ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 3, Issue 12, June 2014

Optimal Preservation Technology Investment, Credit Period and Ordering Policies for Deteriorating Inventory under Credit Period Dependent Quadratic Demand

Nita H. Shah, Digesh Kumar B. Shah and Dushyant Kumar G. Patel
Department of Mathematics, Gujarat University, Ahmedabad-380009, Gujarat, India
Department of Mathematics, L. D. College of Engg., Ahmedabad-380015, Gujarat, India
Department of Mathematics, Govt. Poly. For Girls, Ahmedabad-380015, Gujarat, India

Abstract*—The supplier presents its buyer a credit period to settle the account which attracts more buyers and increases market demand. However, the offer of credit period invites default risk for the supplier. In this paper, we also implement efficient preservation technology for our inventory system of deteriorating items. The demand is considered to be quadratic and is dependent of permissible trade credit. The objective is to maximize the total profit per unit time with respect to optimal investment to be made, credit period and procurement quantity. Numerical example is given to illustrate the theoretical results and concavity of total profit is established. Managerial observations are outlined using sensitivity analysis.

Keywords—**Trade Credit, Deterioration, Preservation Technology Investment, Quadratic Demand.**

I. INTRODUCTION

Harris-Wilson's EOQ model assumes that a supplier pays instantly when he gets delivery for the inventory products. But if the buyer has an option to pay after words without interest charges, then he attracts to buy. And this may be proved to be an excellent strategy in today's competitive scenario.

But at the same time, there may be a chance of default risk for the supplier if buyer can't pay his dues. Obviously, the lengthy delay period increases the default risk. Goyal [5] first derived an inventory model considering delay period. Shah et al. [12] gave a review article on inventory modeling with trade credits. Citations in the review articles are studied from the buyer's point of view when supplier offers a predetermined credit period. Jaggi et al. [8] developed retailer's decision with credit linked demand under permissible delay in payments.

In this model, deterioration added by Shah et al. [11]. Lou and Wang [9] found optimal trade credit and order quantity when trade credit impacts on both demand rate and default risk. They had deterministic constant demand. Teng and Lou [14] analyzed up-stream and down-stream trade credits in EOQ model. Chern et al. [2] gave stackel berg solution in a vendor − buyer supply chain model with permissible delay in payments. Mostly the conclusion from all the articles says that demand rate increases with length of trade credit. Wee [15] defined deterioration as decay, damage, spoilage, evaporation, obsolescence, pilferage, loss of utility or loss of marginal value of commodities that reduces utility. Food grains, fruits and vegetables, beverages, medicines, radioactive chemicals, fertilizers and pesticides, volatile liquids like spray and perfumes, passenger seats are examples of deteriorating products, whose usefulness decreases during the regular storage. Raafat [10], Shah and Shah [13], Goyal and Giri [4], Bakker et al. [1] formulated a review of literature on deteriorating inventory model. Our main goal of the paper is to control deterioration of items by taking appropriate measures viz. architecting efficient preserving technology. This will reduce unnecessary waste, minimize economic losses and improve business rivalry.

Hsu et al. [7] developed an inventory model with preservation technology investment to minimize the deterioration rate of inventory for constant demand. Dye and Hsieh [3] derived an optimal replenishment policy for deteriorating items with effective investment in preservation technology. Hsein and Dye [6] derived a production inventory model incorporating the effect of preservation technology investment when demand is fluctuating with time. In this paper, we analyze an EOQ model for the supplier to integrate facts: (1) trade-credit dependent quadratic demand and (2) default risk due to offer of trade credit. (3) Optimal preservation technology investment for the deteriorating inventories.

Then we set up the necessary condition for finding the optimal solution, check how sensitive the optimal solution is to the various inventory parameters and deduce some managerial insights. The non-linearity of objective function renders us to obtain a closed form solution to the supplier's optimal credit period, purchase quantity and preservation technology investment. An algorithm is presented to obtain the supplier's optimal decision variables. The model is validated by giving the numerical example.

