

# Normalized cut algorithm on curvelet coefficient for digital image segmentation

Dr.Khalil al-saif, Nagham ajeel al-ajeely

*Abstract--In this paper a new approach was suggested for image segmentation depend on the coefficients of curvelet transform segmented by N-cut algorithm to get multi-segments when different levels are used to decompose the image in curvelet transform with different N-cut algorithm parameters. The proposed algorithm was applied on variant digital images with different types and size. Segmentation quality found it is sensitive with S-area parameter of the N-cut algorithm (direct proportional). Overlapping parts in original image provide better segmentation with deep information about the target segment. The proposed algorithm dose not affected by the image size. The proposed algorithm was implemented using Matlab programming language with version (R2013a).*

**Index Terms**— Curvelet transform, Digital image processing, Image segmentation, N-cut algorithm, Ridgelet transforms.

## I. INTRODUCTION

Different fast advance technology in current years have created image data in different areas, such as fashion design, medicine and etc.

An efficiently method was needed for object detection in image data for decision maker [1]. Discrete ridge let Transform is the initial approach of curve let transform. But ridge let based curve let transform is not efficient as it uses complex ridge let transform. Two new forms of curve let transform based on different operations of Fourier samples namely, unequally-spaced fast Fourier transform (USFFT) and wrapping based fast curve let transform was proposed by Candès et al. at 2005[2].

Curvelet transform developed by Struck et al. is a new multi-scale approach to represent edges and other singularities along curves much more efficiently than the traditional transforms using fewer coefficients for a given accuracy of reconstruction [3].

In image processing Curvelet transform has been successfully used as an effective tool in field of denoising/ image decomposition/ texture classification/ image deconvolution / astronomical imaging/ and contrast enhancement , etc[4].

## II. RELATED WORK

In 2013 A. DJIMELI, D. TCHIOTSOP et.al focused on improved edge model based on Curvelet coefficients analysis. Experimental results show that their method brings out details on edges when the decomposition scale increases [5].

Yan Zhang n, TaoLi et.al in 2013 proposed an approach to analysis and segmentation of tire laser shearography image by combining curve let transform and Canny edge detection to detect defects in tire surface. So this method would result in a reconstructed image more convenient for edge detection and the time complexity is reduced [6].

In 2013 Kunal Pandey and Shekhar Suralkar analyze the characteristics of the Fast Discrete Curvelet Transform hybridized by an image fusion algorithm based on Discrete Wavelet Transform and the Fast Discrete Curvelet Transform [7].

Emmanuel Candès, Laurent Demanet et.al in 2006 described two digital implementations of a new mathematical transform, namely, the second generation curvelet transform Which based on the wrapping of specially selected Fourier samples[8].

Mohamed Elhabiby et.al. Present in 2012 an assessment of second generation curvelet transforms as an edge detection tool which introduced and compared with wavelet transform and Canny Edge detector. Results show the power of curve let transform over the wavelet transform through the detection of non vertical oriented edges, with detailed detection of curves and circular boundaries [9].

## III. WAVELET AND CURVELET

The main difference between the two transform can be summarized by table I [4],[10],[9],[11] :

Curvelets can provide solutions to the limitations that are apparent in wavelet transform and summarized as follows:

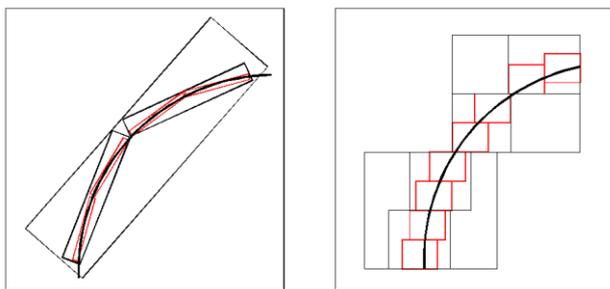
- Curved singularity representation,
- Limited orientation (Vertical, Horizontal and Diagonal)
- Absence of anisotropic element (isotropic scaling) [9].

Figure (1) shows the edge representation capability of wavelet (right) and curvelet transform (left). More wavelets are required for an edge representation using the square shape of wavelets at each scale, compared to the number of required curvelets, which are of an elongated needle shape. The main idea here is that the edge discontinuity is better approximated by curvelets than wavelets. Curve lets can provide solutions for the limitations (curved singularity representation, limited

orientation and absence of anisotropic element) existing in the wavelet transform. [9].

