

# Design of Novel Linear Phase Digital Differentiator

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**Abstract:** - This paper presents the design of novel linear phase digital differentiator. Filter coefficients are calculated based on the globally optimized techniques. Different globally optimized algorithms are genetic algorithm, pattern search, simulating annealing. Different control parameters are defined that are used for the analysis of the digital differentiator. The results show that the response of the proposed differentiator will closely approximate the response of the ideal system. In this paper, we are considering the absolute magnitude response of the differentiator as the parameter to measure the accuracy and the range of operation of the designed differentiator. Simulation result also shows that the designed differentiator is outperforms the all existing digital differentiator in terms of magnitude error but this difference in error is very small. This error can only be visible at higher frequency. Result also reveals that the performance of the proposed differentiator will not approaches the ideal response at low and higher frequency range.

**Keywords:**-Digital differentiator, Frequency response, Simulating Annealing, Pattern Search, digital filter.

## I. INTRODUCTION

Digital differentiator finds a wide range of application in different area of engineering such as control, biomedical engineering and radar processing. It also finds application in the field of image processing with 2-D digital differentiator. The frequency response of an ideal digital differentiator is

$$H_{int}(\omega) = j\omega \quad (1)$$

Where  $j = \sqrt{-1}$  and  $\omega$  is the angular frequency in radians. Differentiation is used to extract information about rapid transients in the signal. Low-pass filters are used to reject noise frequencies higher than the cut-off frequencies of the signal. Low-pass filtering and differentiation can be implemented as a single low-pass differentiator filter [1]. The proposed low-pass differentiators are IIR differentiators. It is shown that fourth-order differentiators compare favourably with the much higher order state of the art finite impulse-response (FIR) low-pass differentiators [1]. The accuracy of the proposed differentiators is comparable to that obtained by higher order filters. An additional advantage is that an almost linear phase is also obtained in the pass band region. The numerator of the transfer function of the IIR filter corresponds to a linear phase filter while the coefficients of the denominator are allowed to vary to

obtain an optimum solution. The resulting differentiators compare favourably with the state of the art of higher order, six or seven times higher, FIR low-pass differentiators. Different approaches for the design of digital integrators are as follows simple linear interpolation between the magnitude responses of the classical rectangular, trapezoidal and Simpson digital integrators [2], linear programming optimisation approach [3], optimising the pole-zero locations [4]-[6], Simulated Annealing, Genetic Algorithms [7], and Fletcher and Powell Optimization. Digital differentiator is obtained from the response of the digital integrator by inverting the transfer function of the digital integrator. Simulated annealing (SA) is a random-search technique which exploits an analogy between the way in which a metal cools and freezes into a minimum energy crystalline structure (the annealing process) and the search for a minimum in a more general system; it forms the basis of an optimisation technique for combinatorial and other problems. Simulated annealing is a popular local search meta-heuristic used to address discrete and, to a lesser extent, continuous optimization problems. The key feature of simulated annealing is that it provides a means to escape local optima by allowing hill-climbing moves in hopes of finding a global optimum. This paper presents the design of novel linear phase digital differentiator. Filter coefficients are calculated based on the globally optimized techniques. Different globally optimized algorithms are genetic algorithm, pattern search, simulating annealing. Different control parameters are defined that are used for the analysis of the digital differentiator. The rest of the paper is organized as follows: In section II, explain the simulating annealing algorithm applied to a discrete optimization problem, the objective function generates values for two solutions. In Section III, different steps of simulating annealing algorithm are presented. Section IV explains designed digital differentiator based on the simulating annealing algorithm. In Section V, shows the experimental results to analysis the performance of the differentiator in terms of magnitude response. Finally, a conclusion is put forward.

## II. SIMULATING ANNEALING

Simulated annealing [8] is a method for solving unconstrained and bound-constrained optimization problems. The method models the physical process of

heating a material and then slowly lowering the temperature to decrease defects, thus minimizing the system energy. At each iteration of the simulated annealing algorithm, a new point is randomly generated. The distance of the new point from the current point, or the extent of the search, is based on a probability distribution with a scale proportional to the temperature. The algorithm accepts all new points that lower the objective, but also, with a certain probability, points that raise the objective. By accepting points that raise the objective, the algorithm avoids being trapped in local minima, and is able to explore globally for more possible solutions. An annealing schedule is selected to systematically decrease the temperature as the algorithm proceeds. As the temperature decreases, the algorithm reduces the extent of its search to converge to a minimum.