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 3, Issue 12, June 2014

II. NOTATIONS AND ASSUMPTIONS

A. Notations

A	Ordering cost per order
C	Purchase cost per unit
P	Selling price per unit ; where $P > C$
h	Holding cost per unit per annum
M	Credit period offered by the supplier to his buyers (a decision variable)
$R(M,t)$	Time and credit period dependent annual demand rate
$I(t)$	Inventory level at any instant of time t, $0 \le t \le T$
θ	Deterioration rate under natural environment, $0 \le \theta \le 1$
u	Preservation technology investment per unit time to reduce deterioration rate (a decision variable)
$f(u)$	Proportion of reduced deterioration rate, where $0 \le f(u) \le 1$
T	Cycle time (a decision variable)
Q	Supplier's purchase quantity
$\pi(M,T,u)$	Supplier's profit per unit time

Assumptions

1. The inventory system has single item.
2. The items in inventory deteriorate at a constant rate. There is no repair or replacement of deteriorated items during the cycle time under consideration.
3. The replenishment rate is infinite.
4. Lead-time is zero or negligible. Shortages are not allowed.
5. In global market, supplier keeps selling price constant to bind his retailers.
6. As stated in Teng and Lou ([14]), trade credit is similar to price discount. We consider demand rate to be function of time and credit period as

$$R(M,t) = a\left(1 + bt - ct^2\right)M^{\beta} \quad (1)$$

Where $a > 0$ is scale demand, $0 \le b < 1$ denotes linear rate of change of demand with respect to time, $0 \le c < 1$ denotes quadratic rate of change of demand and $\beta > 0$ is constant.

7. For supplier, default risk increases if longer credit period is presented to the buyer. Here, the rate of default risk giving the credit period M is taken to be

$$F(M) = 1 - M^{-\delta} \quad (2)$$

where $\delta > 0$ is a constant.

8. The proportion of reduced deterioration rate $f(u)$ is assumed to be continuous, increasing and concave function of capital investment in preservation technology u, i.e. $f'(u) > 0$, $f''(u) < 0$. We can set $f(0) = 0$.

III. MATHEMATICAL MODEL

The supplier's inventory is depleting due to quadratic demand and offer of credit period. The items in the inventory system deteriorate at constant rate θ. When the player devotes u in the preservation technology, the fraction of reduced deterioration rate $f(u)$ will appear. The differential equation governing the inventory level is given by

$$\frac{dI(t)}{dt} = -aM^{\beta}\left(1 + bt - ct^2\right) - \theta\alpha I(t); \text{where } \alpha = 1 - f(u) \quad (3)$$

with $I(T) = 0$.

Then solution of differential equation (3) is given by

$$I(t) = \frac{-M^{\beta}a}{\theta^3\alpha^3}\left(\begin{array}{l}\left(\theta^2\alpha^2\left(cT^2 - Tb - 1\right) - 2\theta\alpha\left(-\frac{b}{2} + Tc\right) + 2c\right)\\ + \left(\theta^2\alpha^2\left(1 + bt - ct^2\right) - \theta\alpha\left(b - 2ct\right) - 2c\right)\end{array}\right) \quad (4)$$

At the start, the supplier has Q units in inventory system i.e. $Q = I(0)$

$$= \frac{-M^{\beta}a}{\theta^3\alpha^3}\left(\begin{array}{l}\left(\theta^2\alpha^2\left(cT^2 - Tb - 1\right) - 2\theta\alpha\left(-\frac{b}{2} + Tc\right) + 2c\right)\\ + \theta^2\alpha^2 - \theta\alpha b - 2c\end{array}\right) \quad (5)$$

The applicable costs per cycle for the supplier are

- Net revenue after default risk:

$$SR = P\int_0^T R(M,t)\,dt\,M^{-\delta} = -\frac{1}{6}PM^{-\delta+\beta}aT\left(2cT^2 - 3Tb - 6\right)$$

- Purchase cost ; $PC = CQ$
- Ordering cost ; $OC = A$
- Holding cost ; $HC = h\int_0^T I(t)\,dt$
- Preservation Technology Investment; $PTI = uT$

Therefore, the supplier's annual profit per unit time is given by

$$\pi(M,T,u) = \frac{1}{T}\left[SR - PC - OC - HC - PTI\right] \quad (6)$$

For making annual profit maximum per unit time with respect to credit period, cycle time and preservation technology investment, the necessary conditions are