**Table I : Difference between curvelet transform and wavelet transform**

|   | Curvelet  | Wavelet   |
|---|---|---|
| 1 | Represent any discontinuity in image more effectively with very few amounts of non-zero coefficients.                                   | Generate non-zero value for three discontinuities: Horizontal, Vertical and diagonal. |
| 2 | Capture more precisely because it localized in scale, position and orientation.   | Capture limited directional information.  |
| 3 | Curvelet pyramid contains elements with a very high degree of directional specification.  | Less directional specification.   |
| 4 | In addition to the time-frequency localization shows anisotropy, and singularities can be well approximated with very few coefficients. | Time-frequency localization.  |
| 5 | Width and length are related by the relation : $width \approx length^2$ that is known as parabolic or anisotropic scaling.              | Equal size frame.   |
| 6 | The edge discontinuity is better approximated.  | The edge discontinuity is less approximated.  |



**Fig. 1 Representation of curved singularities using wavelets (right) and curvelets (left).**

#### IV. CURVELET TRANSFORM

Candes and Donoho in 1999 Initial introduction of Curvelet transforms technique as a result of the increasing which have ability of effective multi-resolution analysis to overcome the drawbacks of wavelet analysis. Transform was designed to represent edges and other singularities along curves much more efficiently than traditional transforms, i.e. using significantly fewer coefficients for a given accuracy of reconstruction. later authors proposed a considerably simpler second-

generation curvelet transform. This second generation curvelet transforms is meant to be easier to understand and use. It is also faster and less redundant compared to its first generation version [9].

The curvelet transform is a multi-scale transform with strong directional character in which elements are highly anisotropic at fine scales. multi-resolution representation provides several features that set the curvelet transform apart from wavelets, multi-wavelets, wavelet suitable for dealing with objects with singularities or variations that are associated with exceptional points (e.g., very high or very low), while curvelet transform is ideally suited for presentation of variations that display discontinuities or edges[12].

Curvelet approach implements the effective parabolic scaling law on the sub bands in the frequency domain to capture curved edges. An oscillating behavior in the direction perpendicular to their orientation in frequency domain can be exhibit in curvelet for achieve higher level of efficiency. curvelet transform is usually implemented in the frequency domain. That is, both the curvelet filter and the image are transformed then multiplied in the Fourier frequency domain. Then inverse the product to obtain the curvelet coefficients. The process can be described as:

Curvelet transform = IFFT [FFT (Curvelet Filter) FFT (Image)]. and the product from the multiplication is a wedge [1].

#### A. CURVELET PROPERTIES

- The curvelet transform exhibits a new kind of pyramid structure,
- Curvelet frame elements exhibit new scaling laws
- Curvelet provides an efficient representation of images with edges [12].

#### B. CURVELET GENERATION

There are two generations of the curvelet transform. The idea of The First Generation Discrete Curvelet Transform (DCTG1) is first to decompose the image into a set of wavelet bands, and analyze each band by a local ridge let transform. It results in a large amount of redundancy. Moreover, this process is very time consuming, which makes it less feasible for facial features analysis in a large database To overcome on the all the drawbacks in the (DCTG1) such as the parabolic scaling ratio  $width=length^2$  is not completely true and time consuming [11]. So ridge let based curve let transform is not efficient as it uses complex ridge let transform [13].

The Second Generation Curvelet Transform (DCTG2) introduced in 2006 is not only simpler, but is **faster and less redundant** compared to its first generation version. Currently two implementations of fast (DCTG2) are available i.e. Unequally-Spaced Fast Fourier Transform (USFFT) Based Curvelet and Frequency Wrapping Based

Curvelet. The difference is the choice of spatial grid used to translate curvelet at each scale and angle. [11].

The second generation curvelet transform is an effective frequency analysis for each sub band being divided in the frequency domain. As higher the frequency, the larger the band- width of the sub band becomes. For each sub band, this is divided further into several regions with different polar angles. [6].

### V. THE NORMALIZED CUTS SEGMENTATION

Shi and Malik proposed the Normalized Cuts for image segmentation problem, which is based on Graph Theory. This algorithm treats an image pixel as a node of graph, and considers segmentation as a graph partitioning problem. The Normalized Cuts algorithm measures both the total dissimilarity between the different groups as well as the total similarity within the groups. Amazingly, the optimal solution of splitting points is easily computed by solving a generalized eigen value problem. [14].

