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Inventory Model for Deteriorating Items with Fixed Life under Quadratic Demand and Nonlinear Holding Cost

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***Abstract:* - In this paper, we analyzed a deterministic inventory model with time-dependent quadratic demand and time-varying holding cost where units have expiry date. The model considered here allows shortages, and the demand is partially backlogged. The model is solved analytically by minimizing the total inventory cost. The results are illustrated with numerical example for the model. The sensitivity analysis is carried out to find critical inventory parameters for the decision maker.**

Keywords: **Deteriorating items; Expiration date; Shortages; Quadratic demand; Time-varying holding cost.**

I. INTRODUCTION

One of the assumptions in the traditional inventory model was that the items preserved their physical characteristics while they were kept stored in the inventory. This assumption is evidently true for most items, but not for all. However, the deteriorating items are subject to a continuous loss in their masses or utility throughout their lifetime due to decay, damage, spoilage, and penalty of other reasons. Owing to this fact, controlling and maintaining the inventory of deteriorating items becomes a challenging problem for decision makers. Harris [10] developed the first inventory model, Economic Order Quantity, which was generalized by Wilson [24] who gave a formula to obtain economic order quantity. Whit in [23] considered the deterioration of the fashion goods at the end of the prescribed shortage period. A model developed by Ghare and Schrader [8] for an exponentially decaying inventory. Dave and Patel [6] were the first to study a deteriorating inventory with linear increasing demand when shortages are not allowed. In this field some of the recent work has been done by Chung and Ting [5]. Also Wee [22] studied an inventory model with deteriorating items. Chang and Dye [4] developed an inventory model with time-varying demand and partial backlogging. Goyal and Giri [9] gave recent trends of modeling in deteriorating item inventory. They classified inventory models on the basis of demand variations and various other conditions or constraints. Ouyang and Cheng [18] developed an inventory model for deteriorating items with exponential declining demand and partial backlogging. The effects of learning and forgetting on the optimal production lot size for deteriorating items with time-varying demand and deterioration rates were studied by Alamri and Balkhi [3]. Dye et al. [7] find an optimal selling price and lot size with a varying rate of deterioration and exponential partial backlogging. They assume that a fraction of customers who backlog their orders increases exponentially as the waiting time for the next replenishment decreases. In 2008, Roy developed a deterministic inventory model when the deterioration rate is time proportional. Holding cost is time dependent and demand rate is a function of selling price. Liao [12] gave an economic order quantity (EOQ) model with non-instantaneous receipt and exponential deteriorating item under two level trade credits. Pareek et al. [19] developed a deterministic inventory model for deteriorating items with salvage value and shortages. Skouri et al. [21] developed an inventory model with ramp-type demand rate, partial backlogging, and Weibull's deterioration rate. Mishra and Singh [14] developed a deteriorating inventory model for waiting time partial backlogging when demand and deterioration rate is constant. They made the work of Abad [1, 2] more realistic and applicable in practice. Mandal [13] gave an EOQ inventory model for Weibull-distributed deteriorating items under ramp-type demand and shortages. Mishra and Singh [15, 16] gave an inventory model for ramp-type demand, time-dependent deteriorating items with salvage value and shortages and deteriorating inventory model for time-dependent demand and holding cost with partial backlogging. Hung [11] gave an inventory model with generalized-type demand, deterioration, and backorder rates. Mishra et al. [17] developed an inventory model for deteriorating items with time-dependent demand and time-varying holding cost under partial backlogging. In classical inventory models, the demand rate and holding cost are assumed to be constant. In reality, the demand and holding cost for physical goods may be time dependent. Time also plays an important role in the inventory system; therefore, in this article, we consider that demand and holding cost are time dependent. In this paper, we considered demand rate as quadratic functions of time and developed an inventory model for deteriorating items which has maximum life time.

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Shortages are allowed and demand is partially backlogged. In the differential equation of on-hand inventory, the cumulative holding cost is considered as a non-linear increasing positive function of time. In the next section the assumptions and notations of the model are introduced. The mathematical model and solution procedure are derived in the mathematical formulation section, and the numerical and graphical analysis is presented after mathematical formulation. The article ends with some concluding remarks and scope of future research.

