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Energy Conversation Law in the Free Atmosphere

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Abstract — ***This paper is presented atmospheric turbulent flow given by three dimensional Navier-Stokes equations. The main purpose of this research is to introduce in the principal forms the difference between incompressible and compressible atmospheric turbulent flows. Due to the mathematical theories of partial differential equations introduced turbulence models based on the transport and continuity equations for appropriate incompressible or compressible turbulent motions. For compressible turbulent flows we analytically consider only an idealized specific case when the density is a function which can be defined at a point. With respect to obtained balance equations for the pressure distribution and external forces were defined significant fundamental properties of the three dimensional Navier-Stokes problem which was*** *described* ***the constitutive relationship between external forces and gradient of pressure in the free atmosphere.***

Index Terms— ***Navier-Stokes equations, incompressible fluid, compressible fluid, potential field, gradient of pressure***

I. INTRODUCTION

The atmospheric flow is governed by the Navier-Stokes equations and the second law of thermodynamics where external forcing is supplied by radioactive forcing from the Sun and Lorentz force, gravitational or dynamic Carioles forcing from the Earth. The Navier-Stokes equations describe interactions between fluctuations and their directions for different wavelengths which have a great interest in mathematical and physical modeling of turbulent processes. Turbulent fluid flow is a nonlinear multiscale phenomenon which poses some difficult fundamental problems in theoretical and mathematical physics. This paper is presented mathematical theory for the Navier-Stokes problem and deals with model problems for common nature atmospheric phenomena of turbulence. Mathematical solution for a practical complex problem requires a perspective using some alternative approach which different from that is needed for studying the general classical issues. It is worth stressing that turbulence is fundamentally interesting and practical importance for engineering models of atmospheric turbulent effects. This importance provides motivation for this research and we have presented a new analytic approach for the Navier-Stokes problem. This result is the first step to mathematical understanding for physical law of the elusive phenomena of turbulence. The Navier-Stokes equations as nonlinear partial differential equations in real natural situation were formulated in 1821 and appeared to give an accurate description of fluid flow including laminar and turbulent features. Concerning the large literature on the Navier-Stokes problem we mention only some papers which consider particularly relevant for our analytic purpose. We have focused on the global existence, uniqueness and smoothness of weak solution for the Navier-Stokes equations. Examples of weak solution were given by Caffarelli [1], Sheffer [2]. A critical analysis for many analytic and numerical solutions of Navier-Stokes equations was given by Fefferman [3]. We will follow this unique idea of existence of weak solution given in [3] by using the energy conservation low for the external force and gradient of pressure. We have to deal with the fundamental problem of fluid dynamics using theoretical prediction and mathematical analysis of turbulence and focus on a rapidly fluctuating solution for the governing equations of a continuous phenomenon that exists on a large range of length and time scales. There exist different scales of fluctuation which energy is transferred from the larger scales to the smaller scales where energy is dissipated into heat by molecular viscosity. We seem to have clear mathematical evidence that conservative force fields effectively makes fluctuation by using the energy conversion process. Without here going into details of the flow field as producing potential and swirling turbulent flows were defined stability conditions for incompressible and compressible air turbulent motion and some fundamental information about behavior of potential, kinetic and static energies which are indicated mechanics of the air turbulent flow.

II. MATHEMATICAL FORMULATION OF THE PROBLEM

Suppose that infinite spaces $\Omega = R^3$, $\Omega_T = R^3 \times (0 < t < \infty)$

We consider the model Navier -Stokes problem for the velocity vector

$$\vec{u}(x,t) = u_1(x,t)\vec{i} + u_2(x,t)\vec{j} + u_2(x,t)\vec{k}$$

and the fluid pressure field $p(x,t)$ in the following form

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} = -\frac{1}{\rho}\nabla p + \nu\Delta\vec{u} + (\frac{\nu}{3} + \eta)\nabla div\vec{u} + \vec{f}(x,t) \quad (1)$$

in Ω_T with the initial conditions

$$\vec{u}\,\Big|_{t=0} = \vec{u}_0(x) \quad \text{on} \quad \Omega \quad (2)$$

Where,

$$\vec{f}(x,t) = f_1(x,t)\vec{i} + f_2(x,t)\vec{j} + f_2(x,t)\vec{k}$$

is the known vector function of an external force (Lorentz, gravity and other forces), ν is the kinematic viscosity, ρ is the fluid density, η is the dynamic viscosity which is related to the kinematic viscosity by $\eta = \rho\nu$, the symbol ∇ denotes the gradient with respect to the function , the symbol Δ denotes the three dimensional Laplace operator.

For the problem (1)-(2) we consider only an idealized specific case when ν , ρ and η are the given known functions which can be defined at a point. We will construct a weak solution for the Navier-Stokes initial value problem (1)-(2). The weak formulation for the Navier-Stokes problem (1)-(2) is based on the introduced technique for an incompressible potential and swirling turbulent flow. There we assume that

$$|\vec{u}| = \sqrt{u_1^2 + u_2^2 + u_3^2} \to 0 \quad for \ |x| = \sqrt{x_1^2 + x_2^2 + x_3^2} \ \to \infty$$

The initial value problem (1)-(2) is concerned with the fundamental solutions for Poisson and heat conduction equations. Particular attention is paid to the integral representation of solutions with their initial values for the turbulent flux which is the basis of hydrodynamics.

Note, that many problems formally exist for any Reynolds numbers and it can have an exact solution, but not all partial differential equation can describe real-nature phenomenon, therefore we will consider the basic model equations of hydrodynamics that correctly can be solved (existence, uniqueness and stability in terms of continuity). The requirement of stability is caused by the fact that physical evidence is usually determined from experiments and approximately, therefore we must be sure that the determined solution is the stability solution. This requirement of stability in terms of continuity seems to be important; therefore we must construct Lyapunov theory for the Navier-Stokes problem which will be a powerful determining method for defining the stability or instability domains for the nonlinear selected systems.

With respect to this requirement let us describe the used method in the proofs of existence and uniqueness of the Navier-Stokes problem for incompressible and compressible flux. The key idea of our research is to exclude the pressure function from the equation (3) by using rotor or divergence operators. According to these transformations we can give the integral representative for the velocity vector and the energy conservation condition for determining the pressure distribution. We involve this method to show that the velocity vector and an external force with respect to the pressure function exist and satisfy the energy conservation law. This idea of construction for the weak solution of the Navier-Stokes problem (3)-(5) we split into three steps. In the first step we claim that we may assume

$$rot \ \vec{f} = 0 \, , \ rot\vec{u}_0 = 0$$

Then we will get the following condition

$$grad \ (\frac{u^2}{2} + \frac{p}{\rho} - \Phi) = 0$$

where

$$grad\Phi(x,t) = -\vec{f}(x,t)$$

Due to this assertion we can find a weak solution for the incompressible fluid. It is proved that under the energy conservation law there exists a unique velocity vector given by the integral representation. Due to appropriate a priory estimate here we get a stable solution for the incompressible Navier-Stokes problem.

In the second step we assume that

$$rot \ \vec{f} \neq 0$$

Due to this assertion we obtain the second kind nonlinear matrix Volterra-Fredholm integral equation which is solved by using the method of successive approximation in the Hilbert space. Under above assumption there exists a unique unstable solution with the appropriate properties.

Mathematical investigation of physical behavior turbulent for the compressible turbulent flux is considered in the further step of the introduced concept. There we have got the integral representation under the identical energy conservation law. This mathematical theory links with the identical energy conservation law and characterizes steady or unsteady behavior of compressible air turbulent motion. Air turbulent motion is supported by the subjected power from some external forces and initial velocity. The shape of turbulent region is determined by the properties which have shown stability or instability of the velocity motion and the pressure distribution. Stabilizing mechanisms can be used to explain features observed in numerical simulations of turbulence.

Consequently, there has been considerable progress in development of mathematical theory for the Navier-Stokes problems which can be classified in steady or unsteady flows with respect to perturbation of the external force and initial condition.