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 3, Issue 12, June 2014

$$\frac{\partial \pi(M,T,u)}{\partial M}=0\ ;\ \frac{\partial \pi(M,T,u)}{\partial T}=0;\frac{\partial \pi(M,T,u)}{\partial u}=0 \qquad (7)$$

Also we check the condition $\begin{vmatrix} \frac{\partial^2 \pi}{\partial M^2} & \frac{\partial^2 \pi}{\partial M \partial T} & \frac{\partial^2 \pi}{\partial M \partial u} \\ \frac{\partial^2 \pi}{\partial T \partial M} & \frac{\partial^2 \pi}{\partial T^2} & \frac{\partial^2 \pi}{\partial T \partial u} \\ \frac{\partial^2 \pi}{\partial u \partial M} & \frac{\partial^2 \pi}{\partial u \partial T} & \frac{\partial^2 \pi}{\partial u^2} \end{vmatrix} < 0$

The non-linearity of equation (6) and its partial derivatives will not allow us to derive closed form of solution. So, we suggest following solution procedure:

Step 1: Allocate values to the inventory parameters.
Step 2: Work out equations in (7) simultaneously with Mathematical software. We have used Maple XIV.
Step 3: Check second order (sufficiency) conditions.
Step 4: Calculate profit $\pi(M,T,u)$ per unit time from Equation (6) and purchase quantity Q using equation (5).

IV. NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS

Example: 1 Consider the reduced deterioration rate as $f(u)=1-e^{-\gamma u}, \gamma>0$ (Hsu *et al.* [7] and Dye and Hsieh [3]).

Let $a=50$ units, $\beta=2$, $b=50\%$, $c=20\%$, $\delta=1.5$, $C=\$10$ per unit, $P=\$20$ per unit, $h=\$2$ per unit per annum, $A=\$50$ per order, $\theta=50\%$ and $\gamma=0.05$.

We obtain optimal credit period $M=0.5332$ year, cycle time $T=1.7035$ years and preservation technology investment $u=\$31.54$ for the supplier. This results in profit of $\$614.01$ with purchase of 32.73 units. The concavity of the total profit per unit time is shown in figure 1 w. r. t. u for $M=0.5332$ and $T=1.7035$, in figure 2 w. r. t. T for $M=0.5332$ and $u=31.54$ and in figure 3 w. r. t. M for $T=1.7035$ and $u=31.54$.

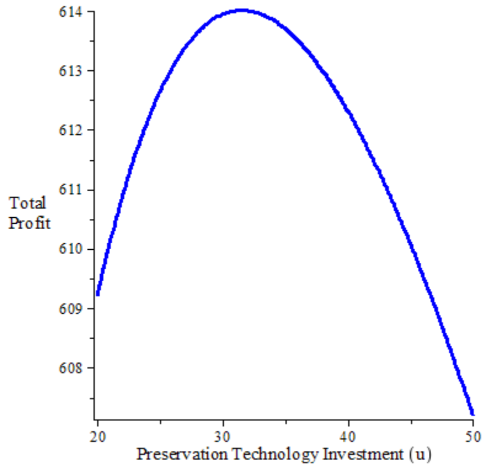

Fig. 1 Concavity of Total Profit w. r. t. Preservation Technology Investment (*u*)

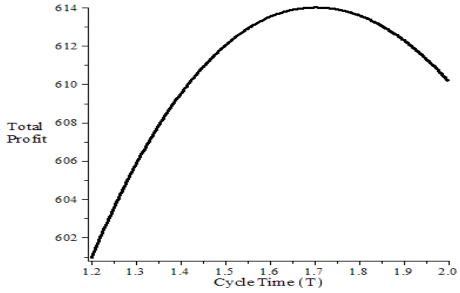

Fig. 2 Concavity of Total Profit w. r. t. Cycle Time (T)

