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Application of Graph Theory in Traffic Management

Darshankumar Dave, Nityangini Jhala
Humanities and Applied Sciences Department, Parul Institute of Technology, Limda. Ta.Waghodia. Dist. Vadodara, Gujarat, INDIA

***Abstract** In order to solve the traffic problems in a city, the use of graph theory concepts has been applied in this paper. The compatibility graph corresponding to the problem and circular arc graphs have been introduced. Compatibility graph corresponding to the problem, spanning sub graph and circular arc graphs then are utilized to reduce our problem to the solution of LP problems. Illustrative examples are included to demonstrate the validity and applicability of the technique.*

I. INTRODUCTION

One of the main features of modern cities is the permanent growth of population and every year new road and highways are built in most of the urban areas to accommodate the growing number of vehicles [2]. This increase in the number of vehicles in urban cities has led to the increase in time losses of traffic participants, the increase of environmental and noise pollution and also increases in the number of traffic accidents. Traffic congestion has become one of the major obstacles for the development of many urban areas, affecting millions of people. Constructing new roads may improve the situation, but it is very costly and in many cases it is impossible due to the existing structures. The only way to control the traffic flow in such a situation is to use the current road network more efficiently. A methodology of handling city traffic in a very efficient way by proper traffic management instead of modifying the road infrastructure is presented in this paper. Recent progress in mathematics especially in its applications caused in observable progress of graph theory, in a way that it is now a proper tool for researches in different fields like Encoding theory, Electrical networks, Operation research and other fields. In this paper we review the applications of circular arc graphs in solving a special category of traffic problems. The paper is organized as follows: in section 2 we describe the required definitions and preliminary. The proposed method has been presented in form of example in section 3. In section 4 we solved an example of the problem with offering method.

II. PRELIMINARIES

Consider a finite family of non-empty sets. The intersection graph of this family is obtained by representing each set by a vertex, two vertices being connected by an edge if and only if the corresponding sets intersect. Intersection graphs have received much attention in the study of algorithmic graph theory and their applications. Well-known special classes of intersection graphs include interval graphs, chordal graphs, circular arc graphs, and so on.[4]

Definition 2.1 A circular arc graph is the intersection graph of a family of arcs on a circle. We say that these arcs are a circular arc representation of the graph.

Definition 2.2 A clique of graph G is a complete sub graph of G.

Definition 2.3 A clique of graph *G* is a maximal clique of *G* if it is not properly contained in another clique of *G*.

III. PROBLEM STATEMENT

The problem is to install traffic lights at a road junction in such a way that traffic flows smoothly and efficiently at the junction. We take a specific example and explain how our problem could be tackled. Figure 1 displays the various traffic streams, namely *a*, *b*, *c*, *d* at Town Hall Circle, Anand.

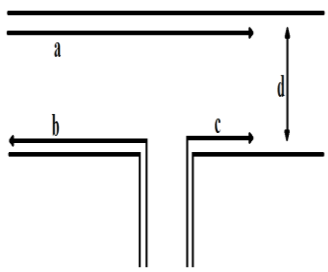

Fig 1: Traffic streams

Vehicles approaching an intersection prepare themselves to perform a certain maneuver i.e. to drive through, turn left, or turn right at an intersection. The vehicles that perform this maneuver represent a flow component. Such an arrival flow component is called a traffic stream [6]. The traffic streams which can move simultaneously at an intersection without any conflict may be termed as compatible. For instance, in Figure 1, streams *a* and *b* are compatible, whereas *a* and *d* are not. The phasing of lights should be such that when the green lights are on for two streams, they should be compatible. We suppose that the total time for the completion of green and red lights during one cycle is two minutes. We form a graph *G* whose vertex set consist of the traffic streams in question and we make two vertices of *G* joined by an edge if, and only if, the corresponding streams are compatible.

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The graph thus obtained is the compatibility graph corresponding to the problem in question. The compatibility graph of Figure 1 is shown in Figure 2.

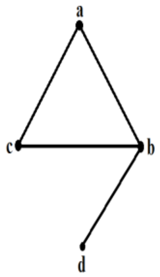


Fig 2: Graph G (Compatibility graph of Figure 1)

We take a circle and assume that its perimeter corresponds to the total cycle period, namely 120 seconds. We may think that the duration when a given traffic stream gets a green light corresponds to an arc of this circle. Hence, two such arcs of the circle can overlap only if the corresponding streams are compatible. The resulting circular arc graph may not be the compatibility graph because we do not demand that two arcs intersect whenever corresponds to compatible flows. (There may be two compatible streams but they need not get a green light at the same time). However, the intersection graph *H* of this circular arc graph will be a spanning sub graph of the compatibility graph. So we have to take all spanning sub graph of *G* in to account and choose from them the spanning sub graph that has the most maximal clique. The proper graph *H* for the above example is shown in Figure 3.

