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Instabilities in Displacement Process through Homogeneous Porous Media

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Abstract— ***in this paper well known phenomenon of fingering (Instabilities) has been discussed which occurs in two immiscible phase flow through homogeneous porous media. The flow of two immiscible fluids in a large medium can be investigated fairly simply if it is unidirectional, in other words if the different values such as pressures, saturations, fluid speeds, etc. vary only in a single space direction corresponding to the movement direction.***

Index Terms—**Instabilities, Homogeneous porous media, S.O.R.**

I. INTRODUCTION

If the porous medium is thicker, the vertical component of the velocities can not be ignored, and the analysis of the forces acting in the porous medium shows that the interfaces are “ fronts ” (front means the zone of the medium where saturation of injecting phase rises sharply) are generally distorted (encroachment) . These encroachment occurs on the scale of the front, called the **tongue phenomenon**, but also a smaller scale, called **fingering** [1]. These encroachments are governed by conditions of stability of instability. Many experiment shows that these instabilities (tongue, fingering) depend in particular on the mobility ratio. More precisely, instabilities are more likely to appear if the mobility ratio (M) is higher than 1. In other words, injected fluids that are more mobile than native fluid can cause harmful instabilities. The phenomenon of fingering occurs due to the difference in viscosities of flowing fluids Most of the earlier authors have completely neglected the capillary pressure. Varma [2] and Mehta [3] who have included capillary pressure in the analysis of fingers. Scheidegger [4] has given an up-to-date review of the topic. This problem has great practical importance for oil production in the oil reservoir engineering. Therefore, it is necessary to stabilize the fingers at time in oil recovery processes. In the present paper, this phenomenon has been considered with capillary pressure. The assumption made is that the individual pressure of the two flowing phases may be replaced by their common mean pressure and the behavior of the fingers is determined by a statistical treatment. The governing law of Darcy, governing equation of continuity and certain basic assumptions yields a nonlinear partial differential equation for motion of saturation of injecting fluid. The solution is obtained using Successive over Relaxation method.

II. STATEMENT OF THE PROBLEM

When a fluid contained in porous medium is displaced by another fluid of lesser viscosity, instead of regular displacement of the whole front, protuberance may occur which shoot through the porous medium at relatively great speed. Those protuberances are called fingers [5] and the phenomenon is called fingering. We consider that a finite cylindrical piece of homogeneous porous medium of length L, fully saturated with oil, which is displaced by injecting water which give rise to fingers (protuberance). Since the entire oil at the initial boundary $x = 0$ (x being measured in the direction of displacement), is displaced through a small distance due to water injection, therefore, it is assumed that complete saturation exists at the initial boundary. Here, an analytical expression for the cross- sectional area occupied by fingers has been obtained. For the mathematical formulation, we consider the governing law which is Darcy’s law, here, as valid for the investigated flow system and assumed further that the macroscopic behavior of fingers is governed by statistical treatment. In the statistical treatment of fingers [6] only the average behavior of the two fluids involved is taken into consideration. It was shown by Scheideger and Johnson [7], that this treatment of motion with the introduction of the concept of fictitious relative permeability become formally identical to the Buckley-Leverett description of two immiscible of injected fluid flow through porous media. The saturation of injected fluid (S_i) is then defined in average cross-sectional area occupied by the injected fluid level x at time t, i.e. $S_i(x,t)$. Thus the saturation of injected fluid in porous medium represents the average cross-sectional area occupied by fingers [8].

III. MATHEMATICAL FORMULATION OF THE PROBLEM

A. FUNDAMENTAL EQUATION

Assuming validity of Darcy’s law, the seepage velocity of water (V_w) and oil (V_o) may be written as;

$$V_w = -\frac{k_w}{\mu_w} k \frac{\partial P_w}{\partial x} \quad \text{......................(1)}$$

$$V_o = -\frac{k_o}{\mu_o} k \frac{\partial P_o}{\partial x} \quad \text{..........................(2)}$$

Where k is the permeability of the homogeneous medium, k_w and k_o are relative permeability’s of water and oil respectively , P_w *and* P_o are the pressures of water and oil respectively and μ_o and μ_w are viscosities of water and oil respectively. Again k_w and k_o are assumed to be functions of water saturation S_w and oil saturation S_o respectively.

The equations of continuity (phase densities are regarded constant) are:

$$\varphi \frac{\partial S_w}{\partial t} + \frac{\partial V_w}{\partial x} = 0 \quad \text{.....................(3)}$$

$$\varphi \frac{\partial S_o}{\partial t} + \frac{\partial V_o}{\partial x} = 0 \quad \text{.....................(4)}$$

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Where φ is the porosity of the medium?

The porous medium is considered to be fully saturated, from the definition of phase saturation, it is evident that

$S_w + S_o = 1$ …..……………..(5)

The capillary pressure (P_c), defined as the pressure discontinuity of the flowing phases across their common interface, may be written as:

$P_c = P_o - P_w$ ………….…… (6)

For definiteness of the mathematical analysis, we assume standard form of an analytical expression due to the relationship between the relative permabilities, phase saturation and capillary pressure as (for instabilities)

$k_w = S_w\ ,\quad k_o = S_o = 1 - S_w$ …………..(7)

$P_c = -\beta S_w$ …………………………………..(8)

(For physical significance, we consider negative sign which shows the direction of saturation of water is opposite to capillary pressure). For definiteness, we consider β to be small parameter.