III. STABLE SOLUTION FOR INCOMPRESSIBLE FLUID

In this part we consider incompressible fluid characterized by the three -dimensional Navier-Stokes problem in the following system of equations:

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} = -\frac{1}{\rho}\nabla p + \nu\Delta\vec{u} + \vec{f}(x,t) \ \text{in}\,\Omega_T \qquad (3)$$

$$div \ \vec{u} = 0 \quad \text{in} \quad \Omega_T \qquad (4)$$

with an initial conditions

$$\vec{u} \ |_{t=0} = \vec{u}_0(x) \quad \text{on} \quad \Omega \qquad (5)$$

Using well-known formula of vector analysis

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$$\frac{1}{2}grad\ \vec{u}^2 = [\vec{u} \times rot\vec{u}] + (\vec{u}\nabla)\vec{u} \qquad (6)$$

and operator $rot\ \vec{u} = \nabla \times \vec{u}$ which is the determinant of the third order

$$rot\ \vec{u} = \nabla \times \vec{u} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x_1} & \frac{\partial}{\partial x_1} & \frac{\partial}{\partial x_2} \\ u_1 & u_2 & u_3 \end{vmatrix}$$

we have got the following equation

$$\frac{\partial \vec{u}}{\partial t} + grad(\frac{p}{\rho} + \frac{1}{2}\vec{u}^2) = [\vec{u} \times rot\vec{u}] + \nu\Delta\vec{u} + \vec{f}(x,t) \qquad (7)$$

Considering the function

$$grad\Phi(x,t) = -\vec{f}(x,t) \qquad (8)$$

which represents potential energy and using the divergence operator we can get an important expression for potential energy

$$\Phi(x,t) = div\vec{f} * \frac{1}{4\pi|x-\xi|}$$

Here symbol $*$ is a convolution between two functions:

$div\vec{f}$ and $\frac{1}{4\pi|x-\xi|}$

Function

$$|x-\xi| = \sqrt{(x_1-\xi_1)^2 + (x_2-\xi_2)^2 + (x_3-\xi_3)^2}$$

represents a distance between the point $\vec{\xi} = (\xi_1, \xi_2, \xi_3)$ and the point $\vec{x} = (x_1, x_2, x_3)$.

Assume that

$$rot\vec{u} = 0 \qquad (9)$$

we have got

$$\frac{\partial \vec{u}}{\partial t} + grad(\frac{p}{\rho} + \frac{1}{2}u^2) = \nu\Delta\vec{u} + \vec{f}(x,t) \qquad (10)$$

Using the divergence operator and condition (9) for the expression (8) we obtain the balance equation

$$\frac{p}{\rho} + \frac{u^2}{2} - div\vec{f} * \frac{1}{4\pi|x-\xi|} = 0 \qquad (11)$$

This balance expression (11) represents the conservation of energy. Using gradient operator for balance equation (11) we have got

$$grad\ (\frac{u^2}{2} + \frac{p}{\rho}) + \vec{f} = 0 \qquad (12)$$

Applying the balance expression (12) to the Navier-Stokes equation (10) we obtain the mathematical problem

$$\frac{\partial \vec{u}}{\partial t} - \nu\Delta\vec{u} = 2\vec{f} \qquad (13)$$

$$div\ \vec{u} = 0 \qquad (14)$$

with an initial condition

$$\vec{u}\ |_{t=0} = \vec{u}_0(x) \qquad (15)$$

Following the classical procedure we can get solutions for the problem (13)-(15) in the integral sum of the parabolic potentials

$$\vec{u} = \int_{R^3} \vec{u}_0(\xi)G(x-\xi,t)d\xi + 2\int_0^t d\tau \int_{R^3} \vec{f}(\xi,\tau)G(x-\xi,t-\tau)d\xi \qquad (16)$$

Where

$$G(x,\xi,t) = \frac{e^{-\frac{(x-\xi)^2}{4\nu t}}}{\left(2\sqrt{\pi\nu t}\right)^3}$$

is the Green's function for the three dimensional infinite space R^3. Here derivations Green's function $G(x,\xi,t)$ have estimations

$$\left|\frac{\partial}{\partial x_i}G(x,\xi,t)\right| \le \frac{e^{-\frac{(x-\xi)^2}{8\nu t}}}{\left(\sqrt{\pi}\right)^3 \nu^2 t^2} (\ i = 1,2,3)$$

Notice that assumption (9) is closely related with the following conditions

$$rot\ \vec{f} = 0,\ rot\vec{u}_0 = 0$$

by using properties of the Green's function $G(x,\xi,t)$ and its derivative evaluations we have got a uniqueness and stable solution (16) satisfying following estimation

$$\|\vec{u}\|_{H^{(2,1)}_{\Omega_T}} \le M_0(\|\vec{u}_0\|_{H^{(1)}_{\Omega}} + 2\|\vec{f}\|_{L_2}) \qquad (17)$$

For positive constant M_0.

Condition (11) for the scalar pressure function $p(x,t)$ predicts a steady feature which introduces the balance equation for a stable turbulent motion. This condition links with the energy conservation law and characterizes steady behavior for the turbulent motion that can be main property for the stability turbulent flows.

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After using the Navier-Stokes equation (7) and (17) has been obtained the estimation with norm on the Hilbert spaces for the pressure function $p(x,t)$

$$\|p\|_{H^{(1,0)}_{\Omega_T}} \le M_1(\|\vec{u}_0\|_{H^{(1)}_{\Omega}} + \|\vec{f}\|_{L_2})$$

with positive constant M_1.

Consequently, we see that a stability of the turbulent flow depends on the condition (11), but fundamental interest is in the study of unsteady features for a unstable swirling motion which characterizes high Reynolds numbers. There is the main mathematical difficulty which lies in the defining the rotational zone with relevant properties respect to appropriate spatial coordinate and time for the turbulent flux. In this case new obtained condition admits solution that can be predicted specific process in terms of unstable flow by increasing the rotation velocity.

IV. VELOCITY COMPONENTS AND PRESSURE FUNCTION FOR INCOMPRESSIBLE FLUID

In this part we have extremely interesting features in terms of the rotation function which belong to the class of bifurcating flow instabilities. Considerable attention has been devoted to the mathematical formalization of unsteady turbulent features which characterize turbulence fluctuations by large scale coherent structures. The organization of these exciting turbulent structures plays a key role in the mass -heat transfer of the unsteady turbulent motion. To see how conservation laws arise from physical principles we will begin by assuming that

$$grad\ \left(\frac{u^2}{2} + \frac{p}{\rho} - \Phi\right) \ne 0 \tag{18}$$

then the Navier-Stokes problem (3)-(5) can be written as follows:

$$\frac{\partial \vec{u}}{\partial t} - \nu\Delta\vec{u} - [\vec{u} \times rot\ \vec{u}] + grad\ \left(\frac{u^2}{2} + \frac{p}{\rho} - \Phi\right) = \vec{f}^* + 2f$$

There vector function $\vec{f}^*$ is a convolution between vector and matrix

$$\vec{f}^8 = \begin{pmatrix} -\frac{\partial^2 f_1}{\partial x_2^2} - \frac{\partial^2 f_1}{\partial x_3^2} + \frac{\partial^2 f_2}{\partial x_1 \partial x_2} + \frac{\partial^2 f_3}{\partial x_1 \partial x_3} \\ \frac{\partial^2 f_1}{\partial x_1 \partial x_2} - \frac{\partial^2 f_2}{\partial x_1^2} - \frac{\partial^2 f_2}{\partial x_3^2} + \frac{\partial^2 f_3}{\partial x_2 \partial x_3} \\ \frac{\partial^2 f_1}{\partial x_1 \partial x_3} + \frac{\partial^2 f_3}{\partial x_1 \partial x_3} - \frac{\partial^2 f_3}{\partial x_1^2} - \frac{\partial^2 f_3}{\partial x_2^2} \end{pmatrix} * \begin{pmatrix} \frac{1}{4\pi|x-\xi|} & 0 & 0 \\ 0 & \frac{1}{4\pi|x-\xi|} & 0 \\ 0 & 0 & \frac{1}{4\pi|x-\xi|} \end{pmatrix}$$