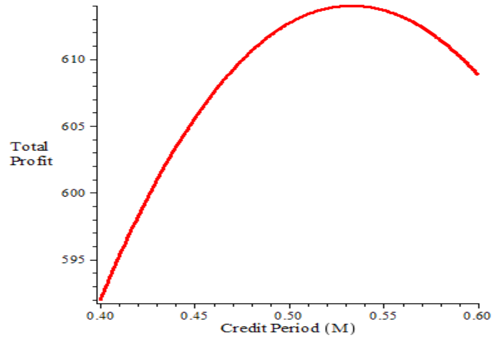

Fig. 3 Concavity of Total Profit w. r. t. Credit Period (*M*)

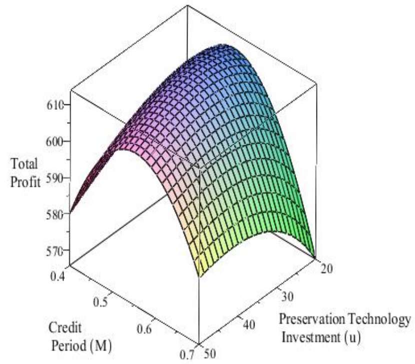

Fig. 4 Concavity of Total Profit w. r. t. Credit Period (*M*) and Preservation Technology Investment (*u*)

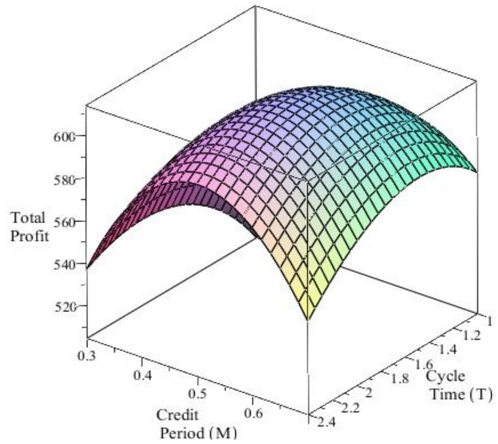

Fig. 5 Concavity of Total Profit w. r. t. Credit Period (*M*) and Cycle Time (*T*)

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 3, Issue 12, June 2014

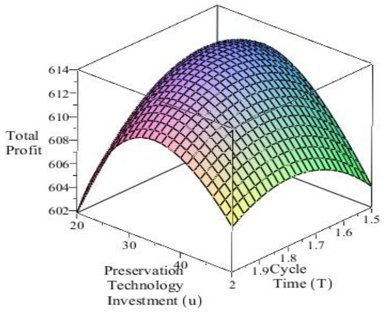

Fig. 6 Concavity of Total Profit w. r. t. Preservation Technology Investment (*u*) and Cycle Time (*T*)

Now, we study changes in optimal solutions and profit function by varying inventory parameters as -40%,-20%,+20%,+40%; one at time. The variations in credit period, cycle time, total profit, purchase quantity and preservation technology investment are shown in figure 7-11 respectively. It is observed from fig. 7 that credit period has large impact of retail price and trade credit elasticity. It decreases for unit purchase cost and rate of change of demand; *b*. This validate so long said argument that delay period increases demand. Fig. 8 shows that increase in rate of change of demand *b* boosts cycle time whereas increase in trade credit elasticity decreases cycle time. Ordering cost has positive impact on cycle time. Fig 9 reveals that increase in retail price, scale demand and rate of change of demand have deep impact on total profit and contrast to this increase in trade credit elasticity and purchase cost reduce total profit. Fig 10 describe that purchase quantity get increase with increase in trade credit elasticity, retail price, rate of change in demand and scale demand whereas it decreases with increase in holding cost and purchase cost. Fig 11 says that we have to invest more in preservation technology investment with increase in trade credit elasticity, rate of change of demand, retail price and the obvious parameter deterioration rate. Whereas the impact of mark up trade credit, holding cost and purchase cost is negative on preservation technology investment. The conduct of the demand with time and delay period is shown in fig. 12.

