

Windowed Based Design of Quadrature Mirror Filter

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Abstract: *In this paper, a windowed based design of Quadrature mirror filter is presented. In the flexible systems, to identify the frequencies of disturbances filter banks and quadrature mirror filter are employed. In this paper, a common approach to construct such filter-banks is to employ a two channel quadrature mirror filter (QMF) bank with FIR analysis and synthesis filters. High order Filters are required to achieve a sharp transition band and a high stop band attenuation, which results in a high computational complexity and a high system delay. QMF is designed using different widow techniques and find out the reconstruction Error and mean square Error and compare them. By comparing the results we conclude that Designing QMF with Blackman window has least Peak reconstruction error and with rectangular window has maximum PRE. The design examples illustrate that the proposed algorithm is superior in term of peak reconstruction error, computation time, and number of iterations. The proposed algorithm is simple, easy to implement, and linear in nature.*

Index Terms: Quadrature mirror filter (QMF), Decimation filters. Peak reconstruction error (PER). Poly-phase structure.

I. INTRODUCTION

During the last two decades, there has been substantial progress in multirate digital filters and filter banks. This includes the design of quadrature mirror filters (QMF). A two-channel QMF bank is extensively used in many signal processing fields such as sub band coding of speech signal, image processing, antenna systems, design of wavelet bases, and biomedical engineering and in digital audio industry [1]-[3]. During last two decades, quadrature mirror filter banks have been extensively used in sub-band coding of speech signals, image processing and trans-multiplexers [4][5]. Originally, the concept of QMF is introduced for removing aliasing distortion in speech coding.

Quadrature mirror filter (PQMF) banks [8] have been widely used to decompose 2-D signals into directional components required for the directional sub band coding of image and video data [1]. QMF filter are used in many speech and communication application [2][3]. It is essential to understand how to efficiently change the sampling rate of band pass signals because it involves M -channel filter banks, which provide the ability to decompose the signal into M sub band components.

In several applications, in order to be able to estimate the frequencies of sinusoids hidden in noise, there is a need to decompose the original signal into its

subcomponents. This process, called “analysis,” is the first part in the signal decomposition with “filter banks.”

The other part, called “synthesis,” is the assembling of the decomposed signal again into one signal. The exactly reversed procedure is performed by the “transmultiplexers”.

Digital filter bank finds wide applications in many areas of signal processing such as TMUX, wireless communication channel, sub-band coding and acoustic echo cancellation [7][8]. Because of such wide application, many researchers have given a lot of attention in efficient designing of filter bank. Filter bank is formed by parallel combination of band pass filters, which decompose incoming signal into number of sub-band signals and process them a tone end, opposite to normal case where the sub- band signals get recombined and reconstructed the output signal. It deals with processing of the signal at different sampling rate at different nodes of system.

In this paper, a windowed based design of Quadrature mirror filter is presented. A common approach to construct such filter-banks is to employ a two channel quadrature mirror filter (QMF) bank with FIR analysis and synthesis filters. QMF is designed using different widow techniques and find out the reconstruction Error and mean square Error and compare them. By comparing the results we conclude that Designing QMF with Blackman window has least Peak reconstruction error and with rectangular window has maximum PRE.

The rest of the paper is organized as follows: In Section II, basic of quadrature mirror filter bank is explained. In section III, quadrature mirror filter bank is designed using poly phase structure. Section IV introduces the Algorithm used for the designing of quadrature mirror filter bank. Steps of QMF design is explained in section V. In Section VI, simulation results will be explained with the help of graphical representation different windowing techniques. Reconstruction of the signal is presented with the help of Hanning and Blackman windows Section VI, conclusions will be put forward.