Considering condition $rot\ \vec{f}^* \ne 0$ and using rotor operator we obtain equation

$$rot\left[\frac{\partial}{\partial t}\ \vec{u} - \ [\vec{u}\ \times rot\ \vec{u}] - \nu\Delta\vec{u}\right] = rot\ \vec{f}^* \tag{19}$$

Denote that

$$\vec{g} = \frac{\partial}{\partial t}\ \vec{u} - \ [\vec{u}\ \times rot\ \vec{u}] - \nu\Delta\vec{u}$$

$$\vec{z} = rot\ \vec{f}^*$$

with respect to (19) we have got vector equation

$$rot\ \vec{g} = \vec{z}$$

Expressing the function $\vec{g}$ in terms of the function $\vec{z}$ we can consider system of equations

$$\begin{aligned} &\frac{\partial g_3}{\partial x_2} - \frac{\partial g_2}{\partial x_3} = z_1 \\ &\frac{\partial g_1}{\partial x_3} - \frac{\partial g_3}{\partial x_1} = z_2 \\ &\frac{\partial g_1}{\partial x_1} + \frac{\partial g_2}{\partial x_2} + \frac{\partial g_3}{\partial x_3} = 0 \end{aligned} \tag{20}$$

Apply to system (19) three-dimensional Fourier transform

$$\begin{aligned} -is_3\tilde{g}_2 + is_2\tilde{g}_3 &= \tilde{z}_1 \\ is_3\tilde{g}_1 - is_1 g_3 &= \tilde{z}_2 \\ is_1\tilde{g}_1 + is_2\tilde{g}_2 + is_3\tilde{g}_3 &= 0 \end{aligned}$$

we can define functions in terms of the transitions

$$\vec{g}_1 = -\frac{(s_1 s_2\tilde{z}_1 + (s_2^2 + s_3^2)\tilde{z}_2)i}{s_3(s_1^2 + s_2^2 + s_3^2)}$$

$$\vec{g}_2 = \frac{((s_1^2 + s_2 s_3)\tilde{z}_1 + s_1 s_2\tilde{z}_2)i}{s_3(s_1^2 + s_2^2 + s_3^2)}$$

$$\vec{g}_3 = \frac{(-s_2 s_3\vec{z}_1 + s_1 s_3\vec{z}_2)i}{s_3(s_1^2 + s_2^2 + s_3^2)}$$

Based on the well known formula for the integration

$$\sqrt{\frac{2}{\pi}}\int_0^\infty \frac{\cos\beta t}{t^2 + \alpha^2} = \frac{\pi}{2\alpha}e^{-|\beta|\alpha}, \alpha > 0$$

and formulas for the Fourier transformations

$$\frac{\pi}{2\sqrt{s^2 + a^2}}\exp(-b\sqrt{s^2 + a^2} \to K_0(a\sqrt{x^2 + b^2}), \quad a,b > 0$$

$$K_0(s\sqrt{a^2 + b^2}) \to \frac{1}{\sqrt{a^2 + b^2 + c^2}}$$

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we obtain the inverse

$$\frac{1}{s_1^2+s_2^2+s_3^2} \to \frac{1}{|x-\xi|}$$

There symbol $\to$ indicates transitions from the representation to the original.

Using representation

$$\tilde{Z}_0(f_1,f_2,f_3)=\begin{cases}\tilde{z}_1=is_2\tilde{f}_3^*-is_3\tilde{f}_2^*\\ \tilde{z}_2=is_3\tilde{f}_1^*-is_1\tilde{f}_3^*\\ \\ 0\end{cases}$$

we obtain the nonlinear heat conduction problem

$$\frac{\partial\vec{u}}{\partial t}-\nu\Delta\vec{u}-[\vec{u}\times rot\ \vec{u}]=\vec{b}(x,t) \qquad (21)$$

with an initial condition

$$\vec{u}\ |_{t=0}=\vec{u}_0(x) \qquad (22)$$

where vector

$$\vec{b}(x,t)=\frac{1}{4\pi}\begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix}$$

has components

$$b_1=(\frac{\partial^3}{\partial x_3\partial^2x_2}+\frac{\partial^3}{\partial^3x_3})\int_{R^3}\frac{1}{|x-\xi|}\int_{-\infty}^{\infty}\theta(\xi_3-\varsigma_3)f_1^*(\xi_1,\xi_2,\varsigma_3,t)d\varsigma_3d\xi-$$

$$-(\frac{\partial^3}{\partial x_3\partial x_2\partial x_1}+\frac{\partial^3}{\partial x_1\partial^2x_2}+\frac{\partial^3}{\partial x_1\partial^2x_3})\int_{R^3}\frac{1}{|x-\xi|}\int_{-\infty}^{\infty}\theta(\xi_3-\varsigma_3)f_2^*(\xi_1,\xi_2,\varsigma_3,t)d\varsigma_3d\xi+$$

$$+\frac{\partial^3}{\partial x_3\partial x_2\partial x_1}\int_{R^3}\frac{1}{|x-\xi|}\int_{-\infty}^{\infty}\theta(\xi_3-\varsigma_3)f_3^*(\xi_1,\xi_2,\varsigma_3,t)d\varsigma_3d\xi$$

$$b_2=-\Big[\frac{\partial^3}{\partial x_3\partial x_2\partial x_1}\int_{R^3}\frac{1}{|x-\xi|}\int_{-\infty}^{\infty}\theta(\xi_3-\varsigma_3)f_1^*(\xi_1,\xi_2,\varsigma_3,t)d\varsigma_3d\xi+$$

$$+(\frac{\partial^3}{\partial x_3\partial^2x_1}+\frac{\partial^3}{\partial x_2\partial^2x_3}+\frac{\partial^3}{\partial x_2\partial^2x_1})\int_{R^3}\frac{1}{|x-\xi|}\int_{-\infty}^{\infty}\theta(\xi_3-\varsigma_3)f_2^*(\xi_1,\xi_2,\varsigma_3,t)d\varsigma_3d\xi+$$

$$+(\frac{\partial^3}{\partial x_2\partial^2x_1}+\frac{\partial^3}{\partial x_3\partial^2x_2})\int_{R^3}\frac{1}{|x-\xi|}\int_{-\infty}^{\infty}\theta(\xi_3-\varsigma_3)f_3^*(\xi_1,\xi_2,\varsigma_3,t)d\varsigma_3d\xi$$

$$b_3=-\Big[\frac{\partial^3}{\partial x_3\partial x_2\partial x_1}\int_{R^3}\frac{1}{|x-\xi|}\int_{-\infty}^{\infty}\theta(\xi_3-\varsigma_3)f_1^*(\xi_1,\xi_2,\varsigma_3,t)d\varsigma_3)d\xi-$$

$$-(\frac{\partial^3}{\partial x_2\partial^2x_3}-\frac{\partial^3}{\partial x_3\partial^2x_1})\int_{R^3}\frac{1}{|x-\xi|}\int_{-\infty}^{\infty}\theta(\xi_3-\varsigma_3)f_2^*(\xi_1,\xi_2,\varsigma_3,t)d\varsigma_3d\xi-$$

$$-\frac{\partial^3}{\partial x_3\partial^2x_2}\int_{R^3}\frac{1}{|x-\xi|}\int_{-\infty}^{\infty}\theta(\xi_3-\varsigma_3)f_3^*(\xi_1,\xi_2,\varsigma_3,t)d\varsigma_3d\xi\ \Big]$$