The steps of N-cut algorithm are explained here [15]:

1. Construct a weighted graph  $G=(V,E)$  by taking each pixel as a node and connecting each pair of pixels by an edge. by using this equation :

$$w_{ij} = e^{-\frac{\|F(i)-F(j)\|_2^2}{\sigma_f^2}} * \begin{cases} e^{-\frac{\|X(i)-X(j)\|_2^2}{\sigma_x^2}} & \text{if } \|X(i)-X(j)\|_2 < r \\ 0 & \text{otherwise.} \end{cases}$$

where  $X(i)$  is the spatial location of node  $i$ , and  $F(i)$  is a feature vector based on intensity, color, or texture information at that node defined as:

- $F(i)=1$ , in the case of segmenting point sets,
- $F(i) = I(i)$  , the intensity value, for segmenting brightness images,
- $F(i)=[v, v.s.\sin(h), v.s.\cos(h)].(i)$ , where  $h, s, v$  are the HSV values, for color segmentation,
- $F(i)=[|I * f_1|; \dots; |I * f_n|].(i)$ , where the  $f_i$  are DOOG filters at various scales and orientations.

2. Solve for the eigenvectors with the smallest eigen values of the system
3. Once the eigenvectors are computed, we can partition the graph into two pieces using the second smallest eigenvector.
4. Decide if the current partition should be subdivided by checking the stability of the cut, and make sure Ncut is below the prespecified value.

### VI. PROPOSED ALGORITHM

The backbone of the algorithm is to decomposed the image using curvelet transformation then the selected coefficient will be segmented using N-cut technique with different parameter, then the segments edge will be

inserted in the original image to make separation between the different segments.

The algorithm can summarized by the following steps:

1. Image Acquisition (should be in gray level)
2. Decompose the image using curvelet transform.
3. Choose one of the image's coefficients to be segmented it using N-cut algorithm (with different parameters).
4. Find the edges of each segment in the selected coefficient using morphological approach.
5. Cross over between edges from step 4 and original image.
6. New result can be obtained by rerun with different coefficients plus new n-cat parameters.

### VII. APPLIED EXAMPLE

A baby image figure (2) selected to be applied with the proposed algorithm.



Fig. 2: The original image

The image was converted to gray level and resized to (400\*400) pixel, then decomposed by curvelet transform with the following parameters number of scales =2, number of angles at the 2<sup>nd</sup> coarsest level =8 to obtained the coefficient. The image coefficients are 9 coefficients (one coefficient in scale one and eight coefficient in scale two) as shown in figure (3).



Fig. 3: curve let coefficients

One of the above coefficients (sequence 6 in figure (3) which represents the fifth coefficient in scale 2) is selected to be segmented using N-cut algorithm, as shown in figure (4).

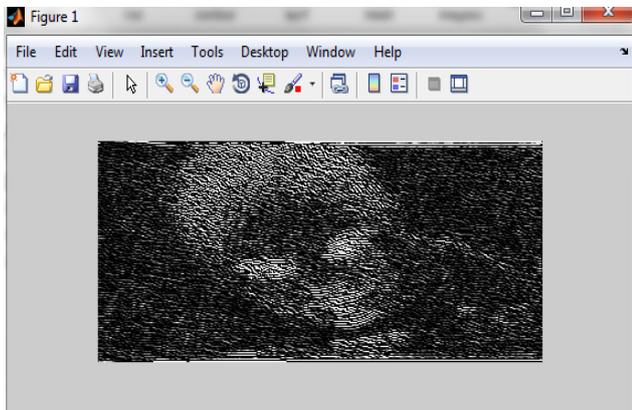


Fig. 4: sequence 6

N-cut segmentation algorithm was applied with following parameters:  $SI = 5$ ;  $SX = 6$ ;  $r = 1.5$ ;  $sNcut = 0.14$   $sAreas = 600$ ;

Where:

"sArea" is a threshold to be the smallest number of points that a segment is allowed to have, say 500 points. This makes sure that each piece we get is not too small.

"sNcut" is a threshold as the largest N-cut(A,B) value for which we will allow a segment to be partitioned more. This makes sure that our algorithm will not try to partition a segment that is highly likely to be one object in the image.