II. QUADRATURE MIRROR FILTER BANK

QMF bank is a two-channel filter bank, which consists of analysis filters, down-samplers at transmission end, and up-samplers and synthesis filters at receiving end. The block diagram of QMF bank is shown in Fig.1. In QMF a discrete-time signal $x[n]$ is split into a two sub-

band signals having equal bandwidth, using low-pass and high-pass analysis filters $H_0(Z)$ and $H_1(Z)$ respectively. These sub-band signals are down sampled by a factor of two to achieve signal compression or to reduce complexity. At output side, the two sub-band signals are interpolated by a factor of two and passed through low-pass and high-pass synthesis filters $F_0(Z)$ and $F_1(Z)$, respectively.

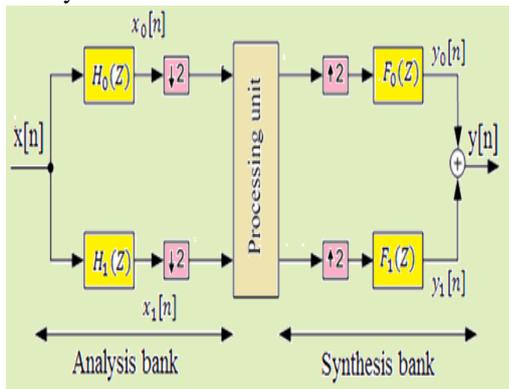


Fig. 1.QMF bank

If the down-sampling and up-sampling factors are equal to or greater than the number of bands of the filter bank, then the output $y[n]$ can be made to retain some or all of the characteristics of the input signal $x[n]$ by choosing appropriately the filters in the structure. It means for perfect reconstruction up-sampling or down-sampling factors should be equal to or greater than the no of sub-band in the QMF. If the up-sampling and down-sampling factors are equal to the number of bands, then the structure is called a critically sampled filter bank. The most common application of this scheme is in the efficient coding of a signal $x[n]$.

A. Frequency response of analysis filters

The analysis filters $H_0(Z)$ and $H_1(Z)$ have typically a low-pass and high-pass frequency responses, respectively, with a cutoff at $\frac{\pi}{2}$.

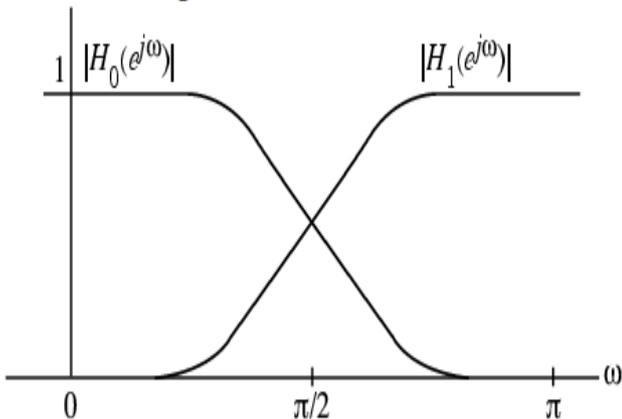


Fig. 2 Frequency response of analysis filters

III. QMF USING POLY-PHASE STRUCTURE

The polyphase structure [6]-[9] of an FIR filter is an interesting structure in its own right. It is also useful in deriving efficient realizations of filters for decimation and interpolation. For a FIR filter of length-M, the polyphase structure consists of I FIR filters in parallel, where M is selected to be an integer multiple of I. That is, I must divide M. Therefore, each parallel FIR filter has $K = M/I$ coefficients. To derive the polyphase structure, consider a simple example of a length-6 FIR filter ($M = 6$) given by

$$H_0(z) = h(0) + h(1)z^{-1} + h(2)z^{-2} + h(3)z^{-3} + h(4)z^{-4} + h(5)z^{-5}$$

If we let $I = 2$, then $H(z)$ can be written as

$$H_0(z) = (h(0) + h(2)z^{-2} + h(4)z^{-4}) + (h(1)z^{-1} + h(3)z^{-3} + h(5)z^{-5})$$

Where we have grouped even and odd terms. Define the following sub filters

$$H_0(z) = (h(0) + h(2)z^{-2} + h(4)z^{-4})$$

$$H_1(z) = (h(1)z^{-1} + h(3)z^{-3} + h(5)z^{-5})$$

Then the original filters can be written in terms of the sub filters as

$$H(z) = H_0(z^2) + z^{-1}H_1(z^2)$$

The polyphase structure of FIR filter using 2 sub bands is shown in Fig. 3.