Expanding the bracket $[\vec{u}\times rot\ \vec{u}]$ we obtain the following expression

$$[\vec{u}\times rot\ \vec{u}]=$$

$$=u_2(\frac{\partial u_2}{\partial x_1}-\frac{\partial u_1}{\partial x_2})\vec{i}+u_3(\frac{\partial u_3}{\partial x_2}-\frac{\partial u_2}{\partial x_3})\vec{j}+u_1(\frac{\partial u_1}{\partial x_3}-\frac{\partial u_3}{\partial x_1})\vec{k}-$$

$$-u_3(\frac{\partial u_1}{\partial x_3}-\frac{\partial u_3}{\partial x_1})\vec{i}-u_1(\frac{\partial u_2}{\partial x_1}-\frac{\partial u_1}{\partial x_2})\vec{j}-u_2(\frac{\partial u_3}{\partial x_2}-\frac{\partial u_2}{\partial x_3})\vec{k}$$

which can be written as

$$[\vec{u}\times rot\ \vec{u}]=\Big[u_2(\frac{\partial u_2}{\partial x_1}-\frac{\partial u_1}{\partial x_2})-u_3(\frac{\partial u_1}{\partial x_3}-\frac{\partial u_3}{\partial x_1})\Big]\vec{i}+$$

$$+\Big[u_3(\frac{\partial u_3}{\partial x_2}-\frac{\partial u_2}{\partial x_3})-u_1(\frac{\partial u_2}{\partial x_1}-\frac{\partial u_1}{\partial x_2})\Big]\vec{j}+$$

$$+\Big[u_1(\frac{\partial u_1}{\partial x_3}-\frac{\partial u_3}{\partial x_1})-u_2(\frac{\partial u_3}{\partial x_2}-\frac{\partial u_2}{\partial x_3})\Big]\vec{k}$$

For convenience, let $[\vec{u}\times rot\ \vec{u}]$ denote that

$$u_1^{**}=u_2(\frac{\partial u_2}{\partial x_1}-\frac{\partial u_1}{\partial x_2})-u_3(\frac{\partial u_1}{\partial x_3}-\frac{\partial u_3}{\partial x_1})$$

$$u_2^{**}=u_3(\frac{\partial u_3}{\partial x_2}-\frac{\partial u_2}{\partial x_3})-u_1(\frac{\partial u_2}{\partial x_1}-\frac{\partial u_1}{\partial x_2})$$

$$u_3^{**}=u_1(\frac{\partial u_1}{\partial x_3}-\frac{\partial u_3}{\partial x_1})-u_2(\frac{\partial u_3}{\partial x_2}-\frac{\partial u_2}{\partial x_3})$$

Problem (21)-(22) is closely related with the nonlinear integral equation satisfying

$$\vec{u}=\vec{u}^{**}*G+\vec{F}$$

where

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$$\vec{F} = \vec{u}_0 * G + \vec{b} * G$$

$$G(x,\xi,t) = \frac{e^{-\frac{(x-\xi)^2}{4\nu t}}}{\left(2\sqrt{\pi\nu t}\right)^3}$$

is the Green's function in three dimensional whole space R^3. Properties of the Green's function and its derivative evaluation allow solving the nonlinear matrix Volterra - Fredholm integral equation by using successive approximations. Using Betta function

$$B(n+\frac{1}{2},\frac{1}{2}) = \frac{(n-1)!\,(\sqrt{\pi})^2}{n!} = \frac{\pi}{n}$$

and well-known properties of Green's function we have got estimations

$$\|u^{(0)}\|^2 \le M_0(\|u_0\|^2 + t\|b\|^2)$$

$$\|u^{(1)}\|^2 \le M_0(\|u_0\|^2 + t\|b\|^2 + \frac{M_0^2\sqrt{t}}{2}(\|u_0\|^2 + t\|b\|^2)^2$$

$$\|u^{(2)}\|^2 \le \frac{M_0(\|u_0\|^2 + t\|b\|^2)}{1!} + \frac{M_0^2\sqrt{t}(\|u_0\|^2 + t\|b\|^2)^2}{2!} + \frac{M_0^3 t^{3/2}(\|u_0\|^2 + t\|b\|^2)^4}{3!}$$

....

$$\|u^{(n)}\|^2 \le \frac{M_0(\|u_0\|^2 + t\|f\|^2)}{1!} + \frac{M_0^2\sqrt{t}(\|u_0\|^2 + t\|b\|^2)^2}{2!} + \frac{M_0^3 t^{3/2}(\|u_0\|^2 + t\|b\|^2)^4}{3!} + \ldots + \frac{M_0^n t^{n+1/2}(\|u_0\|^2 + t\|b\|^2)^{2n}}{n!}$$

Due to this fact we have the unique solution of the problem (21)-(22)

$$\vec{u} = \int_0^t d\tau \int_\Omega R\left[A(\vec{F}(\xi,\tau))\Gamma(x-\xi,t-\tau)\right]d\Omega + \vec{F}(x,t)$$

$$R[\cdot] = F(x,t) + \int_0^t d\tau \int_{R^3} A[F(\xi,\tau)]\Gamma(x-\xi,t-\tau)d\xi +$$

$$+\int_0^t d\tau \int_{R^3} A\left[\int_0^\tau d\tau_1 \int_{R^3} A[F(\xi,\tau)]\Gamma(\xi-\zeta,\tau-\tau_1)d\zeta\right]\Gamma(x-\xi,t-\tau)d\xi +$$

$$+\int_0^t d\tau \int_{R^3} A\left[\ldots\left[\int_0^\tau d\tau_1 \int_{R^3} A[F(\xi,\tau)]\Gamma(\xi-\zeta,\tau-\tau_1)d\zeta\right]\ldots\right]\Gamma(x-\xi,t-\tau)d\xi + \ldots$$

$$A(F) = \begin{pmatrix} \frac{1}{2}(F_1^2+F_2^2+F_3^2) & -F_1F_2 & -F_1F_3 \\ -F_1F_2 & \frac{1}{2}(F_1^2+F_2^2+F_3^2) & -F_2F_3 \\ -F_1F_3 & -F_2F_3 & \frac{1}{2}(F_1^2+F_2^2+F_3^2) \end{pmatrix}$$

where

$$\vec{F} = \vec{b} * G + u_0 * G$$

$$\Gamma(x-\xi,t) = \begin{pmatrix} \frac{\partial G(x-\xi,t)}{\partial x_1} \\ \frac{\partial G(x-\xi,t)}{\partial x_2} \\ \frac{\partial G(x-\xi,t)}{\partial x_3} \end{pmatrix}$$

Vector function $\vec{F}(x,t)$ satisfies following estimation

$$\|\vec{F}(x,t)\| \le C(\|\vec{u}_0\| + t\|\vec{b}\|)$$

Using the well-known properties of Green's functions we have got estimation for the vector velocity in the space $L_2(R^3 \times [0,T]$

$$\|\vec{u}\|_{L_2}^2 \le (\|\vec{u}_0\|_{L_2}^2 + t\|\vec{b}\|_{L_2}^2)\Big[1 + M_0(\|\vec{u}_0\|_{L_2}^2 + t\|\vec{b}\|_{L_2}^2)e^{(\|\vec{u}_0\|_{L_2}^2 + t\|\vec{b}\|_{L_2}^2)^2}\Big] \quad (23)$$

Following the classical procedure we get the uniqueness and stability of solution for the problem (3)-(5). Also we obtain equation for the pressure function

$$\frac{u^2}{2} + \frac{p}{\rho} - div\vec{f} * \frac{1}{4\pi|x-\xi|} - divf^{**} * \frac{1}{4\pi|x-\xi|} = 0 \quad (24)$$

where

$$\vec{f}^{**} = \vec{b} - \vec{f}^{*} - 2f$$

$$\|p\|_{L_2(\Omega_T)} \le C_0(\|\vec{u}_0\|_{L_2(\Omega)} + t\|\vec{\Psi}\|_{H_{\Omega_T}^{(1,0)}})\Big[1 + C_1\sqrt{t}(\|\vec{u}_0\|_{L_2(\Omega)} + t\|\vec{\Psi}\|_{H_{\Omega_T}^{(1,0)}})e^{C_2 t(\|\vec{u}_0\|_{L_2(\Omega_T)} + t\|\vec{\Psi}\|_{H_{\Omega_T}^{(1,0)}})^2}\Big]$$

$$\|\Psi\|_{L_2} = \sqrt{(\Psi_1)^2 + (\Psi_2)^2 + (\Psi_3)^2}$$

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$$\vec{\Psi}(x,t)=\begin{pmatrix}\Psi_1\\ \Psi_2\\ \Psi_3\end{pmatrix}\equiv\begin{pmatrix}\int_{-\infty}^{\infty}\theta(x_1-\xi_1)f_1(\xi_1,x_2,x_3,t)d\xi_1\\ \int_{-\infty}^{\infty}\theta(x_2-\xi_2)f_2(x_1,\xi_2,x_3,t)d\xi_2\\ \int_{-\infty}^{\infty}\theta(x_3-\xi_3)f_3(x_1,x_2,\xi_3,t)d\xi_3\end{pmatrix}$$

$\theta(z)$ is Heaviside step function.

