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Image Security by Using Fuzzy Graph

Gamil R.S. Qaid, Sanjay N. Talbar, Ali Abdo M.AL – Kubati

[1,2] *Electronics & Telecom. Dept. S.G.G.S. Institute of Engineering and Technology, Nanded, India*

[3] *Computer Engineering Dept. Computer Science & Engineering Faculty, Hodeidah University, Yemen.*

***Abstract* — In the current time, especially in the computer world, the security of data becomes an important issue. This paper presents a novel fast method for encrypting and decrypting images, based on fuzzy graph (FG) mapping technique. The Fuzzy graphs are obtained from a matrix of image's pixels, and then they are used to encrypt an image. The experimental results show that this method is more efficient, high level security, low loss less and high speed.**

Index: **Fuzzy graphs, fuzzy mappings, image, encryption, decryption, membership functions.**

I. INTRODUCTION

Fuzzy Set Theory finds in image processing a growing application domain. This may be explained not only by its ability to model the inherent imprecision of images together with expert knowledge, but also by the large and powerful toolbox it offers for dealing with spatial information under imprecision [1]. Various fuzzy related algorithms in the domain of image processing and pattern recognition have been discussed in [2-4]. Here, some sketches are introduced for obtaining fuzzy graphs such as fuzzy triangle, sigmoid fuzzy graphs from a square matrix of size n and discussed in detail how multiple numbers of fuzzy graphs are generated for a single given matrix and single given membership functions. Later, this idea of generating multiple fuzzy graphs [FG] is applied as a technique in image encryption. Graph theory is proved to be tremendously useful in modelling the essential features of systems with finite components. Graphical models are used to represent telephone network, railway network, communication problems, traffic network etc. Graph theoretic models can sometimes provide a useful structure upon which analytic techniques can be used. A graph is also used to model a relationship between a given set of objects. Each object is represented by a vertex and the relationship between them is represented by an edge if the relationship is unordered and by means of a directed edge if the objects have an ordered relation between them. Relationship among the objects need not always be precisely defined criteria; when we think of an imprecise concept, the fuzziness arises. [5] Fuzzy graph theory was introduced by Azriel Rosenfeld in 1975. Though he is very young, he has been growing fast and has numerous applications in various fields [6]. Later, Bhattacharya [7] gave some remarks on fuzzy graphs. Mordeson and Peng introduced some operations on fuzzy graphs. A new concept, namely, graph structure was introduced by E. Sampathkumar [9].

Definition

A fuzzy graph is a pair $G(\sigma, \mu)$ where σ is a fuzzy subset of S, μ is a symmetric fuzzy relation on σ. The elements of S are called the vertices or nodes of G and the pair of vertices as edges in G. The underlying crisp graph of the fuzzy graph $G(\sigma, \mu)$ is denoted as G*: (S, E) where $E \subseteq S \times S$. The crisp graph (S, E) is a special case of the fuzzy graph G with each vertex and edge of (S, E) having degree of membership.

Types of fuzzy graph:

1. Regular Fuzzy Graph

A. Nagoor Gani and K.Radha (2008) introduced regular fuzzy graphs on paper "On Regular Fuzzy Graphs". In their paper, they introduced regular fuzzy graphs, total degree and totally regular fuzzy graphs. Regular fuzzy graphs and totally regular fuzzy graphs are compared through various examples. A sufficient and necessary condition for equivalence and characterization of regular fuzzy graph was provided [10].

2. Fuzzy Dual Graph

Nuha abdul-jabbar, Jehan H. Naoom , Eman H. Ouda (Dec 2009) introduced Fuzzy dual graph. In their paper, the definition of fuzzy dual graphs was considered with the following properties:

a) The dual of the fuzzy graph is the fuzzy graph itself.

b) The dual of fuzzy bipartite graph is Eulerian fuzzy graph [11].

3. Complementary Fuzzy Graph

Moderson (1994) introduced the concept of complement of fuzzy graphs, and M. S. Sunitha and A. Vijay Kumar (2001) gave a modified definition of complement of fuzzy graph [12]. R. Sattanathan and S. Lavanya (2009) conducted a study on complementary fuzzy graphs and fuzzy chromatic number. In their paper, they found the fuzzy chromatic number of complement of fuzzy graphs and gave the bounds for sum and product of fuzzy chromatic number of fuzzy graph and its complement [13].

4. Antipodal Fuzzy Graph

A. Nagoor Gani and J. Malarvizhi (2010) defined concept of Antipodal Graph. In crisp graph theory, the concept of antipodal graph of a given graph G was introduced by Smith [13].

5. Constant intuitionistic fuzzy graphs

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M. G. Karunambigai, R. Parvathi and R. Buvaneshwari (2011) introduced Constant intuitionistic fuzzy graphs. In their work, Constant Intuitionistic Fuzzy Graphs (IFGs), and totally constant IFGs were introduced. A necessary and sufficient condition for equivalence was provided. A characterization of constant IFGs on a cycle was also given. Some properties of constant IFGs with suitable illustrations were also discussed [14].