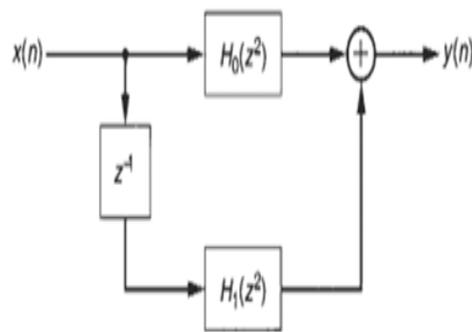


Fig. 3 polyphase structure of FIR filter using 2 sub bands

| Window Technique | Peak reconstruction error(db) | Mean square Error |
|------------------|-------------------------------|-------------------------|
| Blackman | 0.0261 | 2.4603×10^{-5} |
| Hanning | 0.0982 | 5.9826×10^{-4} |
| Rectangular | 0.3491 | 1.37×10^{-2} |

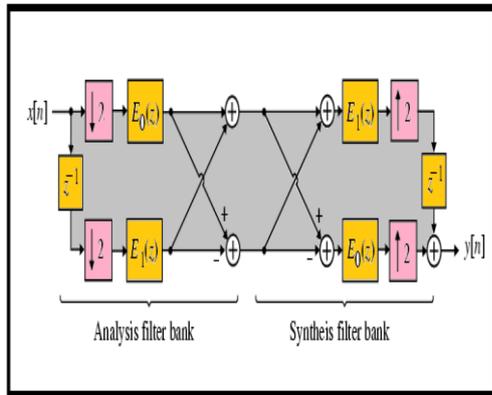


Fig. 4 QMF structure

Making use of the cascade equivalences, the QMF structure [10] can be further simplified as shown Fig. 4.

IV. ALGORITHM

Over past few years, many methods have been proposed to minimize the reconstruction error (amplitude distortion) so that perfect reconstruction could be achieved. For perfect reconstruction, prototype filter in QMF bank must satisfy these conditions.

$$|H_0(e^{j\omega})|^2 + |H_0(e^{j(\pi-\frac{\pi}{2})})|^2 = 1 \text{ For } 0 < \omega < \pi/2$$

$$|H_0(e^{j\omega})| = 0 \text{ For } \omega > \pi/2$$

In the proposed algorithm, objective function is formulated using condition given in Eqn.

$$T\left(e^{j\frac{\pi}{2}}\right) = \left|H_0\left(e^{j\frac{\pi}{2}}\right)\right|^2 + \left|H_0\left(e^{-j\frac{\pi}{2}}\right)\right|^2 = 1 \text{ at } \omega = \pi/2$$

and cut-off frequency is varied to satisfy the objective function with use of optimization algorithm with some modification. In the proposed algorithm the Filter order and stop-band attenuation are fixed, and cut-off frequency is iteratively adjusted so that filter coefficients are near to 0.707 at $\omega = 0.5\pi$.

$$T\left(e^{j\frac{\pi}{2}}\right) = \left|H_0\left(e^{j\frac{\pi}{2}}\right)\right|^2 + \left|H_0\left(e^{j\left(-\frac{\pi}{2}\right)}\right)\right|^2 = 1 \text{ At } \omega = \frac{\pi}{2}$$

Specify tolerance at $\omega = 0.5\pi$ and magnitude response in ideal condition and design the prototype using initial cut off frequency and filter order using different windows and Chebyshev [2]. If tolerance is not satisfied, initial cut-off frequency is varied using step size. Prototype filter is

redesigned using new cut-off frequency, same order and stop-band attenuation. In every iteration, step size is halved. There is no problem of initial start point and convergence. Therefore, the proposed approach can be effectively used for prototype filter of larger taps that results in low reconstruction error so that perfect reconstruction could be achieved.