V. STABLE SOLUTION FOR COMPRESSIBLE FLUID

In this part we introduce the mathematical description for compressible fluid given by governing equations: momentum conservation and energy conservation. The development of our idea is the most challenging task due to behavior of expression

$$(\frac{\nu}{3}+\eta)\nabla div\vec{u}$$

in the Navier-Stockes equations (1)-(2) which can change physical properties and characteristics of turbulent interaction effects.

Let us consider following problem

$$\frac{\partial\vec{u}}{\partial t}+(\vec{u}\cdot\nabla)\vec{u}=-\frac{1}{\rho}\nabla p+\nu\Delta\vec{u}+(\frac{\nu}{3}+\eta)\nabla div\vec{u}+\vec{f}(x,t) \quad (1)$$

$$grad\ (\frac{u^2}{2}+\frac{p}{\rho})+\vec{f}=0 \quad (2)$$

in Ω_T with an initial conditions

$$\vec{u}\ |_{t=0}=\vec{u}_0(x) \quad \text{on} \quad \Omega \quad (3)$$

Problem (1)-(3) deals with flows having the vector velocity with condition

$$div\vec{u}\neq 0 \quad (25)$$

Using formula (6) we have

$$\frac{\partial\vec{u}}{\partial t}-[\vec{u}\times rot\ \vec{u}]=\nabla(\frac{1}{\rho}p+\frac{u^2}{2})+\nu\Delta\vec{u}+(\frac{\nu}{3}+\eta)\nabla div\vec{u}+\vec{f}(x,t) \quad (26)$$

Assume that

$$rot\vec{u}=0$$

then the equation (26) can be written as

$$\frac{\partial\vec{u}}{\partial t}=\nabla(\frac{1}{\rho}p+\frac{u^2}{2})+\nu\Delta\vec{u}+(\frac{\nu}{3}+\eta)\nabla div\vec{u}+\vec{f}(x,t) \quad (27)$$

Using the energy conservation law which predicts a stable turbulent motion given by (2) we have got following system

$$\frac{\partial\vec{u}}{\partial t}=\nu\Delta\vec{u}+(\frac{\nu}{3}+\eta)\nabla div\vec{u}+2\vec{f}(x,t) \quad (28)$$

$$\vec{u}\ |_{t=0}=\vec{u}_0(x) \quad \text{On } \Omega \quad (29)$$

Using the divergence operator we have obtained

$$\frac{\partial div\vec{u}}{\partial t}=\nu\Delta div\vec{u}+(\frac{\nu}{3}+\eta)div(\nabla div\vec{u})+2div\vec{f}(x,t)$$

Suppose that

$$\vec{U}=div\vec{u}$$

we have got some analogies for the heat problem

$$\frac{\partial\vec{U}}{\partial t}=(\frac{4\nu}{3}+\eta)\Delta\vec{U}+2div\vec{f}(x,t) \quad (30)$$

with the initial conditions

$$\vec{U}\ |_{t=0}=div\vec{u}_0(x) \quad (31)$$

There exists a unique stable periodic solution of the system (30)-(31)

$$\vec{U}=\int_{R^3}div\vec{u}_0(\xi)G_\alpha(x-\xi,t)d\xi+2\int_0^t d\tau\int_{R^3}div\vec{f}(\xi,\tau)G_\alpha(x-\xi,t-\tau)d\xi$$

or

$$\vec{U}=div\vec{u}_0*G_\alpha^*+2div\vec{f}*G_\alpha^* \quad (32)$$

where

$$G_\alpha(x,\xi,t)=\frac{e^{-\frac{(x-\xi)^2}{4\alpha t}}}{\left(2\sqrt{\pi\alpha t}\right)^3}$$

$$\alpha=\frac{4\nu}{3}+\eta$$

Using solution (32) and properties of the Green's function $G_\alpha(x,\xi,t)$ we have got solution for the problem (30)-(31)

$$\vec{u}=grad\left\{div\vec{u}_0*G_\alpha^*+2div\vec{f}*G_\alpha^*\right\} \quad (33)$$

where

$$G_\alpha^*(x,\xi,t)=G_\alpha(\xi,\zeta,t)*\frac{1}{4\pi|x-\zeta|}$$

Using properties of the Green's function $G_\alpha^*(x,\xi,t)$

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and its derivative evaluations we have got uniqueness and stable solution (33) satisfying following estimations

$$\|\vec{u}\|_{L_2(\Omega_T)} \le M_0(\|\vec{u}_0\|_{L_2(\Omega)} + 2\sqrt{t}\|\vec{f}\|_{L_2(\Omega_T)})$$

$$\|\vec{u}\|_{H^{(2,1)}_{\Omega_T}} \le M_0(\|\vec{u}_0\|_{H^{(1)}_{\Omega}} + 2\|\vec{f}\|_{H^{(1,0)}_{\Omega_T}})$$

$$\|p\|_{H^{(1,0)}_{\Omega_T}} \le M_0(\|\vec{u}_0\|_{H^{(1)}_{\Omega}} + 2\|\vec{f}\|_{H^{(1,0)}_{\Omega_T}})$$

Here M_0 is positive constant.

Compressible phenomena involve heat transfer, species and charge transport. Note that $\rho = \rho(x,t)$ is the density and respect to expression

$$\sum_{i=1}^{n} c_i(x,t) = \rho(x,t)$$

Matrix formulation of the constitutive equations for multicomponent diffusion can be solved in terms of the molar density $c_i(x,t)$ for the n species by using the Maxwell-Stefan and generalized Flick's equations.

For defining the velocity vector $\vec{u}$ in this case we must investigate the Navier-Stockes problem (1)-(2) with variable coefficient

$$\alpha(x,t) = \nu(x,t)(\frac{4}{3} + \rho(x,t))$$

We will approach to this type of the Navier-Stockes problem which would be possible to uniquely define the weak solution with the Holder condition for the variable coefficient $\alpha(x,t)$.

VI. UNSTABLE SOLUTION FOR COMPRESSIBLE FLUID

Unstable compressible swirling motion which characterizes high Reynolds numbers also represents fundamental interest in the study of unsteady features. There new obtained condition of the turbulent motion admits solution that can be predicted in terms of the rotation function which is concerned unstable fluid motion. To see how conservation laws arise from the vector velocity and the pressure gradient we will begin by assuming that

$$grad\ (\frac{u^2}{2} + \frac{p}{\rho} - \Phi) \neq 0,$$

then the Navier-Stokes equations (1)-(3) for the compressible turbulent motion can be written as follows:

$$\frac{\partial \vec{u}}{\partial t} - \nu\Delta\vec{u} - (\frac{\nu}{3} + \eta)\nabla div\vec{u} - [\vec{u} \times rot\ \vec{u}] + grad\ (\frac{u^2}{2} + \frac{p}{\rho} - \Phi) = \vec{f}^* + 2f$$

There vector function $\vec{f}^*$ is a convolution between vector and matrix

$$\vec{f}^* = \begin{pmatrix} -\frac{\partial^2 f_1}{\partial x_2^2} - \frac{\partial^2 f_1}{\partial x_3^2} + \frac{\partial^2 f_2}{\partial x_1 \partial x_2} + \frac{\partial^2 f_3}{\partial x_1 \partial x_3} \\ \frac{\partial^2 f_1}{\partial x_1 \partial x_2} - \frac{\partial^2 f_2}{\partial x_1^2} - \frac{\partial^2 f_2}{\partial x_3^2} + \frac{\partial^2 f_3}{\partial x_2 \partial x_3} \\ \frac{\partial^2 f_1}{\partial x_1 \partial x_3} + \frac{\partial^2 f_3}{\partial x_1 \partial x_3} - \frac{\partial^2 f_3}{\partial x_1^2} - \frac{\partial^2 f_3}{\partial x_2^2} \end{pmatrix} * \begin{pmatrix} \frac{1}{4\pi|x-\xi|} & 0 & 0 \\ 0 & \frac{1}{4\pi|x-\xi|} & 0 \\ 0 & 0 & \frac{1}{4\pi|x-\xi|} \end{pmatrix}$$