6. Fuzzy Graph Structures

T. Dinesh and T.V. Ramakrishnan (2011) introduced the concept of a fuzzy graph structure based on the concept of graph structure. A new concept, namely, graph structure was introduced by E. Sampathkumar which, in particular, is a generalization of the notions like graphs, signed graphs and edge-coloured graphs with the colourings. According to him, G = (V,R1, R2, ..., Rk) is a graph structure if V is a nonempty set and R1, R2, ..., Rk are relations on V which are mutually disjoint such that each Ri, i = 1,2,3,...,k, is symmetric and irreflexive. This is the motivation for the study of fuzzy graph structures. New concepts like ρi−edge, ρi−cycle, ρi−tree, ρi−forest, fuzzy ρi−cycle, fuzzy ρi−tree, fuzzy ρi−forest, ρi−connectedness etc. are introduced and studied [15].

7. Bipolar Fuzzy Hypergraphs

In 1994, Zhang [16] initiated the concept of bipolar fuzzy sets as a generalization of fuzzy sets. Bipolar fuzzy sets are an extension of fuzzy sets whose range of membership degree is [-1,1]. In bipolar fuzzy set, membership degree 0 of an element means that the element is irrelevant to the corresponding property, the membership degree [0, 1] of an element indicates that the element somewhat satisfies the property, and the membership degree [-1, 0] of an element indicates the element somewhat satisfies the implicit counter property. S. Samanta and M. Pal (2012) defined some basic concepts of bipolar fuzzy hyper graphs, cut level bipolar fuzzy hyper graphs, dual bipolar fuzzy hyper graphs and bipolar fuzzy transversal. Also, some basic theorems related to the stated graphs have been presented [17].

8. Irregular Fuzzy Graphs

A. Nagoor Gani and S. R. Latha (2012) defined Neighbourly total irregular fuzzy graphs, highly irregular fuzzy graphs and highly total irregular fuzzy graphs. A comparative study between neighbourly irregular and highly irregular fuzzy graphs was made. Also, some results on neighbourly irregular fuzzy graphs were studied [18].

9. Irregular Bipolar Fuzzy Graphs

S. Samanta and M. Pal (2012) defined irregular bipolar fuzzy graphs and its various classifications. Size of regular bipolar fuzzy graphs was derived. The relation between highly and neighbourly irregular bipolar fuzzy graphs were established. Some basic theorems related to the stated graphs were presented [19].

10. Fuzzy Labelling Graph

A. Nagoor Gani and D. Rajalaxmi (a) Subahashini (2012) introduced a new concept of fuzzy labelling. A graph is said to be a fuzzy labeling graph if it has fuzzy labeling. Fuzzy sub graph, union, fuzzy bridges, fuzzy end nodes, fuzzy cut nodes and weakest arc of fuzzy labeling graphs have been discussed. And number of weakest arc, fuzzy bridge, cut node and end node of a fuzzy labeling cycle has been found. It is proved that $\Delta(G\omega)$ is a fuzzy cut node and $\delta(G\omega)$ is a fuzzy end node of fuzzy labelling graph. Also, it was proved that If Gω is a connected fuzzy labelling graph then there exists a strong path between any pair of nodes [20].

11. Interval-valued Fuzzy Graph

Muhammad Akram and Wieslaw A. Dudek (2012) defined the operations of Cartesian product, composition, union and join on interval-valued fuzzy graphs and investigate some properties. They gave concept of isomorphism (resp. Weak isomorphism) between interval-valued fuzzy graphs. Also, they introduced the notion of interval-valued fuzzy complete graphs and presented some properties of self complementary and self weak complementary interval-valued fuzzy complete graphs [21].

II. PROBLEM STATEMENT

This proposed method investigates a new approach for security image using the concept of Fuzzy Graph (FG) which is used to output new encryption method by exploitation the feature membership of pixel and interval-valued fuzzy graphs.

III. THE PROPOSED ALGORITHM

The overall procedures and steps of encrypting and decrypting an image are summarized as follows:

1. Loading an image.
2. Determining the height and width of image (H and W).
3. Checking the result of H mod 8 and W mod 8. If they are equal to 0 then go to 4th step otherwise doing H = H+(8- (H mod 8)) and W = W+(8- (W mod 8)).
4. Dividing the image into sets of block, and the size of each block is (8*8).
5. Considering a block B(w *h) where w and h are width and height of B.
6. Finding the membership for each pixel by dividing the pixel by 255.