II. NOTATION AND ASSUMPTIONS

The following notations and assumptions will be used in this paper:

i. Deterioration rate is time proportional.

ii. $\theta(t)=\dfrac{1}{1+m-t}, m>0, T\le m$, $\theta(t)$ denotes rate of deterioration of units in inventory.

iii. Demand rate is time dependent and quadratic, i.e., $D(t)=a\left(1+bt+ct^2\right), a>0, 0<b,c<1$.

iv. Shortages are allowed and partially backlogged.

v. π is the shortage cost per unit per unit time.

vi. β is the backlogging rate; $0\le\beta\le 1$.

vii. During time t_1 the inventory is depleted due to the deterioration and demand of item. At time t_1, the inventory becomes zero and shortage starts occurring.

viii. There is no repair or replenishment of deteriorating item during the period under consideration.

ix. Replenishment is instantaneous; lead time is zero.

x. T is the length of the cycle.

xi. Holding cost $h(t)$ per unit time is time dependent and is assumed $h(t)=h(1+\alpha t)$ where α denotes increment with respect to t and lies between 0 and 1.

xii. C is the unit cost of an item.

xiii. IB is the maximum inventory level during the shortage period.

xiv. I_0 is the maximum inventory level during (0, T).

xv. π_L is the lost sale cost per unit.

xvi. A is cost of placing one order.

TC is the total cost of an inventory system $=\dfrac{1}{T}$ [cost of placing one order (A) + purchase cost (PC)+ the holding cost (HC) + the shortage cost (SC) + lost sale cost (LSC)]

III. MODEL FORMULATION

The rate of change of the inventory during the positive stock period $(0,t_1)$ and shortage period (t_1,T) is governed by the following differential equations:

$$\frac{dI_1(t)}{dt}=-D(t)-\theta(t)I_1(t),\quad 0\le t\le t_1 \qquad (1)$$

$$\frac{dI_2(t)}{dt}=-\beta D(t),\quad t_1\le t\le T \qquad (2)$$

The initial inventory level is I_0 unit at time t = 0; from t = 0 to $t=t_1$, the inventory level reduces, owing to both demand and deterioration, until it reaches zero level at time $t=t_1$. At this time, shortage is accumulated which is partially backlogged at the rate β. At the end of the cycle, the inventory reaches a maximum shortage level so as to clear the backlogged and again raises the inventory level to I_0 (Figure 1).

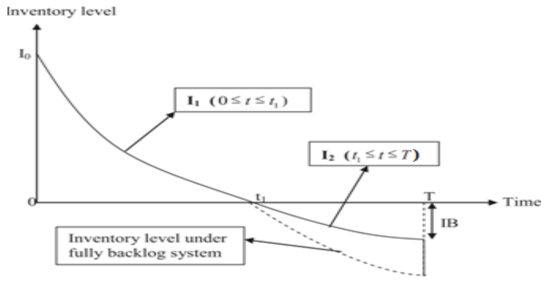

Fig 1. Graphical representation of the state of inventory system

Thus, boundary conditions are as follows:

$$I_1(0)=I_0,\quad I_1(t_1)=0,\quad I_2(t_1)=0.$$

The solutions of Equations 1 and 2 with boundary conditions are as follows:

$$I_1(t)=\frac{(1+m-t)}{1+m}\left(\frac{1}{1+m}\left(a\left(\left(1+b(1+m)+c(1+m)^2\ln\left(\frac{1+m}{1+m-t_1}\right)-\frac{1}{2}ct_1^2-(b+c)t_1-cmt_1\right)\right)\right)-\frac{1}{1+m}\left(a\left(\left(1+b(1+m)+c(1+m)^2\right)\ln\left(\frac{1+m}{1+m-t}\right)-\frac{1}{2}ct^2-(b+c)t-cmt\right)\right)\right) \qquad (3)$$

$$I_2(t)=\beta a\left(t_1+\frac{1}{2}bt_1^2+\frac{1}{3}ct_1^3\right)-\beta a\left(t+\frac{1}{2}bt^2+\frac{1}{3}ct^3\right) \qquad (4)$$

Using Equation 3, we get the following:

$$I_0=\frac{1}{1+m}\left(a\left(\left(1+b(1+m)+c(1+m)^2\ln\left(\frac{1+m}{1+m-t_1}\right)-\frac{1}{2}ct_1^2-(b+c)t_1-cmt_1\right)\right)\right) \qquad (5)$$

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Inventory is available in the system during the time interval $(0, t_1)$. Hence, the cost for holding inventory in stock is computed for time period $(0, t_1)$ only.

Holding cost is as follows:

$$HC = \int_0^{t_1} h(t) I_1(t)\, dt \quad (6)$$

Shortage due to stock out is accumulated in the system during the interval (t_1, T).

The optimum level of shortage is at $t = T$; therefore, the total shortage cost during this time period is as follows:

$$SC = \pi \int_{t_1}^{T} -I_2(t)\, dt$$

$$SC = \pi \left(\frac{1}{12}\beta ac\left(T^4 - t_1^4\right) + \frac{1}{6}\beta ab\left(T^3 - t_1^3\right) + \frac{1}{2}\beta a\left(T^2 - \right. \right. \\ \left. -\beta a\left(t_1 + \frac{1}{2}bt_1^2 + \frac{1}{3}ct_1^3\right)\left(T - t_1\right) \right) \quad (7)$$

Due to stock out during (t_1, T), shortage is accumulated, but not all customers are willing to wait for the next lot size to arrive. Hence, this results in some loss of sale which accounts to loss in profit.