Considering condition $rot\ \vec{f}^* \neq 0$ and denote that

$$\vec{g} = \frac{\partial}{\partial t}\ \vec{u} - [\vec{u}\ \times rot\ \vec{u}] - \nu\Delta\vec{u} - (\frac{\nu}{3} + \eta)\nabla div\vec{u} \quad (34)$$

$$\vec{z} = rot\ \vec{f}^*$$

With respect to expression (34) we have got vector equation

$$rot\ \vec{g} = \vec{z} \quad (35)$$

Expressing the function $\vec{g}$ in terms of the function $\vec{z}$ we can consider system of equations

$$\frac{\partial g_3}{\partial x_2} - \frac{\partial g_2}{\partial x_3} = z_1$$
$$\frac{\partial g_1}{\partial x_3} - \frac{\partial g_3}{\partial x_1} = z_2 \quad (36)$$
$$\frac{\partial g_1}{\partial x_1} + \frac{\partial g_2}{\partial x_2} + \frac{\partial g_3}{\partial x_3} = z_3^*$$

where

$$z_1 = rot\ f_1^*$$
$$z_2 = rot\ f_2^*$$
$$z_3^* = div\vec{u}_0 * G_\alpha^* + 2div\vec{f} * G_\alpha^*$$

Apply three-dimensional Fourier transform to system (36) we have got

$$-is_3\tilde{\vec{g}}_2 + is_2\tilde{\vec{g}}_3 = \tilde{\vec{z}}_1$$
$$is_3\tilde{\vec{g}}_1 - is_1\tilde{g}_3 = \tilde{\vec{z}}_2$$
$$is_1\tilde{\vec{g}}_1 + is_2\tilde{\vec{g}}_2 + is_3\tilde{\vec{g}}_3 = \tilde{\vec{z}}_3^*$$

We can define functions

$$\tilde{\vec{g}}_1 = -\frac{(s_1s_2\tilde{\vec{z}}_1 + (s_2^2 + s_3^2)\tilde{\vec{z}}_2 + s_1s_3\tilde{\vec{z}}_3^*)i}{s_3(s_1^2 + s_2^2 + s_3^2)}$$

$$\tilde{\vec{g}}_2 = \frac{((s_1^2 + s_2s_3)\tilde{\vec{z}}_1 + s_1s_2\tilde{\vec{z}}_2 - s_2s_3\tilde{\vec{z}}_3^*)i}{s_3(s_1^2 + s_2^2 + s_3^2)}$$

$$\tilde{\vec{g}}_3 = \frac{(-s_2s_3\tilde{\vec{z}}_1 + s_1s_3\tilde{\vec{z}}_2 - s_3^2\tilde{\vec{z}}_3^*)i}{s_3(s_1^2 + s_2^2 + s_3^2)}$$

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Using representation

$$\tilde{Z}_0(f_1, f_2, f_3) = \begin{cases} \tilde{z}_{11} = is_2\tilde{f}_3^* - is_3\tilde{f}_2^* \\ \tilde{z}_2 = is_3\tilde{f}_1^* - is_1\tilde{f}_3^* \\ \tilde{z}_3 = \tilde{z}_3^* \end{cases}$$

we have obtained the following vector equation

$$\frac{\partial\vec{u}}{\partial t} - \nu\Delta\vec{u} - (\frac{\nu}{3} + \eta)\nabla div\vec{u} - [\vec{u}\times rot\ \vec{u}] = \vec{b}^*(x,t) \quad (37)$$

where vector

$$\vec{b}^*(x,t) = \frac{1}{4\pi}\begin{pmatrix} b_1^*(f_1^*, f_2^*, f_3^*, z_3^*) \\ b_2^*(f_1^*, f_2^*, f_3^*, z_3^*) \\ b_3^*(f_1^*, f_2^*, f_3^*, z_3^*) \end{pmatrix}$$

Using divergence operator for the equation (37) and using properties of function

$$G_\alpha(x,\xi,t) = \frac{e^{-\frac{(x-\xi)^2}{4\alpha t}}}{\left(2\sqrt{\pi\alpha t}\right)^3}$$

we obtain we the expression

$$div\vec{u} = div[\vec{u}\times rot\ \vec{u}] * G_\alpha + div\vec{u}_0 * G_\alpha + div\vec{b}^* * G_\alpha$$

or

$$div[\vec{u} - [\vec{u}\times rot\ \vec{u}] * G_\alpha] = F_\alpha^* \quad (38)$$

where

$$F_\alpha^* = div\vec{u}_0 * G_\alpha + div\vec{b}^* * G_\alpha$$

Donate that

$$grad\ w = \vec{u} - [\vec{u}\times rot\ \vec{u}] * G_\alpha$$

and using equation (38) we have got the following expression

$$\vec{u} - [\vec{u}\times rot\ \vec{u}] * G_\alpha = grad\left\{F_\alpha^* * \frac{1}{4\pi|x-\xi|}\right\} \quad (39)$$

Considering equation (39) in the following form

$$\vec{u} = \vec{u}^{**} * G_\alpha + \vec{F}^*$$

where

$$u_1^{**} = u_2(\frac{\partial u_2}{\partial x_1} - \frac{\partial u_1}{\partial x_2}) - u_3(\frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1})$$

$$u_2^{**} = u_3(\frac{\partial u_3}{\partial x_2} - \frac{\partial u_2}{\partial x_3}) - u_1(\frac{\partial u_2}{\partial x_1} - \frac{\partial u_1}{\partial x_2})$$

$$u_3^{**} = u_1(\frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1}) - u_2(\frac{\partial u_3}{\partial x_2} - \frac{\partial u_2}{\partial x_3})$$

$$\vec{F}^* = \vec{u}_0 * G_\alpha + \vec{b}^{**} * G_\alpha$$

$$\vec{b}^{**} = grad\left\{F_\alpha^* * \frac{1}{4\pi|x-\xi|}\right\}$$

we have got a unique solution for the problem (1)-(2)

$$\vec{u} = \int_0^t d\tau \int_\Omega R[A(\vec{F}^*(\xi,\tau))\Gamma_\alpha(x-\xi,t-\tau)]d\Omega + \vec{F}^*(x,t)$$

where

$$R[\cdot] = F^*(x,t) + \int_0^t d\tau \int_{R^3} A[F^*(\xi,\tau)]\Gamma_\alpha(x-\xi,t-\tau)d\xi +$$

$$+\int_0^t d\tau \int_{R^3} A\left[\int_0^\tau d\tau_1 \int_{R^3} A[F^*(\xi,\tau)]\Gamma_\alpha(\xi-\zeta,\tau-\tau_1)d\zeta\right]\Gamma_\alpha(x-\xi,t-\tau)d\xi +$$

$$+\int_0^t d\tau \int_{R^3} A\left[\ ...\left[\int_0^\tau d\tau_1 \int_{R^3} A[F^*(\xi,\tau)]\Gamma_\alpha(\xi-\zeta,\tau-\tau_1)d\zeta\right]...\ \right]\Gamma_\alpha(x-\xi,t-\tau)d\xi + ...$$

$$A(\vec{F}^*) = A(\vec{F})\ |_{\vec{F}=\vec{F}^*}$$

$$\Gamma_\alpha(x-\xi,t) = \begin{pmatrix} \frac{\partial G_\alpha(x-\xi,t)}{\partial x_1} \\ \frac{\partial G_\alpha(x-\xi,t)}{\partial x_2} \\ \frac{\partial G_\alpha(x-\xi,t)}{\partial x_3} \end{pmatrix}$$