V. STEPS OF ALGORITHM

Step 1: Specify Stop-band attenuation (As), Pass band ripples

Step2: Initialize pass band frequency, step size (step), Magnitude Response (MRI)=.707 (ideal condition) Tolerance and counter.

Step3: Design prototype filter using Kaiser/chebyshev window, and calculate mag res (MRC)

$$\left|H_0 e^{j\omega}\right| \text{ at } \omega=0.5\pi \text{ and also calculate error } = \text{MRI-MRC.}$$

Step:4 A).If error is less than tolerance than design another filter.

- If $MRC > MRI$,
- then increase $\omega_c = \omega_c + \text{step}$
- Otherwise: $\omega_c = \omega_c - \text{step}$

B). If tolerance is satisfied design other filter using step 2 and 3

Step 5: Redesign prototype filter with new ω_c and same N. calculate error.

Step 6: counter= counter +1

Step 7: Step=Step/2; go to step 4 till tolerance is not satisfied.

VI. SIMULATION RESULTS

Rectangular Window: The weighting function for the rectangular window [10] is given by

$$w(n) = \begin{cases} 1, & \text{for } 0 \leq n \leq M \\ 0, & \text{otherwise} \end{cases}$$

Bartlett window: The weighting function for the Bartlett window [10] is given by

$$w(n) = 1 - \frac{2\left|n - \frac{M}{2}\right|}{M}$$

Blackman window: The weighting function for the Blackman window [10] is given by

$$w(n) = 0.42 - .5\cos\frac{2\pi n}{M} + 0.08\cos\frac{4\pi n}{M}$$

Hamming window: The weighting function for the Hamming window [10] is given by

$$w(n) = 0.54 - 0.46\cos\frac{2\pi n}{M}$$

Hanning Window: The weighting function for the Hanning window [10] is given by

$$w(n) = 0.5 - 0.5\cos\frac{2\pi n}{M}$$

Kaiser Window: In Kaiser Window, the side lobe level can be controlled with respect to the main lobe peak by varying a parameter beta. The width of the main lobe can be varied by adjusting the length of filter. Kaiser window function [10] is used for the implementation of prototype filter as follows

$$w(n) = \frac{I_0 \left\{ \beta \sqrt{1 - \left(\frac{n - \frac{M}{2}}{\frac{M}{2}} \right)^2} \right\}}{I_0 \beta}$$

Fig. 5 shows the Magnitude Response of Analysis filter using hanning window. Using the following parameters as $A_s=60\text{db}$, $N=115$, $r_p=0.00005\text{db}$. We Reconstruction of signal using these parameter. Fig.6 shows the Reconstruction of signal using Hanning window. Fig. 7 shows the Magnitude Response of Analysis filter using Blackman window. Table 1 shows the Comparison of Result for reconstruction Error and mean square Error by using Different Window techniques