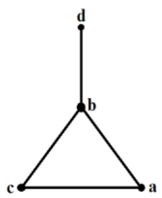
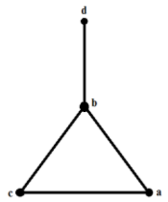


Fig 3: Graph H (Intersection graph)

The efficiency of our phasing may be measured by minimizing the total red light time during a traffic cycle that is the total waiting time for all the traffic streams during a cycle. For the sake of concreteness, we may assume that at the time of starting, all lights are red. The maximal clique of H are $k_1 = \{a, b, c\}$ and $k_2 = \{b, d\}$. Each clique $k_i, 1 \le i \le 2$, corresponds to a phase during which all streams in that clique receive green lights. In phase 1, traffic streams *a*,*b* and *c* receive a green light; in phase 2, *b* and *d* receive a green light. Suppose we assign to each phase k_i a duration d_i. Our aim is to determine the d_i's (≥ 0) so that the total waiting time is minimum. Further, we may assume that the minimum green light time for any stream is 20 seconds. Traffic stream *a* gets a red light when the phase k_2 receive a green light. Hence *a*'s total red light time is d_2. Similarly, the total red light time of all streams *c* and *d*, respectively are d_2 and d_1.

Therefore the total red light time of all streams in one cycle is $Z = 2d_2 + d_1$. Our aim is to minimize Z subject to $d_i \ge 0, 1 \le i \le 2$ and $d_1 \ge 20$ $d_2 \ge 20$, $d_1 + d_2 \ge 20$ & $d_1 + d_2 = 120$. The optimal solution to this LP problem is $d_1 = 100$, $d_2 = 20$, $\min Z = 140$. The phasing that corresponds to this least value would then be the best phasing of the traffic lights.



IV. EXAMPLE

The Figure 4 displays the various traffic streams, namely *a*,*b*,..., *g* . We apply the proposed method for this example. The compatibility graph corresponding to the problem is shown in Figure 5.

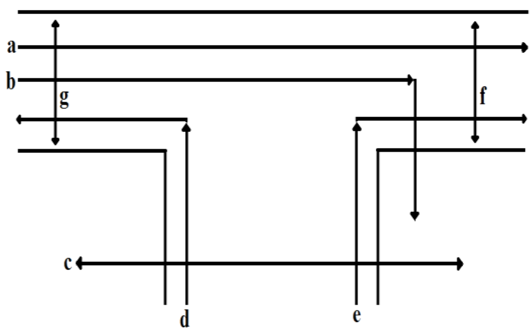


Fig 4: Traffic streams

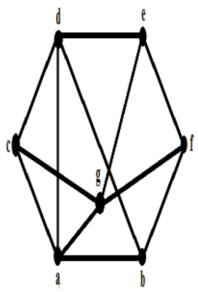


Fig 5: Graph G (Compatibility graph)

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The proper graph (the sub graph of G that has the most maximal clique) *H* for this example is shown in Figure 6.

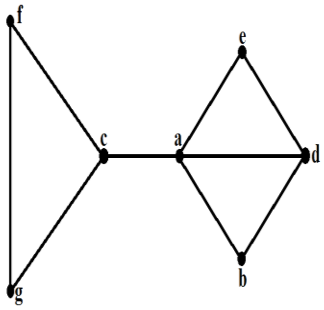

Fig 6: Graph H (Intersection graph)

The maximal clique of H are $k_1 = \{a, e, d\}$, $k_2 = \{a, b, d\}, k_3 = \{a, c\}$ and $k_4 = \{c, f, g\}$. Our aim is to minimize $Z = 4d_1 + 4d_2 + 4d_3 + 3d_4$ subject to $d_i \geq 0, \quad 1 \leq i \leq 4$ and $d_1 \geq 20$ $d_2 \geq 20$, $d_1 + d_2 \geq 20$, $d_1 + d_2 + d_3 \geq 20$ $d_3 + d_4 \geq 20$ $d_4 \geq 20$ & $d_1 + d_2 + d_3 + d_4 = 120$. The optimal solution to this LP problem is $d_1 = 80$, $d_2 = 20$, $d_3 = 0, d_4 = 20 \min Z = 460$seconds.

V. CONCLUSION

In this paper using the concept of circular arc graphs, we tried to solve the problem of phasing of traffic lights at a junction such that traffic signals can be used more efficiently to avoid long time waiting at junctions and congestions. Illustrative example is included to demonstrate the validity and applicability of the technique.

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