Vector function $\vec{F}^*(x,t)$ satisfies following estimation

$$\|\vec{F}^*(x,t)\| \leq C(\|\vec{u}_0\| + \sqrt{t}\|\Psi\|)$$

Using the properties of Green's functions we have got estimation for the vector velocity in the space $L_2(R^3\times[0,T]$

$$\|\vec{u}\|^2_{L_2(\Omega_T)} \leq (\|\vec{u}_0\|^2_{L_2(\Omega)} + \sqrt{t}\|\Psi\|^2_{H^{1,0}(\Omega_T)})\Big[\ 1 +$$

$$+ M_0(\|\vec{u}_0\|^2_{L_2(\Omega)} + \sqrt{t}\|\Psi\|^2_{H^{1,0}(\Omega_T)})e^{(\|\vec{u}_0\|^2_{L_2(\Omega)} + \sqrt{t}\|\Psi\|^2_{H^{1,0}(\Omega_T)})^2}\ \Big]$$

Also we have obtained the equation for the pressure function

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$$\frac{u^2}{2}+\frac{p}{\rho}-div\vec{f}*\frac{1}{4\pi|x-\xi|}-div\vec{F}^{**}*\frac{1}{4\pi|x-\xi|}=0$$

where

$$\vec{F}^{**}=\vec{b}^{**}-\vec{f}^{*}-2f$$

$$\|p\|^2_{H^{1,0}(\Omega_T)}\le(\|\vec{u}_0\|^2_{H^{1,0}(\Omega)}+\|\Psi\|^2_{H^{2,0}(\Omega_T)})\Big[\ 1+$$

$$+M_0(\|\vec{u}_0\|^2_{H^{1,0}(\Omega)}+\|\Psi\|^2_{H^{2,0}(\Omega_T)})e^{(\|\vec{u}_0\|^2_{H^{1,0}(\Omega)}+\|\Psi\|^2_{H^{2,0}(\Omega_T)})^2}\Big]$$

Note that some difficulties in this way arise in solving of the Navier-Stokes problem which was encountered in studying turbulent behavior for unstable motion. There we have some analogies of bifurcating instabilities for compressible unstable motion where Navier-Stokes equations represent the evolution for the governing distribution functions, which depend on the velocity vector in the position of particles as a result of thermal excitation at any finite turbulent energy.

VII. RESULTS AND DISCUSSION

Let us gather and formulate main results about properties of the vector velocity and the scalar function of pressure. Recall the notations $\Omega=R^3$ and $\Omega_T=R^3\times(0<t<\infty)$ we look for periodic solution for the problem (1)-(3).We assume that functions $f_i(x,t)$ and $u_{0i}(x,t)$ satisfy

$$u_{0i}(x)=u_{0i}(x+k_j),\quad f_i(x,t)=f_i(x+k_j,t)$$

for $1\le j\le 3$, where $k_j=j^{th}$ is unit vector in R^3.

Theorem 1. Let $u_{0i}(x,t)\in H^{(2)}(\Omega)$ and $f_i(x,t)\in L_2(\Omega_T^{(i)})$ be periodic functions and $rot\ \vec{f}=0$, $rot\vec{u}_0=0$.

Then there for the Navier-Stokes problem (3) - (5) exists a unique stable periodic solution

$$\vec{u}=\int_{R^3}\vec{u}_0(\xi)G(x-\xi,t)d\xi+2\int_0^t d\tau\int_{R^3}\vec{f}(\xi,\tau)G(x-\xi,t-\tau)d\xi$$

and a unique scalar function of pressure $p(x,t)$ which satisfies energy conservation law

$$\frac{u^2}{2}+\frac{p}{\rho}-div\vec{f}*\frac{1}{4\pi|x-\xi|}=0 \quad (11)$$

Moreover, there exists positive constant M_0 such that for all functions $\vec{u}(x,t)\in H^{(2,1)}(\Omega_T)$ and $p(x,t)\in H^{(1,0)}(\Omega_T)$ satisfy the following estimates

$$\|\vec{u}\|_{H^{(2,1)}_{\Omega_T}}\le M_0(\|\vec{u}_0\|_{H^{(1)}_{\Omega}}+2\|\vec{f}\|_{L_2})$$

$$\|p\|_{H^{(1,0)}_{\Omega_T}}\le M_0(\|\vec{u}_0\|_{H^{(1)}_{\Omega}}+2\|\vec{f}\|_{L_2})$$

We can formulize simple result that Bernoulli's equation is an consequence important of the formula (11).

Remark 1. Assume that $rot\ \vec{f}=0$, $rot\vec{u}_0=0$ are satisfied. If $\vec{f}=C\vec{x}+\vec{d}$, where C matrix

$$C=\begin{pmatrix}\frac{c_1}{m} & 0 & 0\\ 0 & \frac{c_2}{m} & 0\\ 0 & 0 & -gh\end{pmatrix}$$

$\vec{d}$ - a numerical vector, m - a body's mass, c_1,c_2 are independent constants which satisfy the condition $c_1+c_2\ge 0$, g is the acceleration of gravity, h is the height. Then fluid flow can be considered to be an incompressible flow which satisfies Bernoulli's equation

$$\frac{mp}{\rho}+\frac{mu^2}{2}+mgh=const \quad (40)$$

Here $\frac{mp}{\rho}$ is a binding energy of the mass elements, $\frac{mu^2}{2}$ is a kinetic energy , mgh is a potential energy .

The next theorem provides the result about unstable motion.

Theorem 2. Let $u_{0i}(x,t)\in H^{(2)}(\Omega)$ and $f_i(x,t)\in H^{(1,0)}(\Omega_T^{(i)})$ be periodic functions and $rot\ \vec{f}\ne 0$. Under this assumption there exists a unique unstable periodic solution of the Navier-Stokes problem (3) - (5)

$$\vec{u}=\int_0^t d\tau\int_\Omega R\big[A(\vec{F}(\xi,\tau))\Gamma(x-\xi,t-\tau)\big]d\Omega+\vec{F}(x,t)$$

$$\vec{F}=\vec{b}*G+\vec{u}_0*G$$

and a unique scalar function of pressure $p(x,t)$ which satisfies

$$\frac{u^2}{2}+\frac{p}{\rho}-div\vec{f}*\frac{1}{4\pi|x-\xi|}-div\vec{f}^{**}*\frac{1}{4\pi|x-\xi|}=0 \quad (41)$$

where

$$\vec{f}^{**}=\vec{b}-\vec{f}^{*}-2f$$

Moreover, there exist positive constants C_0,C_1,C_2 such that for all functions $\vec{u}(x,t)\in H^{(2,1)}(\Omega_T)$ and $p(x,t)\in H^{(1,0)}(\Omega_T)$ satisfy the following estimates

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$$\|\vec{u}\|_{H^{(2,1)}_{\Omega T}} \le C_0(\|\vec{u}_0\|_{H^{(1)}_{\Omega}} + \|\vec{\Psi}\|_{H^{(1,0)}_{\Omega T}})\Big[\ 1 + C_1(\|\vec{u}_0\|_{H^{(1)}_{\Omega}} + \|\vec{\Psi}\|_{H^{(3,0)}_{\Omega T}})e^{C_2 t(\|\vec{u}_0\|_{H^{(1)}_{\Omega}} + \|\vec{\Psi}\|^2_{H^{(1,0)}_{\Omega T}})^2}\Big]$$

$$\|p\|_{H^{(1,0)}_{\Omega T}} \le C_0(\|\vec{u}_0\|_{H^{(1)}_{\Omega}} + \|\vec{\Psi}\|_{H^{(1,0)}_{\Omega T}})\Big[\ 1 + C_1(\|\vec{u}_0\|_{H^{(1)}_{\Omega}} + \|\vec{\Psi}\|_{H^{(1,0)}_{\Omega T}})e^{C_2 t(\|\vec{u}_0\|_{H^{(1)}_{\Omega}} + \|\vec{\Psi}\|_{H^{(1,0)}_{\Omega T}})^2}\Big]$$

where

$$\vec{\Psi}(x,t) = \begin{pmatrix} \int_{-\infty}^{\infty} \theta(x_3 - \varsigma_3) f_1(x_1, x_2, \varsigma_3, t) d\varsigma_3 \\ \int_{-\infty}^{\infty} \theta(x_3 - \varsigma_3) f_2(x_1, x_2, \varsigma_3, t) d\varsigma_3 \\ \int_{-\infty}^{\infty} \theta(x_3 - \varsigma_3) f_3(x_1, x_2, \varsigma_3, t) d\varsigma_3 \end{pmatrix}$$

$\theta(z)$ is Heaviside step function.