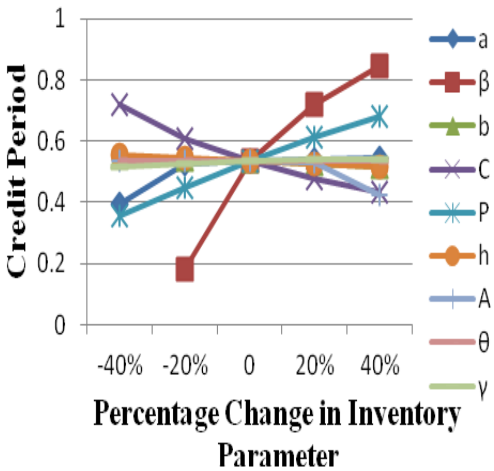

Fig. 7 Sensitivity analysis for credit period (*M*)

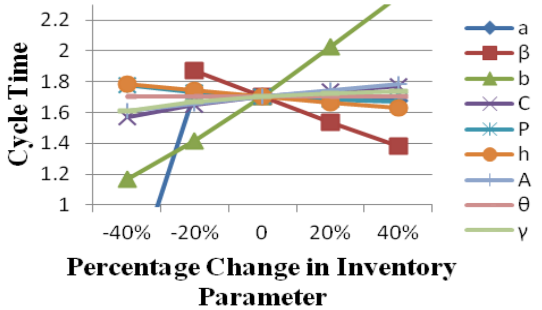

Fig. 8 Sensitivity analysis for cycle time (*T*)

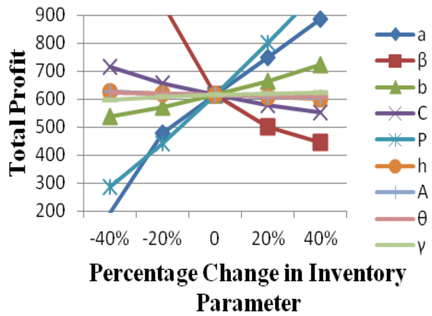

Fig. 9 Sensitivity analysis for total profit

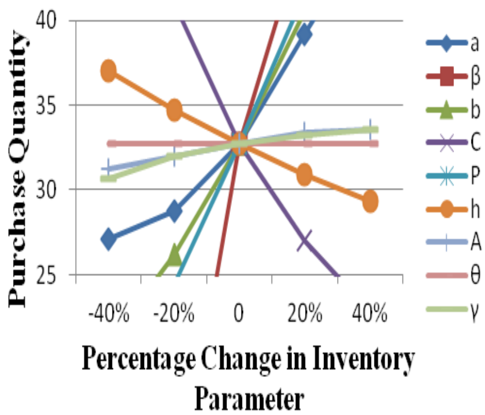

Fig. 10 Sensitivity analysis for Purchase quantity (*Q*)

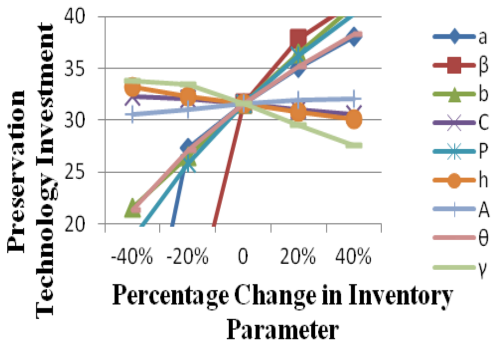

Fig. 11 Sensitivity analysis for preservation technology investment (*u*)

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 3, Issue 12, June 2014

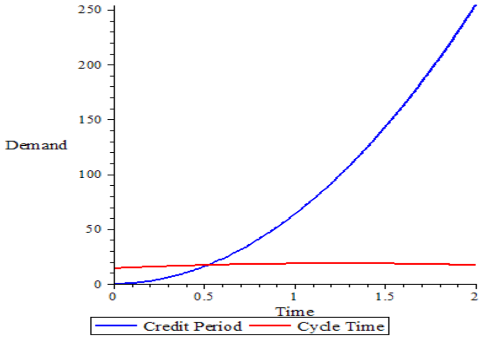

Fig. 12 Demand vs. Time

V. CONCLUSIONS

The judgment about setting allowable credit period is tough for the supplier. The longer credit period involves higher default risk. This paper discusses above concerns. The non-linear profit function is derived. The inventory system deals with quadratic demand. The necessary conditions are shaped to search out optimal solution based on the data. This research will help extensively to the supplier for setting optimal credit period and for deploying optimal investment in preservation technology to cope up with deterioration. For future research, this concept can be extended for stochastic demand or fuzzy demand. One can think of allowing shortages or can add risk analysis to give more realistic results.