The simulated annealing algorithm [9] was originally inspired from the process of annealing in metal work. Annealing involves heating and cooling a material to alter its physical properties due to the changes in its internal structure. As the metal cools its new structure becomes fixed, consequently causing the metal to retain its newly obtained properties. In simulated annealing we keep a temperature variable to simulate this heating process. We initially set it high and then allow it to slowly 'cool' as the algorithm runs. While this temperature variable is high the algorithm will be allowed, with more frequency, to accept solutions that are worse than our current solution. This gives the algorithm the ability to jump out of any local optimums it finds itself in early on in execution. As the temperature is reduced so is the chance of accepting worse solutions, therefore allowing the algorithm to gradually focus in on a area of the search space in which hopefully, a close to optimum solution can be found. This gradual 'cooling' process is what makes the simulated annealing algorithm remarkably effective at finding a close to optimum solution when dealing with large problems which contain numerous local optimums. Hill climbers [8] can be surprisingly effective at finding a good solution; they also have a tendency to get stuck in local optimums. As we previously determined, the simulated annealing algorithm is excellent at avoiding this problem and is much better on average at finding an approximate global optimum. To help better understand let's quickly take a look at why a basic hill climbing algorithm is so prone to getting caught in local optimums.

### III. SOLUTION TO FILTER COEFFICIENT USING PROBLEM SIMULATED ANNEALING

**Step: 1** First we need set the initial temperature and create a random initial solution.

**Step: 2** Then we begin looping until our stop condition is met. Usually either the system has sufficiently cooled, or a good-enough solution has been found.

**Step: 3** From here we select a neighbour by making a small change to our current solution.

**Step: 4** we then decide whether to move to that neighbour solution.

**Step: 5** finally, we decrease the temperature and continue looping

### IV. PROPOSED DIFFERENTIATOR

Several methods have been used for their design. Proposed second order differentiator is obtained as

$$H_{Diff\_SA\_2}(z) = \frac{1.1538 - 0.5408z^{-1} - 0.613z^{-2}}{1 + 0.7121z^{-1} + 0.067z^{-2}} \quad (2)$$

For the third order differentiator, the zeros and poles of the second order differentiator are used as starting points. Thus, the differentiator becomes

$$H_{Diff\_SA\_3}(z) = \frac{1.1555 - 0.3582z^{-1} - 0.714z^{-2} - 0.0833z^{-3}}{1 + 0.8662z^{-1} + 0.1612z^{-2} + 0.0028z^{-3}} \quad (3)$$

The fourth order differentiator was built upon using the third order

$$H_{Diff\_SA\_4}(z) = \frac{1.1540 + 0.2290z^{-1} - 0.8794z^{-2} - 0.4486z^{-3} - 0.0549z^{-4}}{1 + 1.3788z^{-1} + 0.623z^{-2} + 0.1059z^{-3} + 0.0059z^{-4}} \quad (4)$$

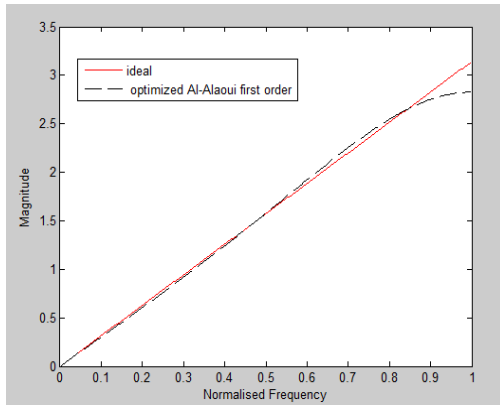
### V. EXPERIMENTAL RESULTS

The filter coefficient of IIR differentiators is obtained using simulated annealing algorithm. Simulation has been done in MATLAB and optimization toolbox of matlab. Following control parameters are too considered while optimizing the filter coefficients as shown in table 1.