Let $\|\vec{y}\|$ represent the positive function

$$\|\vec{y}\| = \|\vec{u}_0\|_{H^{(1)}_{\Omega}} + \|\vec{\Psi}\|_{H^{(1,0)}_{\Omega T}}$$

then for the velocity vector we have the following estimate

$$\|\vec{u}\|_{H^{(1,0)}_{\Omega T}} \le C_0\|\vec{y}\|(1 + c_1\|\vec{y}\|e^{C_3\|\vec{y}\|^2})$$

where function in the right hand side

$$\|\vec{y}^*\| = C_0\|\vec{y}\|(1 + c_1\|\vec{y}\|e^{C_2\|\vec{y}\|^2})$$

describes behavior of bifurcating flow instabilities.

Theorem 3. Let $u_{0i}(x,t) \in H^{(2)}(\Omega)$ and $f_i(x,t) \in H^{(1,0)}(\Omega_T^{(i)})$ be periodic functions and $rot\ \vec{f} = 0$, $rot\vec{u}_0 = 0$. Then there exists a unique stable periodic solution

$$\vec{u} = grad\{div\vec{u}_0 * G^*_{\alpha} + 2div\vec{f} * G^*_{\alpha}\}$$

$$G_{\alpha}(x,\xi,t) = \frac{e^{-\frac{(x-\xi)^2}{4\alpha t}}}{(2\sqrt{\pi\alpha t})^3}$$

is the Green's function, where α is a coefficient

$$\alpha = \frac{4\nu}{3} + \eta$$

for the Navier-Stokes problem (1) - (2) and a unique scalar function of pressure $p(x,t)$ which satisfies

$$\frac{u^2}{2} + \frac{p}{\rho} - div\vec{f} * \frac{1}{4\pi|x-\xi|} = 0 \quad (11)$$

Moreover, there exists positive constant M_0 such that for all functions $\vec{u}(x,t) \in H^{(2,1)}(\Omega_T)$ and $p(x,t) \in H^{(1,0)}(\Omega_T)$ satisfy the following estimates

$$\|\vec{u}\|_{H^{(2,1)}_{\Omega T}} \le M_0(\|\vec{u}_0\|_{H^{(1)}_{\Omega}} + 2\|\vec{f}\|_{H^{(1,0)}_{\Omega T}})$$

$$\|p\|_{H^{(1,0)}_{\Omega T}} \le M_0(\|\vec{u}_0\|_{H^{(1)}_{\Omega}} + 2\|\vec{f}\|_{H^{(1,0)}_{\Omega T}})$$

Theorem 4. Let $u_{0i}(x,t) \in H^{(2)}(\Omega)$ and $f_i(x,t) \in H^{(2,0)}(\Omega_T^{(i)})$ be periodic functions and $rot\ \vec{f} \ne 0$. Under this assumption there exists a unique unstable periodic solution of the Navier-Stokes problem (1) - (2)

$$\vec{u} = \int_0^t d\tau \int_{\Omega} R\big[A(\vec{F}^*(\xi,\tau))\Gamma_{\alpha}(x-\xi,t-\tau)\big]d\Omega + \vec{F}^*(x,t)$$

$$\vec{F}^* = \vec{b}^{**} * G_{\alpha} + \vec{u}_0 * G_{\alpha}$$

and a unique scalar function of pressure $p(x,t)$ which satisfies

$$\frac{u^2}{2} + \frac{p}{\rho} - div\vec{f} * \frac{1}{4\pi|x-\xi|} - div\vec{F}^{**} * \frac{1}{4\pi|x-\xi|} = 0$$

where

$$\vec{F}^{**} = \vec{b}^{**} - \vec{f}^* - 2f$$

Moreover, there exist positive constants C_0, C_1, C_2 such that for all functions $\vec{u}(x,t) \in H^{(2,1)}(\Omega_T)$ and $p(x,t) \in H^{(1,0)}(\Omega_T)$ satisfy the following estimates

$$\|\vec{u}\|_{H^{(2,1)}_{\Omega T}} \le C_0(\|\vec{u}_0\|_{H^{(2)}_{\Omega}} + \|\vec{\Psi}\|_{H^{(2,0)}_{\Omega T}})\Big[\ 1 + C_1(\|\vec{u}_0\|_{H^{(2)}_{\Omega}} + \|\vec{\Psi}\|_{H^{(2,0)}_{\Omega T}})e^{C_2\sqrt{t}(\|\vec{u}_0\|_{H^{(2)}_{\Omega}} + \|\vec{\Psi}\|^2_{H^{(2,0)}_{\Omega T}})^2}\Big]$$

$$\|p\|_{H^{(1,0)}_{\Omega T}} \le C_0(\|\vec{u}_0\|_{H^{(2)}_{\Omega}} + \|\vec{\Psi}\|_{H^{(2,0)}_{\Omega T}})\Big[\ 1 + C_1(\|\vec{u}_0\|_{H^{(2)}_{\Omega}} + \|\vec{\Psi}\|_{H^{(2,0)}_{\Omega T}})e^{C_2\sqrt{t}(\|\vec{u}_0\|_{H^{(2)}_{\Omega}} + \|\vec{\Psi}\|_{H^{(2,0)}_{\Omega T}})^2}\Big]$$

With respect our method there we have shown steady and unsteady behavior involving fundamental properties of the turbulent flows which demonstrate technological and principal importance at the forefront of classical approach where expression of turbulent fluid energy

$$div\vec{f}^{**} * \frac{1}{4\pi|x-\xi|}$$

represents a departure from the average energy of the fluid known as eddy energy. Due to this fact we formalize the balance relation between components of the velocity vector and the pressure function given by the energy conservation low (11).

VIII. CONCLUSION

Assume that a mathematical model of atmospheric turbulent flow was written as an initial value problem we have presented new analytic method which can be classified by stability balance condition when eddy turbulent flows are resulted from the velocity vector and external force which are expected to exist in all infinite domains. There are two unknown independent thermodynamic parameters (the velocity vector and the scalar function of pressure) which play a prominent role in the obtained integral representation of the velocity distribution for the description of the turbulent behavior of fluid motion.The Navier-Stokes equations have been the basis for description of all turbulent phenomena where experimental selection of the regime turbulent fluctuation is costly and sometimes not always realizable process, therefore important argument for analytic research of the Navier-Stokes equation is developing of an mathematical conception which is based on the Green's function and required a good deal with the parabolic and elliptic potential theory. In processes dealing with governing equations the main point stressed that the velocity vector and the pressure function satisfy their balance criteria of stability motion which is the energy conservation law. This mathematical difficulty for determining the velocity vector associates with the nonlinear of Volterra-Fredholm matrix integral equation. In this research is submitted convenient procedure to investigate the Navier-Stokes equations which allows to use 'a priori' estimates for proof existence and uniqueness of weak solution. Weak formulation for the Navier-Stokes problem is based on the extension of idea to the case where the energy falls in the critical domain, due to the pressure transition.There we use this essential feature of the Navier-Stokes equations. Moreover, basic concept our research is based on the weak formulation for the turbulent flows and introduced technique has been investigated in the Hilbert space. In this paper author endeavors to define conditions that govern the flow and transport of flux which need in the construction of Green's function and determination of the energy conservation law for the every model initial value problem. This research can be applied to engineering models for demonstrating technological applications of new analytic approach for modeling multicomponent / multiphase flux which belong to the class of bifurcating flow instabilities and characterize turbulence fluctuations by large scale coherent structures and plays a key role in the mass-heat transfer of the turbulent motion. Introduced approach leads to the conclusion that this submitted analytic solution would have been used for visualization the basic mechanism and the significant physical structure on the unsteadiness of turbulence effects for turbulent influence from the pressure distribution and external force in the considered areas.

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