The value of pressure of oil can be written as (Oroveanu [10])

$P_o = \bar{P} + \frac{P_c}{2}\ ,\quad \bar{P} = \frac{P_c + P_w}{2}$ ……………..….(9)

Where $\bar{P}$ is the mean pressure which is constant?

B. EQUATION FOR MOTION FOR SATURATION

The equation for motion for saturation can be obtained by substituting the values of V_w and V_o from equations (1) and (2) into (3) and (4)

$\varphi\frac{\partial S_w}{\partial t} = \frac{\partial}{\partial x}\left[\frac{k_w}{\mu_w}k\frac{\partial P_w}{\partial x}\right]$ ………………………..(10)

$\varphi\frac{\partial S_o}{\partial t} = \frac{\partial}{\partial x}\left[\frac{k_o}{\mu_o}k\frac{\partial P_o}{\partial x}\right]$ …………………….……..(11)

Eliminating $\frac{\partial P_w}{\partial x}$ from equations (11) and (6), we get

$\varphi\frac{\partial S_w}{\partial t} = \frac{\partial}{\partial x}\left[\frac{k_w}{\mu_w}k\left\{\frac{\partial P_o}{\partial x} - \frac{\partial P_c}{\partial x}\right\}\right]$ ………….……(12)

Combining equations (11) and (12) by using equation (5), we obtain

$\frac{\partial}{\partial x}\left[\left\{\frac{k_w}{\mu_w} + \frac{k_o}{k_o}\right\}k\frac{\partial P_o}{\partial x} - \frac{k_w}{\mu_w}k\frac{\partial P_c}{\partial x}\right] = 0$ …………(13)

Integrating equation (13) with respect to x, we get

$\left[\left\{\frac{k_w}{\mu_w} + \frac{k_o}{k_o}\right\}k\frac{\partial P_o}{\partial x} - \frac{k_w}{\mu_w}k\frac{\partial P_c}{\partial x}\right] = -A$ ……..……(14)

Where A is the constant of integration (negative sign on right hand side is considered for our convenience). Simplifying equation (14), we get

$\frac{\partial P_o}{\partial x} = -\frac{A}{\left\{\frac{k_w}{\mu_w} + \frac{k_o}{k_o}\right\}} + \frac{k_w}{\mu_w\left\{\frac{k_w}{\mu_w} + \frac{k_o}{k_o}\right\}}\frac{\partial P_c}{\partial x} = \frac{1}{2}\frac{\partial P_c}{\partial x}$ …………..(15)

Substituting this value in equation (14), we get the value of A as:

$A = \left\{\frac{k_w}{\mu_w} - \frac{k_o}{\mu_o}\right\}\frac{k}{2}\frac{\partial P_c}{\partial x}$ …………………………....(16)

Now substituting the value of $\frac{\partial P_o}{\partial x}$ from equation (15) into equation (12), we have

$\varphi\frac{\partial S_w}{\partial t} + \frac{\partial}{\partial x}\left[\frac{k_o}{\mu_o}k\frac{\partial P_c}{\partial x}\frac{1}{\left\{1 + \frac{k_o\mu_w}{k_w\mu_o}\right\}} + \frac{A}{\left\{1 + \frac{k_o\mu_w}{k_w\mu_o}\right\}}\right] = 0$ ……....(17)

Substituting the value of A from (16) into (17), we get

$\varphi\frac{S_w}{\partial t} + \frac{\partial}{\partial x}\left[k\frac{k_w}{2\mu_w}\frac{\partial P_c}{\partial x}\right] = 0$

$\varphi\frac{S_w}{\partial t} + \frac{\partial}{\partial x}\left[k\frac{k_w}{2\mu_w}\frac{dP_c}{dS_w}\frac{\partial S_w}{\partial x}\right] = 0$ …………………..…. (18)

From equations (7), (8) and (18), we have

$\varphi\frac{\partial s_w}{\partial t} - \frac{\beta k}{4\mu_w}\frac{\partial^2 s_w}{\partial x^2} = 0$ ……… (19)

Let $k = c_0\tau\frac{\varphi^3}{M_s(1-\varphi)^2}\ ,\quad \tau = \left(\frac{L}{L_e}\right)^2$ …………………(20)

Where φ is porosity

M_s is specific surface area

c_0 Kozeny constant

L_e Effective length of the path of the fluid

From (19) and (20), we have

$\frac{\partial s_w}{\partial t} - \frac{\beta c_0\tau\varphi^2}{4\mu_w M_s(1-\varphi)^2}\frac{\partial^2 s_w}{\partial x^2} = 0$ ………………………..(21)

This is a non-linear partial differential equation of motion for the saturation of the injected fluid through the homogeneous porous media.

