.7	1	7	X