Lost sale cost is calculated as follows:

$$LSC = \pi_L \int_{t_1}^{T} (1-\beta) D(t)\, dt$$

$$LSC = \pi_L \left(\frac{1}{3}(1-\beta)ac\left(T^3 - t_1^3\right) + \frac{1}{2}(1-\beta)ab\left(T^2 - t_1^2\right) + (1-\beta)a\left(T - t_1\right) \right) \quad (8)$$

Purchase cost is as follows:

$$PC = C\left(I_0 + \int_{t_1}^{T} \beta D(t)\, dt \right)$$

$$PC = C\left(\frac{1}{1+m}\left(a\left(\left(1 + b(1+m) + c(1+m)^2\right) \ln\left(\frac{1+m}{1+m-t_1} \right) - \frac{1}{2}ct_1^2 - (b+c)t_1 - cm \right. \right. \right. \\ \left. + \frac{1}{3}\beta ac\left(T^3 - t_1^3\right) + \frac{1}{2}\beta ab\left(T^2 - t_1^2\right) + \beta a\left(T - t_1\right) \right) \quad (9)$$

Total cost is as follows:

$$TC = \frac{1}{T}\left(A + PC + HC + SC + LSC \right) \quad (10)$$

Differentiating Equation 10 with respect to t_1 and T, we get the following:

$$\frac{\partial TC}{\partial t_1} \text{ and } \frac{\partial TC}{\partial T}.$$

To minimize the total cost $TC(t_1, T)$ per unit time, the optimal value of T and t_1 can be obtained by solving the following equations:

$$\frac{\partial TC}{\partial t_1} = 0 \text{ and } \frac{\partial TC}{\partial T} = 0 \quad (11)$$

provided that Equation 10 satisfies the following conditions:

$$\left(\frac{\partial^2 TC}{\partial t_1^2} \right)\left(\frac{\partial^2 TC}{\partial T^2} \right) - \left(\frac{\partial^2 TC}{\partial t_1 \partial T} \right)^2 > 0 \quad \text{and} \quad \frac{\partial^2 TC}{\partial t_1^2} > 0 \quad (12)$$

By solving Equation 11, the value of T and t_1 can be obtained, and with the use of this optimal value, Equation 10 provides the minimum total inventory cost per unit time of the inventory system. Since the nature of the cost function is highly nonlinear, thus the convexity of the function is shown graphically in the next section.

IV. NUMERICAL EXAMPLE

The following numerical values of the parameter in proper unit were considered as input for numerical and graphical analysis of the model,

$a = 70, b = 0.5, c = 0.15, C = 30, h = 5, \pi = 18, \pi_L = 4, m = 2, \beta = 0.25, A = 2000, \alpha = 0.10$.

The output of the model by using maple mathematical software (the optimal value of the total cost, the time when the inventory level reaches zero, and the time when the maximum shortages occur) is TC =$ 1818.591992, t_1 = 1.840170256 years, and T = 1.862187271 years. The total cost function in Equation 10 with some values of t_1 and T such that fixed T at 1.862187271 and t_1 varies from 1.5 to 2.5, fixed t_1 at 1.840170256 and T varies from 1 to 3, and t_1 =1.5 to 2 with equal interval T = 1 to 3, establishes the strictly convexity of total cost function (*TC*). (See Figures 2, 3, and 4, respectively)

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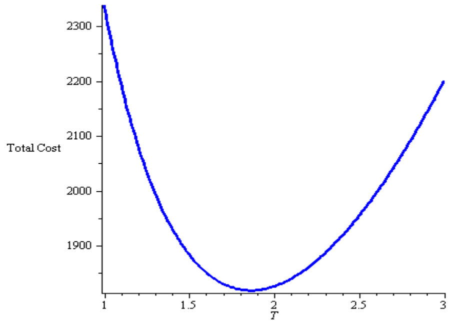

Fig 2. Total cost vs. *T* at t_1 = 1.840170256.

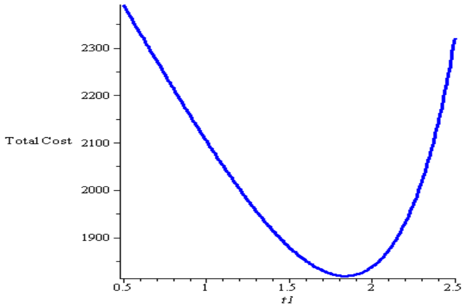

Fig 3. Total cost vs. t_1 at *T* = 1.862187271.