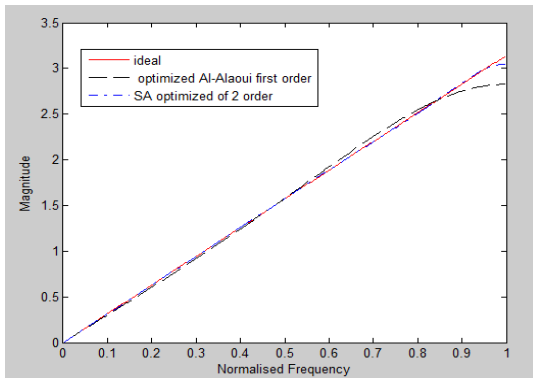
**Table: 1**

Parameter	Simulating Annealing
Population size	1000
Starting pont	.01
Lower bound	-1
Upper bound	1

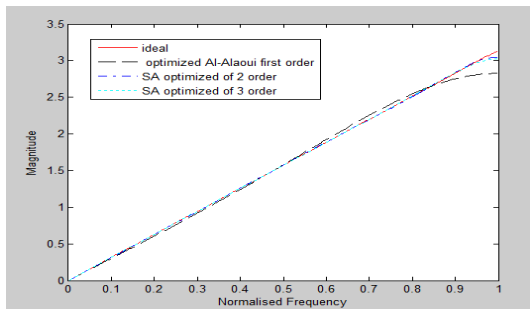
Fig. 1 shows the magnitude response of the ideal and optimized Al-Alaoui first orders differentiator. Fig. 2 depicts the comparative magnitude response of the ideal, optimized Al-Alaoui first order and Simulating Annealing Algorithm optimized second order differentiator. Fig. 3 demonstrate the comparative magnitude response of the ideal, optimized Al-Alaoui first order, Simulating Annealing Algorithm optimized second order and Simulating Annealing Algorithm optimized third order differentiator. Fig. 4 shows the comparative magnitude response of the ideal, optimized Al-Alaoui first order, Simulating Annealing Algorithm optimized second order, Simulating Annealing Algorithm optimized third order and Simulating Annealing Algorithm optimized fourth order differentiator.



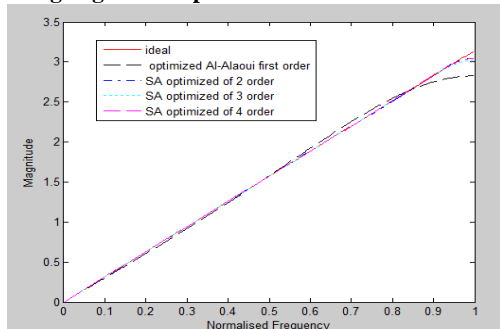
**Fig. 1** magnitude response of the ideal and optimized Al-Alaoui first order differentiator



**Fig. 2** comparative magnitude response of the ideal, optimized Al-Alaoui first order and Simulating Annealing Algorithm optimized second order differentiator.



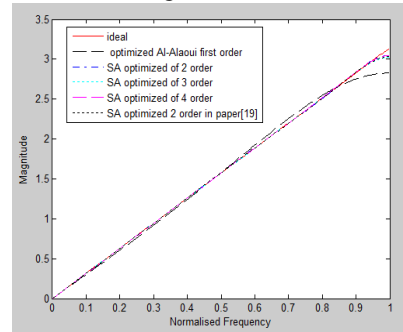
**Fig. 3** comparative magnitude response of the ideal, optimized Al-Alaoui first order, Simulating Annealing Algorithm optimized second order and Simulating Annealing Algorithm optimized third order differentiator.



**Fig. 4** comparative magnitude response of the ideal, optimized Al-Alaoui first order, Simulating Annealing Algorithm optimized second order, Simulating Annealing Algorithm optimized third order and Simulating Annealing Algorithm optimized fourth order differentiator.

**Algorithm optimized third order and Simulating Annealing Algorithm optimized fourth order differentiator**

Fig. 5 depicts the comparative magnitude response of the ideal, optimized Al-Alaoui first order, Simulating Annealing Algorithm optimized second order, Simulating Annealing Algorithm optimized third order, Simulating Annealing Algorithm optimized fourth order differentiator and existing differentiator.



**Fig. 5** comparative magnitude response of the ideal, optimized Al-Alaoui first order, Simulating Annealing Algorithm optimized second order, Simulating Annealing Algorithm optimized third order, Simulating Annealing Algorithm optimized fourth order differentiator and existing differentiator

**VI. CONCLUSIONS**

In this paper, an approach to designing linear phase recursive digital differentiators is presented. We are applying the linear phase properties of the FIR filters and the steeper magnitude roll-off properties of the IIR filters to obtain IIR low-pass digital differentiators. The proposed differentiators have shorter transition regions, and thus better ability to suppress high frequency noise, for much lower order filters, than the corresponding FIR filters. This approach could well be employed to design other types of IIR filters that approximate linear phases in the pass bands and meet the magnitude specifications at a lower computational cost than the corresponding FIR filters. Results show the effectiveness of the proposed differentiator.

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