"r" value will not only influence the performance, but also the computation time. In general, the smaller r is, the sparser the weight matrix will be, and therefore the smaller storage and shorter computational time will be needed.

if "SX" is much smaller than SI, then the algorithm will tend to cut the image into squares instead of tracking the edges.

If "SI" is chosen too small compared with SX, then the algorithm will tend to focus on details more, namely, a big object is more likely to be cut into small pieces. [16]. Figure (5) shows the segments obtained when applied N-cut algorithm with Sarea=600, after the boundary for each segment was detected and encircled with red color. An important notice is when Sarea increase, the number of segments will decrease and vice versa .

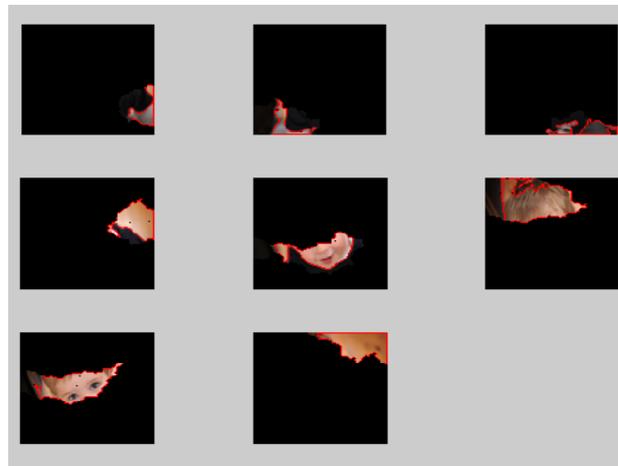


Fig. 5: segments with boundary

Finally, all segments were grouped to obtain the original image as shown in figure 6.



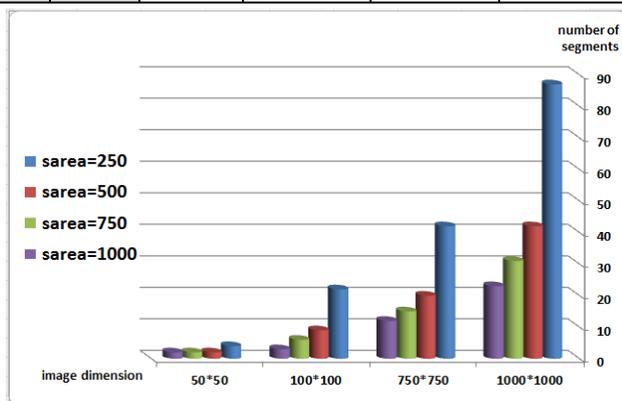
Fig. 6: Original image shows detected segments

The proposed algorithm was applied on images with different dimension ( 50\*50, 100\*100, 150\*150, 200\*200 ) using different values for the parameter Sarea (250, 500, 750, 1000).As a result the algorithm keep the stability of the number of segments in Extrusive form with increasing image dimension. Also, the number of segments relation is Inversely with Sarea. This result is feasible because if segment size (Sarea parameter) in N-cut algorithm is small, this will lead to more number of segments which mean obtain high accuracy to extract the demanded object from the segment in original image.

Table II and figure (7) show that number of segment produced by the proposed algorithm is directly proportion with the variation between s-area and image dimension.

**Table II: Number of segment versus image dimension and S-area**

|        |      | Image dimension |         |         |         |
|--------|------|-----------------|---------|---------|---------|
|        |      | 50*50           | 100*100 | 150*150 | 200*200 |
| S-area | 250  | 4               | 22      | 42      | 87      |
|        | 500  | 2               | 9       | 20      | 42      |
|        | 750  | 2               | 6       | 15      | 31      |
|        | 1000 | 2               | 3       | 12      | 23      |



**Fig. 7: variance of image dimension with number of segment at different s-area**

### VIII. CONCLUSIONS AND FUTURE WORKS

The important conclusion from the discussion of above results is the proposed hybrid operation between curve let transform and N-cut segmentation algorithm gives high accuracy to manipulate with very detailed information in the component of the original image.

The image decomposition to many coefficient using curvelet transform gives flexibility in selection the suitable coefficient to segment it using normalized cut algorithm.

The same results accuracy are obtained when the proposed algorithm was applied on many types of images such as (natural, civil, mechanic).

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