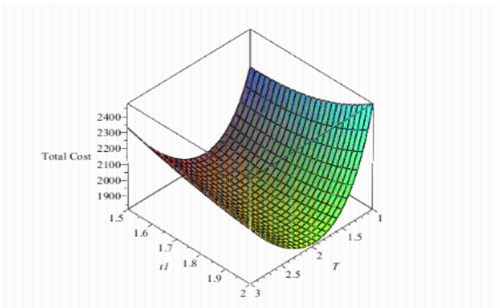

Fig 4. Total cost vs. t_1 and *T*.

V. SENSITIVITY ANALYSIS

The graphical representation of the variations in cycle time *T*, positive inventory cycle time t_1 and total cost *TC* with respect to -20% to 20% variation in different parameters is given in Figures 5, 6, 7 respectively.

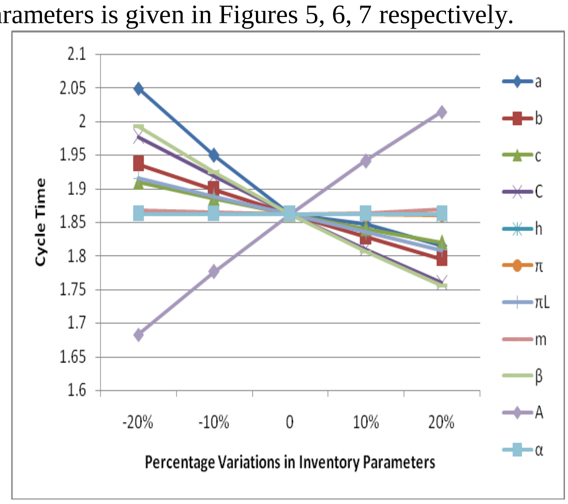

Fig 5. The variations in Cycle time

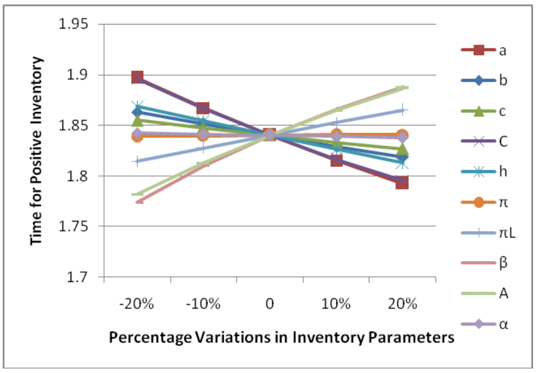

Fig 6. Positive inventory cycle time

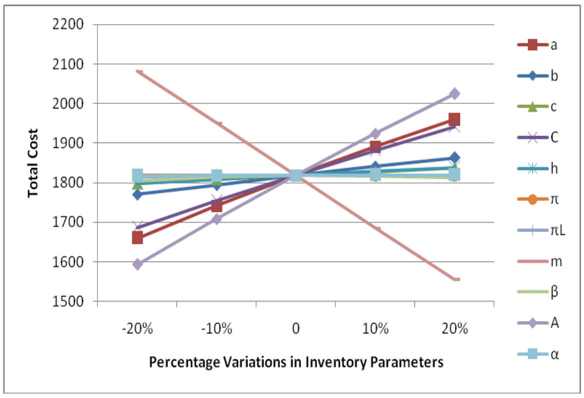

Fig 7. Total cost variations

A. *Observations*

- It is observed from Figure 5 that the cycle time *T* is slightly sensitive to changes in parameters α, m, π, h , moderately sensitive to changes in b, c, C, π_L, β and highly sensitive to changes in a, A.
- From Figure 6 it is seen that that the positive cycle time t_1 is slightly sensitive to changes in parameters α, π, c , moderately sensitive to changes in b, C, π_L, h and highly sensitive to changes in a, A, β.
- From Figure 7 it is observed that that the total cost *TC* is slightly sensitive to changes in parameters $\alpha, \beta, c, h, \pi, \pi_L$, moderately sensitive to changes in b, C and highly sensitive to changes in a, A, m.

VI. CONCLUSION

This paper presents an inventory model of application to the business enterprises that consider the fact that the storage item is deteriorated during storage periods and in which the demand, deterioration, and holding cost depend upon the time. In this paper, we developed a deterministic inventory model with quadratic demand and time-varying holding cost where deterioration is time proportional. The model allows for shortages, and the demand is partially backlogged. The model is solved analytically by minimizing the total inventory cost. By the numerical and

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graphical analysis the proposed model has been verified finally. The obtained results indicate the validity and stability of the model. The proposed model can further be enriched by taking more realistic assumptions such as finite replenishment rate, probabilistic demand rate, etc.

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