Here, the capillary pressure co-efficient, from our assumption, it is small enough to consider. It is a perturbation parameter. Again β is multiplied to the highest derivative in equation (21), therefore, the problem (21) is a singular perturbation problem. Such problem together with appropriate conditions has been solved analytically or numerically.

A set of boundary conditions are written as

$s_w(0,t) = s_{w_0}\ ,\ s_w(L,t) = s_{w_1}$ ……………(22)

$\frac{\partial}{\partial x}s_w(L,t) = 0,\ 0 \le x \le L$ ……………...(23)

Where s_{w_0} is the saturation at $x = 0$

s_{w_1} is the saturation at $x = L$

Also we assume that there is no flow across the face $x = L$(because the face at $x = L$ is assumed to be imperable), that is,

Let $X = \frac{x}{L}\ ,\quad T = \frac{c_0\tau\varphi^2}{4\mu_w M_s(1-\varphi)^2 L^2}t$

From (21) , we have

$\frac{\partial s_w}{\partial T} - \beta\frac{\partial^2 s_w}{\partial X^2} = 0$ ……………………………….…(24)

With $s_w(0,T) = s_{w_0}\ ,\ s_w(1,T) = s_{w_1}$ ……..……..(25)

$\frac{\partial}{\partial X}s_w(1,T) = 0,\ 0 \le X \le 1$ …………………….…(26)

IV. MATHEMATICAL SOLUTION

Using S.O.R. method [11, 12], we have

$$s_{w_{i,j+1}} = s_{w_{i,j}} + \frac{\beta k}{2h^2}\left(s_{w_{i+1,j}} - 2s_{w_{i,j}} + s_{w_{i-1,j}} + s_{w_{i+1,j+1}} - 2s_{w_{i,j+1}} + s_{w_{i-1,j+1}}\right)$$

Let $\mathbf{r} = \frac{\mathbf{k}}{\mathbf{h^2}}$, $\mathbf{c_m} = s_{w_{i,j}} + \frac{\beta r}{2}\left(s_{w_{i+1,j}} - 2s_{w_{i,j}} + s_{w_{i-1,j}}\right)$

$$s_{w_{i,j+1}} = (1-\omega)s_{w_{i,j}} + \omega\left[\frac{\beta r}{2(1+\beta r)}\left(s_{w_{i+1,j}} + s_{w_{i-1,j+1}}\right) + \frac{c_m}{(1+\beta r)}\right]$$

Choose k =0.1. h=0.1, β=0.1, $\omega = 1.8$, $s_{w_0} = 1$,

$s_{w_1} = s_{w_{i,j+1}} =$

$-0.8s_{w_{i,j}} + 1.8\left[0.25\left(s_{w_{i+1,nj}} + s_{w_{i-1,j+1}}\right) + \frac{c_m}{(2)}\right]$

Where $c_m = 0.5\left(s_{w_{i+1,j}} + s_{w_{i-1,j}}\right)$

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Figures and Tables

X	T= 0.1	T=0.2	T=0.3	T=0.4
	s_w			
0	1	1	1	1
0.1	0.9	0.5445	0.90545	0.67798
0.2	0.405	0.49005	0.55815	0.61314
0.3	0.18225	0.33079	0.38569	0.45623
0.4	0.08201	0.19848	0.26411	0.32338
0.5	0.03691	0.11164	0.17311	0.22345
0.6	0.01661	0.06028	0.1084	0.15045
0.7	0.00747	0.03165	0.06524	0.09888
0.8	0.00336	0.01628	0.03843	0.03899
0.9	0.00151	0.00872	0.01764	0.02073
1	0	0	0	0

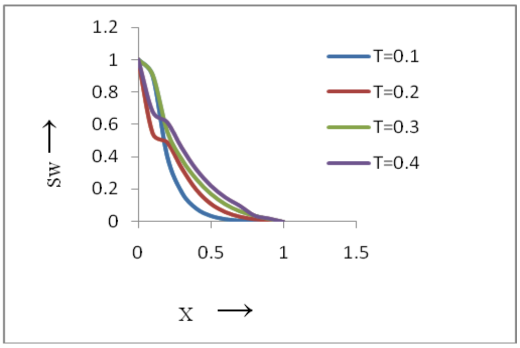

Fig -I

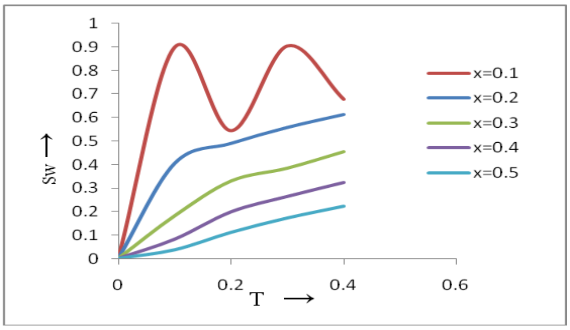

Fig -II

V. CONCLUSION

From figure –I, we can say that as x increases the saturation (S_w) decreases parabolically. Also the governing equation is parabolic. From figure -II, it is clear that as T increases saturation (S_w) increases.

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