REFERENCES

[1] Bakker, M., Riezebos, J. and Teunter, R. H., "Review of inventory systems with deterioration since 2001," European Journal of Operational Research. 221, 125-138, 2012.

[2] Chern, M. S., Pan, Q., Teng, J. T., Chan, Y. L. and Chen, S. C., "Stackelberg solution in a vendor–buyer supply chain model with permissible delay in payments," International Journal of Production Economics 144, 397-404, 2013.

[3] Dye, C. Y. and Hsieh, T. P., "An optimal replenishment policy for deteriorating items with effective investment in preservation technology," European Journal of Operational Research 218, 106-112, 2012.

[4] Goyal, S. K. and Giri, B. C., "Recent trend in modeling of deteriorating inventory," European Journal of the Operational Research Society 134, 1-16, 2001.

[5] Goyal, S. K., "Economic order quantity under conditions of permissible delay in payments," Journal of the Operational Research Society, 36(4), 335-338, 1985.

[6] Hsieh, T. P. and Dye, C. Y., "A production inventory model incorporating the effect of preservation technology investment when demand is fluctuating with time," Journal of Computational and Applied Mathematics 239, 25-36, 2013.

[7] Hsu P. H., Wee, H. M. and Teng, H. M., "Preservation technology investment for deteriorating inventory." International Journal of Production Economics 124, 338-394, 2010.

[8] Jaggi, C. K., Goyal, S. K. and Goel S. K., "Retailer's optimal replenishment decisions with credit-linked demand under permissible delay in payments," European Journal of Operational Research 190(1), 130-135, 2008.

[9] Lou, K. R. and Wang, W.C., "Optimal trade credit and order quantity when trade credit impacts on both demand rate and default risk," Journal of the Operational Research Society 64, 1551-1556, 2013.

[10] Raafat, F., "Survey of literature on continuously deteriorating inventory models," Journal of the Operational Research Society 40, 27-37, 1991.

[11] Shah, N. H., Shukla, K. T. and Shah, B. J., "Deteriorating inventory model for two-level credit linked demand under permissible delay in payments," Australian Society of Operations Research Bulletin 28(4), 27-36, 2009.

[12] Shah, N. H., Soni, H. N. and Jaggi, C. K., "Inventory model and trade credit: Review," Control and Cybernetics, 39(3), 867-884, 2010.

[13] Shah, Nita H. and Shah, Y. K., "Literature survey on inventory models for deteriorating items," Economic Annals 44, 221-237, 2000.

[14] Teng, J. T. and Lou, K. R., "Seller's optimal credit period and replenishment time in a supply chain with up-stream and down-stream trade credits," Journal of Global Optimization 53(3), 417-430, 2012.

[15] Wee, H. M., "Economic production lot size model for deteriorating items with partial back ordering," Computers & Industrial Engineering 24, 449-458, 1993.

AUTHOR'S PROFILE

Dr. Nita H. Shah is a professor in the Department of Mathematics, Gujarat University, Ahmadabad, India. She received her Ph.D. in inventory control management, operations research. Currently, she is engaged in research in inventory control and management, supply chain management, forecasting and information technology and information systems, neural networks, sensors and image processing. She has more than 275 papers published in international and national journals. She is author of four books. She is serving as a member of the editorial board of Investigation Operational, Journal of Social Science and Management, International Journal of Industrial Engineering and Computations and Mathematics Today.

Mr. Digeshkumar B. Shah is an assistant professor in the Department of Mathematics and Humanities, L.D. College of Engineering, Ahmedabad. He has published 15 research papers in international journals viz. International Journal of System Science (IJSS), OPSEARCH, Journal of Mathematical Modeling and Algorithms in Operations Research (JMMA), British Journal of Applied Science & Technology, etc. His areas of interest include operations research and mostly inventory models.

Mr. Dushyantkumar G. Patel is a lecturer in mathematics at Government Polytechnic for Girls, Ahmedabad. He has published 12 research papers in international journals viz. Mexican Journal of Operations Research, ISRN Operations Research, International Journal of Mathematics and Mathematical sciences (IJMMS) etc. His area of interest includes inventory models.