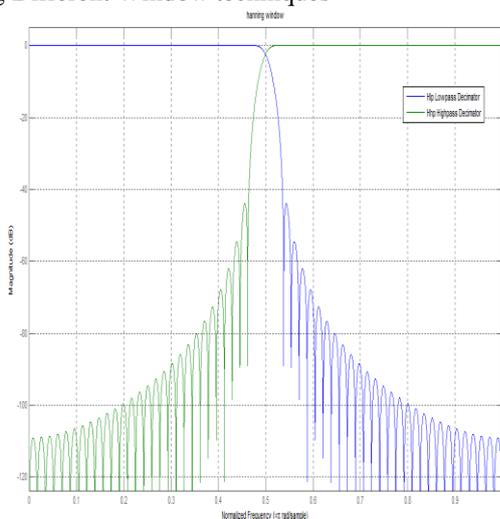


Fig. 5 Magnitude Response of Analysis filter using hanning window

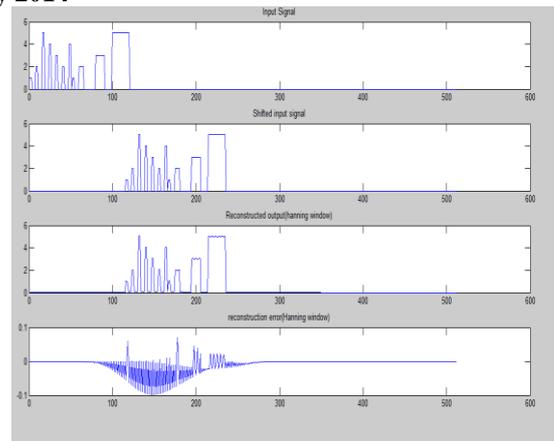


Fig.6 Reconstruction of signal using Hanning window

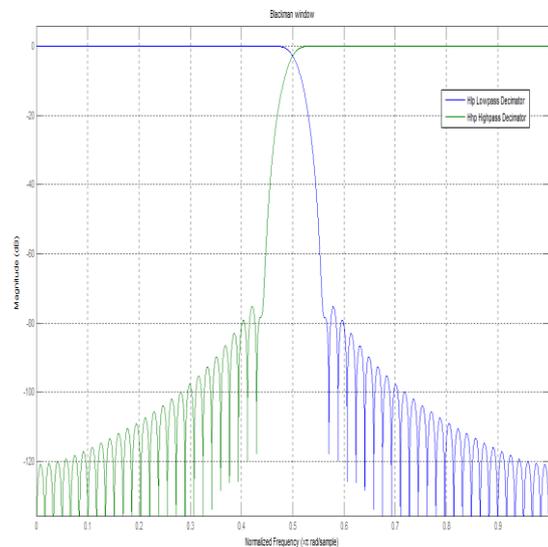


Fig. 7 Magnitude Response of Analysis filter using Blackman window

Using the following parameters as $A_s = 60\text{db}$, $N = 115$, $r_p = 0.00005\text{db}$. We Reconstruction of signal using these parameter. Fig.8 shows the Reconstruction of signal using Blackman window.

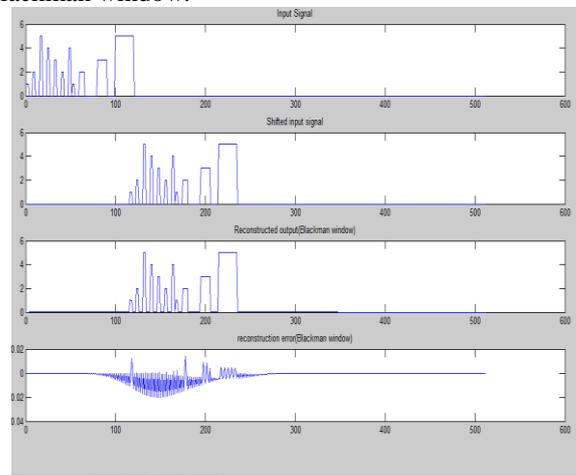


Fig.8 Reconstruction of signal using Blackman window

VII. CONCLUSIONS

In this paper, a windowed based design of Quadrature mirror filter is presented. QMF is designed using different window techniques and find out the reconstruction Error and mean square Error and compare them. We conclude that QMF bank, designed using Blackman window techniques has least peak reconstruction Error (PRE). Rectangular window shows maximum peak reconstruction Error. The proposed algorithm is simple, easy to implement, and linear in nature. We conclude that Designing QMF with Blackman window has least Peak reconstruction error and with rectangular window has maximum PRE.

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