6.1. Doing an interval-valued fuzzy graphs for each block as:

$$X = \left\{ \left(\frac{PR(P_1, P_2, P_2 \ldots\ldots P_n)}{\mu(P_1, P_2, P_2 \ldots\ldots P_n)}\right), \left(\frac{PG(P_1, P_2, P_2 \ldots\ldots P_n)}{\mu(P_1, P_2, P_2 \ldots\ldots P_n)}\right), \left(\frac{PB(P_1, P_2, P_2 \ldots\ldots P_n)}{\mu(P_1, P_2, P_2 \ldots\ldots P_n)}\right) \right\}, \tag{1}$$

$$Y = \left\{ \left(\frac{PR(P_1\ P_2, P_2\ P_3, P_3 P_4, \ldots, P_{n-1} P_n)}{\mu(R)} \right), \left(\frac{PG(P_1\ P_2, P_2\ P_3, P_3 P_4, \ldots, P_{n-1} P_n)}{\mu(G)} \right), \left(\frac{PB(P_1\ P_2, P_2\ P_3, P_3 P_4, \ldots, P_{n-1} P_n)}{\mu(B)} \right) \right\}. \quad (2)$$

7. Getting the encrypted image.

Where X and Y are an interval-valued fuzzy set, $P_1, P_2, P_3, \ldots, P_n$ are the pixels in the vector, PR, PG and PB are the colors values for each pixel system, $\mu(P_1, P_2, P_3, \ldots, P_n)$ is the membership of each pixel which calculated by dividing the value of pixel by 255. And $\mu(R), \mu(G), \mu(B)$ is the membership of pixel's color which calculated by dividing the value of color for each pixel by 255 also.

IV. EXPERIMENTAL ANALYSIS

The proposed method in this study helps in increasing the efficiency of the method in terms of computation time required and complexity to attack. It uses the concept of fuzzy graph and membership function for each pixel In this section, the proposed method is applied on different sizes and types of images. The test of images employed here shows positive result. The implementation of this encryption algorithm was carried out using MATLAB, in core 2 duo of 2.66 GHz machine. The decryption process takes less than 60 micro seconds to get executed. It is shown in the next tables.

Table 1: Images Properties

Original image	Type	Dimensions	Size before encryption	Size after decryption
a	Tif	259 * 194	140 KB	139.2 KB
b	bmp	2050*1153	145 KB	144 KB
c	Jpg	255*255	12KB	12 KB

Table 2: Time of Encryption and Decryption Processes

Image	Time of encryption(ms)	Time of decryption (ms)
a	35	38
b	52	59
c	19	21

Security analysis

A secure encryption method should resist various types of cryptanalysis such as histogram analysis and correlation coefficients. In this paper, the security analysis of the proposed encryption method discussed by experimental tests with many of images; table 1 shows the sample of these images.

a) Histogram

It is a very important feature in analysing images. Figures (1-3) present histograms of RGB colors for the original, encrypted, and decrypted image of Lena. It is apparent that histogram of the encrypted image is nearly uniform and significantly different from the histogram of the original image. It does not give any evidence to employ any statistical analysis attack on the encrypted image. Figures (1-3) confirm that statistical attacks based on the histogram analysis can't give any clue to break the algorithm as all the statistical information of the original image are lost after the encryption.

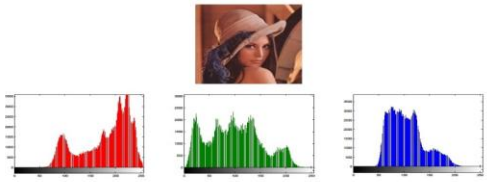

Fig 1. Original Image of Lena and Its RGB Histogram

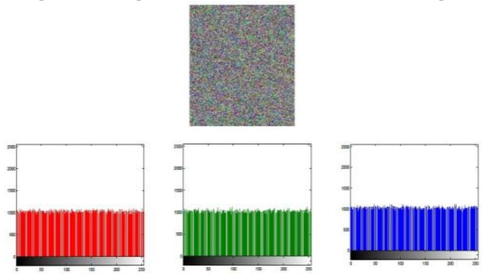

Fig 2. Encrypted Image of Lena and Its RGB Histogram

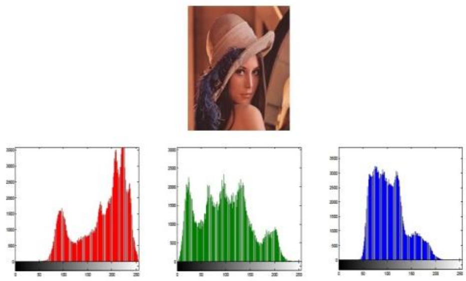

Fig 3. Decrypted Image of Lena and Its RGB Histogram

b) Correlation of the Two Adjacent Pixels

This method involves calculating three adjacent Pixels correlation for each plain cipher image: vertically, horizontally, and diagonally. Figure (4) is the horizontal relevance of adjacent elements in image before and after encryption. It shows significant reduction in relevance of adjacent elements.

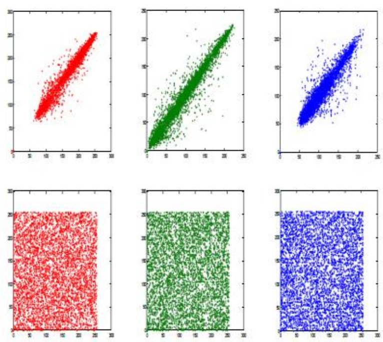

Fig4. Correlation analysis of two horizontally adjacent pixels in the plain and encrypted image of Lena

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V. CONCLUSION

This paper introduces a new method of digital image encryption based on fuzzy graph. This proposed method can be classified as a smart method which has more efficient, high level security, low lossless and high speed. The proposed method has been tested on several images different in formats and sizes and showed good results.

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